Scatter and Glow - Perturbative Testing

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- **Mixing AH and PH**

  \[
  \text{mix1} = \text{ToPH}[3, S[\text{sigma}[3, 1], \text{sigma}[3, 2]]] \\
  S[\text{Ar}[0, 1] \rightarrow \text{Ar}[0, 1] + Y[0, 3, 1, \text{PH}\left[1 - \frac{1}{2} x[3] z + \frac{1}{6} x[3]^2 z^2 + O(z)^3\right]], \\
  \text{Ar}[0, 2] \rightarrow \text{Ar}[0, 2] + Y[0, 3, 2, \text{PH}\left[1 - \frac{1}{2} x[3] z + \frac{1}{6} x[3]^2 z^2 + O(z)^3\right]], \\
  \text{Ar}[0, 3] \rightarrow \text{Ar}[0, 3] + Y[0, 3, 1, \text{PH}\left[-1 + \frac{1}{2} x[3] z - \frac{1}{6} x[3]^2 z^2 + O(z)^3\right]] + \\
  Y[0, 3, 2, \text{PH}\left[-1 + \frac{1}{2} x[3] z - \frac{1}{6} x[3]^2 z^2 + O(z)^3\right]], \\
  \text{Ar}[1, 0] \rightarrow \text{Ar}[1, 0] + Y[1, 3, 0, \text{PH}\left[-1 + \frac{1}{2} x[3] z - \frac{1}{6} x[3]^2 z^2 + O(z)^3\right]], \\
  \text{Ar}[2, 0] \rightarrow \text{Ar}[2, 0] + Y[2, 3, 0, \text{PH}\left[-1 + \frac{1}{2} x[3] z - \frac{1}{6} x[3]^2 z^2 + O(z)^3\right]]] \\
  \text{mix2} = \text{ToPH}[3, S[\text{sigma}[3, 1]]] ** S[\text{sigma}[3, 2]] \\
  S[\text{Ar}[0, 1] \rightarrow \text{Ar}[0, 1] + Y[0, 3, 1, \text{PH}\left[1 - \frac{1}{2} x[3] z + \frac{1}{6} x[3]^2 z^2 + O(z)^3\right]], \\
  \text{Ar}[0, 2] \rightarrow \text{Ar}[0, 2] + Y[0, 3, 2, \text{AH}\left[-1 + e^{-x[3]}\right]], \\
  \text{Ar}[0, 3] \rightarrow \text{Ar}[0, 3] + Y[0, 3, 1, \text{PH}\left[-1 + \frac{1}{2} x[3] z - \frac{1}{6} x[3]^2 z^2 + O(z)^3\right]] + Y[0, 3, 2, \text{AH}\left[-1 + e^{-x[3]}\right]], \\
  \text{Ar}[1, 0] \rightarrow \text{Ar}[1, 0] + Y[1, 3, 0, \text{PH}\left[-1 + \frac{1}{2} x[3] z - \frac{1}{6} x[3]^2 z^2 + O(z)^3\right]], \\
  \text{Ar}[2, 0] \rightarrow \text{Ar}[2, 0] + Y[2, 3, 0, \text{AH}\left[-1 + e^{x[3]}\right]]] \\
  \text{Test}[\text{ToPH}[3, \text{mix1} == \text{mix2}]] \\
  \text{True}
\]

- **The braid group on two strands is commutative:**

  \[
  \text{Expect[Ar[1, 2],} \\
  \text{Ar[1, 2] // ToPH[5, S[\text{sigma}[1, 2]]]} \\
  \text{]}
  \]

  \text{Ar[1, 2]}
Reidemeister 2

\[
\begin{align*}
\text{Expect} &\left[\text{SnG}[S[], 0], \right. \\
&\left. \text{ToPH}[5, \text{SnG}[\text{sigma}[1, 2], \text{sigbar}[1, 2]] \right] \\
\text{SnG}[S[], 0]
\end{align*}
\]

- **Locality in Scale (global over local)**

\[
\begin{align*}
\text{ToPH}[3, S[\text{sigma}[3, 1], \text{sigma}[3, 2]]] \\
S[A[0, 1] &\rightarrow A[0, 1] + Y[0, 3, 1, \text{PH}\left[1 - \frac{1}{2} x[3] z + \frac{1}{6} x[3]^{2} z^{2} + O(z)^{3}\right]]], \\
A[0, 2] &\rightarrow A[0, 2] + Y[0, 3, 2, \text{PH}\left[1 - \frac{1}{2} x[3] z + \frac{1}{6} x[3]^{2} z^{2} + O(z)^{3}\right]], \\
A[0, 3] &\rightarrow A[0, 3] + Y[0, 3, 1, \text{PH}\left[-1 + \frac{1}{2} x[3] z + \frac{1}{6} x[3]^{2} z^{2} + O(z)^{3}\right]] + \\
&\quad Y[0, 3, 2, \text{PH}\left[-1 + \frac{1}{2} x[3] z + \frac{1}{6} x[3]^{2} z^{2} + O(z)^{3}\right]], \\
A[1, 0] &\rightarrow A[1, 0] + Y[1, 3, 0, \text{PH}\left[-1 + \frac{1}{2} x[3] z + \frac{1}{6} x[3]^{2} z^{2} + O(z)^{3}\right]], \\
A[2, 0] &\rightarrow A[2, 0] + Y[2, 3, 0, \text{PH}\left[-1 + \frac{1}{2} x[3] z + \frac{1}{6} x[3]^{2} z^{2} + O(z)^{3}\right]]
\end{align*}
\]

\[
\begin{align*}
\text{Expect} &\left[\{A[1, 2], A[2, 1]\}, \right. \\
&\left. \text{ToAH}[ \{A[1, 2], A[2, 1]\}] \right] // \text{ToPH}[9, S[\text{sigma}[3, 1], \text{sigma}[3, 2]]]
\end{align*}
\]

- **Overcrossings Commute**

\[
\text{ocl} = \text{ToPH}[3, \text{SnG}[\text{sigma}[1, 2], \text{sigma}[1, 3]]]
\]

\[
\begin{align*}
\text{SnG}[S[A[0, 1] &\rightarrow A[0, 1] + Y[0, 1, 2, \text{PH}\left[-1 + \frac{1}{2} x[1] z + \frac{1}{6} x[1]^{2} z^{2} + O(z)^{3}\right]]], \\
&\quad Y[0, 1, 3, \text{PH}\left[-1 + \frac{1}{2} x[1] z + \frac{1}{6} x[1]^{2} z^{2} + O(z)^{3}\right]], \\
A[0, 2] &\rightarrow A[0, 2] + Y[0, 1, 2, \text{PH}\left[1 - \frac{1}{2} x[1] z + \frac{1}{6} x[1]^{2} z^{2} + O(z)^{3}\right]], \\
A[0, 3] &\rightarrow A[0, 3] + Y[0, 1, 3, \text{PH}\left[1 - \frac{1}{2} x[1] z + \frac{1}{6} x[1]^{2} z^{2} + O(z)^{3}\right]], \\
A[2, 0] &\rightarrow A[2, 0] + Y[1, 2, 0, \text{PH}\left[1 - \frac{1}{2} x[1] z + \frac{1}{6} x[1]^{2} z^{2} + O(z)^{3}\right]], \\
A[3, 0] &\rightarrow A[3, 0] + Y[1, 3, 0, \text{PH}\left[1 - \frac{1}{2} x[1] z + \frac{1}{6} x[1]^{2} z^{2} + O(z)^{3}\right]], \{A[1, 2], A[1, 3]\}
\end{align*}
\]
\text{oc2} = \text{ToPH}[3, \text{SnG}[\text{sigma}[1, 3], \text{sigma}[1, 2]]];
\text{Test[oc1 = oc2]}
True

Reidemeister 3

\text{r31} = \text{ToPH}[4, \text{CanonicalForm}[\text{SnG}[\text{sigma}[1, 2], \text{sigma}[1, 3], \text{sigma}[2, 3]]]]

\text{SnG[S[Ar[0, 1] \rightarrow Ar[0, 1] + Y[0, 1, 2, \text{PH}[1 \frac{1}{2} x[1] z \frac{1}{6} x[1]^2 z^2 + \frac{1}{24} x[1]^3 z^3 + O[z]^4]] +
Y[0, 1, 3, \text{PH}[1 \frac{1}{2} x[1] z \frac{1}{6} x[1]^2 z^2 + \frac{1}{24} x[1]^3 z^3 + O[z]^4]],
Ar[0, 2] \rightarrow Ar[0, 2] + Y[0, 1, 2, \text{PH}[1 \frac{1}{2} x[1] z \frac{1}{6} x[1]^2 z^2 + \frac{1}{24} x[1]^3 z^3 + O[z]^4]] +
Y[0, 2, 3, \text{PH}[\frac{1}{2} x[2] z \frac{1}{6} x[2]^2 z^2 + \frac{1}{24} x[2]^3 z^3 + O[z]^4]],
Ar[0, 3] \rightarrow Ar[0, 3] + Y[0, 1, 3, \text{PH}[\frac{1}{2} x[1] z \frac{1}{2} x[1] + \frac{1}{6} (x[1]^2 + 3 x[1] x[2] + 3 x[2]^2) z^2 +
Y[0, 2, 3, \text{PH}[\frac{1}{2} x[2] z \frac{1}{6} x[2]^2 z^2 + \frac{1}{24} x[2]^3 z^3 + O[z]^4]],
Ar[2, 0] \rightarrow Ar[2, 0] + Y[1, 2, 0, \text{PH}[\frac{1}{2} x[1] z \frac{1}{6} x[1]^2 z^2 + \frac{1}{24} x[1]^3 z^3 + O[z]^4]],
Ar[3, 0] \rightarrow Ar[3, 0] + Y[1, 2, 0, \text{PH}[\frac{1}{2} x[1] x[3] - \frac{1}{2} x[1] x[3] - \frac{1}{2} x[2] x[3]] z^2 +
Y[1, 3, 0, \text{PH}[\frac{1}{2} x[1] z \frac{1}{6} (x[1]^2 + 3 x[1] x[2] + 3 x[2]^2) z^2 +
Y[2, 3, 0, \text{PH}[\frac{1}{2} x[2] z \frac{1}{6} x[2]^2 z^2 + \frac{1}{24} x[2]^3 z^3 + O[z]^4]]], \text{Ar[1, 2] + Ar[1, 3] + Ar[2, 3]}
\text{r32} = \text{ToPH}[4, \text{CanonicalForm}[\text{SnG}[\text{sigma}[2, 3], \text{sigma}[1, 3], \text{sigma}[1, 2]]]];
\text{Test[r31 = r32]}
True
Commutators Commute

\[ \text{cc11} = \text{ToPH}[3, \text{SnG}[\sigma[2, 1], \sigma[3, 1], \text{sigbar}[2, 1], \text{sigbar}[3, 1]]] \]

\[ \text{SnG} \left[ \text{Ar}[0, 1] \to \text{Ar}[0, 1] + \gamma[0, 2, 1, \text{PH} \left[ -x[3] z + \left( -\frac{1}{2} x[2] x[3] - \frac{x[3]^2}{2} \right) z^2 + O[z]^3 \right] \right] + \gamma[0, 3, 1, \text{PH} \left[ x[2] z + \frac{1}{2} \left( x[2]^2 + x[2] x[3] \right) z^2 + O[z]^3 \right] \right], \right. \]

\[ \text{Ar}[0, 2] \to \text{Ar}[0, 2] + \gamma[0, 2, 1, \text{PH} \left[ x[3] z + \frac{1}{2} \left( x[2] x[3] + x[3]^2 \right) z^2 + O[z]^3 \right] \right], \]

\[ \text{Ar}[0, 3] \to \text{Ar}[0, 3] + \gamma[0, 3, 1, \text{PH} \left[ -x[2] z + \left( -\frac{1}{2} x[2]^2 - \frac{1}{2} x[2] x[3] \right) z^2 + O[z]^3 \right] \right], \]

\[ \text{Ar}[1, 0] \to \text{Ar}[1, 0] + \gamma[1, 2, 0, \text{PH} \left[ x[3] z + \frac{1}{2} \left( x[2] x[3] + x[3]^2 \right) z^2 + O[z]^3 \right] \right] + \gamma[1, 3, 0, \text{PH} \left[ -x[2] z + \left( -\frac{1}{2} x[2]^2 - \frac{1}{2} x[2] x[3] \right) z^2 + O[z]^3 \right] \right], \]

\[ \gamma[2, 3, 1, \text{PH} \left[ 2 + \frac{3}{2} \left( x[2] + x[3] \right) z + \left( \frac{2 x[2]^2}{3} + x[2] x[3] + \frac{2 x[3]^2}{3} \right) z^2 + O[z]^3 \right] \right] \]

\[ \text{cc12} = \text{ToPH}[3, \text{SnG}[\sigma[4, 1], \sigma[5, 1], \text{sigbar}[4, 1], \text{sigbar}[5, 1]]]; \]

\[ \text{Test}[(\text{cc11} ** \text{cc12}) = (\text{cc12} ** \text{cc11})] \]

True
cc21 = ToPH[4, SnG[sigma[2, 1], sigma[3, 1], sigbar[2, 1], sigbar[3, 1]]]


cc22 = ToPH[4, SnG[sigma[3, 1], sigma[4, 1], sigbar[3, 1], sigbar[4, 1]]];
Test[(cc21 ** cc22) == (cc22 ** cc21)]

True
cc31 = ToPH[4, SnG[sigma[1, 2], sigma[3, 1]] ** SnG[sigbar[1, 2], sigbar[3, 1]]]

SnG[S[Ar[0, 1] → Ar[0, 1] + Y[0, 1, 2],
Ar[0, 2] → Ar[0, 2] + Y[0, 1, 2, PH[x[3] z + \frac{1}{2} \left( x[1] x[3] - x[3]^2 \right) z^2 +
\frac{1}{12} \left( 2 x[1]^2 x[3] - 3 x[1] x[3]^2 + 2 x[3]^3 \right) z^3 + O(z^4)],
Ar[0, 3] → Ar[0, 3] + Y[0, 1, 2, PH[x[1] z + \frac{1}{2} \left( x[1]^2 - x[1] x[3] \right) z^2 +
\frac{1}{12} \left( 2 x[1]^3 - 3 x[1]^2 x[3] + 2 x[1] x[3]^2 \right) z^3 + O(z^4)],
Y[1, 3, 2, PH[-2 - \frac{3}{2} \left( x[1] - x[3] \right) z + \frac{1}{3} \left( -2 x[1]^2 + 3 x[1] x[3] - 2 x[3]^2 \right) z^2 -

cc32 = ToPH[4, SnG[sigma[1, 4], sigma[5, 1], sigbar[1, 4], sigbar[5, 1]]];
Test[(cc31 ** cc32) == (cc32 ** cc31)]

True
Commutators Commutators are Central (along strand 1)

\[
\text{(ccc = SnG[}
\begin{array}{llllll}
\sigma[1, 2], & \sigma[3, 1], & \sigma[1, 2], & \sigma[3, 1], \\
\sigma[4, 1], & \sigma[5, 1], & \sigma[4, 1], & \sigma[5, 1], \\
\sigma[3, 1], & \sigma[1, 2], & \sigma[3, 1], & \sigma[1, 2], \\
\sigma[5, 1], & \sigma[4, 1], & \sigma[5, 1], & \sigma[4, 1]
\end{array}
\text{])} \quad \text{// Last}
\]

\[
Y[1, 4, 2, AH \left( \frac{1}{x[1] \cdot x[4]} \right) - \frac{e^{-x[3]}}{x[1] \cdot x[3]} \cdot \left( -e^{x[1]} \cdot x[1] + e^{x[1]} \cdot x[3] \cdot x[1] + e^{x[1]} \cdot x[4] \cdot x[1] - e^{x[1]} \cdot x[3] \cdot x[4] \cdot x[1] + e^{x[1]} \cdot x[5] \cdot x[1] - e^{x[1]} \cdot x[3] \cdot x[4] \cdot x[5] \cdot x[1] - \right) 
\]

\[
Test[ccc ** SnG[\sigma[6, 1]] = SnG[\sigma[6, 1]] ** ccc]
\]

True

\[
Test[ccc ** SnG[\sigma[1, 6]] = SnG[\sigma[1, 6]] ** ccc]
\]

True

Tails Commute and 4T

\[
\text{Der[Ar[1, 2] + Ar[1, 3]] [Ar[2, 4]]}
\]

\[
Y[1, 2, 4, AH[-1]]
\]

\[
\text{Expect[0, Der[Ar[1, 3]] [Ar[1, 2]]]}
\]

0

\[
\text{Expect[0, Der[Ar[1, 2] + Ar[1, 3]] [Ar[2, 3]]]}
\]

0
Der[Ar[1, 2]][Ar[1, 3] + Ar[2, 3]]
Y[1, 2, 3, AH[-1]]

Expect[0,
  Der[Ar[1, 2]][Ar[3, 1] + Ar[3, 2]]
]
0

Expect[{{0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}}],
ToAH[Table[
    ReducePrimitives[
      Der[ToPH[3, Y[1, 2, 3, AH[1]]]]@Ar[i, j] + Der[Ar[i, j]]@Y[1, 2, 3, ToPH[3, PH[1]]]
    ], {i, 4}, {j, 4}
  ]]
]}

Antisymmetry of Der

Scattering by Exponentials

ToPH[3, S[Exp[Ar[1, 2]]]]

S[Ar[0, 1] → Ar[0, 1] + Y[0, 1, 2, PH[1 - x[1] z + 1/6 x[1]^2 z^2 + O[z]^3]]],

Ar[0, 2] → Ar[0, 2] + Y[0, 1, 2, PH[1 - x[1] z + 1/6 x[1]^2 z^2 + O[z]^3]]],

Ar[2, 0] → Ar[2, 0] + Y[1, 2, 0, PH[1 + x[1] z + 1/6 x[1]^2 z^2 + O[z]^3]]]

Test[
  CanonicalForm[ToPH[9, S[Exp[Ar[1, 2]]]]] == CanonicalForm[ToPH[9, S[sigma[1, 2]]]]
]
True
The BCH Formula

```
ToPH[5, S[sigma[1, 3], sigma[2, 3]]] // Short
S[Ar[0, 1] -> Ar[0, 1] + Y[0, 1, 3, PH[1 + <<4>> + O[z]^5]], <<1>>, <<1>>, Ar[3, 0] -> <<1>>]

unknowns = DeclareSeries[bc[x[1], x[2]], 4]
{bc[0, 0], bc[1, 0], bc[0, 1], bc[2, 0],
 bc[1, 1], bc[0, 2], bc[3, 0], bc[2, 1], bc[1, 2], bc[0, 3]}

PH[bc]
PH[bc[0, 0] + (bc[1, 0] x[1] + bc[0, 1] x[2])] z +
\(1 \frac{bc[2, 0]}{2} x[1]^2 + bc[1, 1] x[1] x[2] + \frac{bc[0, 2]}{2} x[2]^2\) z^2 +
\(1 \frac{bc[3, 0]}{6} x[1]^3 + \frac{bc[2, 1]}{2} x[1]^2 x[2] + \frac{bc[1, 2]}{2} x[1] x[2]^2 + \frac{bc[0, 3]}{6} x[2]^3\) z^3 + O[z]^4

S[Ar[0, 1] ->
Ar[0, 1] + Y[0, <<2>>, PH[1 + \(\frac{x[1]}{2} + \frac{x[2]}{2} + bc[0, 0] x[2]\)] z + (\(<<10>> + bc[0, 1] x[2]^2\)] z^2 +
\(<<1>>^3 \frac{1}{24} + <<16>> + \frac{1}{2} <<1>> <<3>>\) <<1>> <<1>> +
\(- \frac{1}{120} x[1]^4 - \frac{1}{30} x[1]^3 x[2] - \frac{bc[0, 0] x[1]^3 x[2]}{24} + <<30>> +
\(\frac{1}{6} bc[0, 1] x[2]^4 - \frac{1}{4} bc[0, 2] x[2]^4 + \frac{1}{6} bc[0, 3] x[2]^4\)] z^4 + O[z]^5\)]], <<3>>
```
eq = Coefficient[
  Ar[0, 1] // S[Exp[Ar[1, 3] + Ar[2, 3] + Y[1, 2, 3, PH[bc]]]],
  Y[0, 1, 3]
] = Coefficient[
  Ar[0, 1] // S[sigma[1, 3], sigma[2, 3]],
  Y[0, 1, 3]
]

PH[\(-1 + \left(\frac{x[1]}{2} + \frac{x[2]}{2} + bc[0, 0] x[2]\right) z + \left(-\frac{1}{6} x[1]^2 - \frac{1}{3} x[1] x[2] - \frac{1}{2} bc[0, 0] x[1] x[2] +
\left(\frac{x[1]^3}{24} - \frac{1}{8} x[1]^2 x[2] + \frac{1}{6} bc[0, 0] x[1]^2 x[2] - \frac{1}{2} bc[1, 0] x[1]^2 x[2] + \frac{1}{2} bc[2, 0] x[1]^2 x[2] +
  \frac{1}{6} x[1] x[2]^3 + \frac{1}{3} bc[0, 0] x[1] x[2]^3 - \frac{1}{2} bc[0, 1] x[1] x[2]^3 + \frac{1}{2} bc[1, 0] x[1] x[2]^3 +
  bc[1, 1] x[1] x[2]^3 + \frac{x[3]^3}{24} + \frac{1}{6} bc[0, 0] x[2]^3 - \frac{1}{2} bc[0, 1] x[2]^3 + \frac{1}{2} bc[1, 0] x[2]^3\right) z^3 +
\left(-\frac{1}{120} x[1]^4 - \frac{1}{30} x[1]^3 x[2] - \frac{1}{24} bc[0, 0] x[1]^3 x[2] - \frac{1}{6} bc[1, 0] x[1]^3 x[2] -
  \frac{1}{4} bc[2, 0] x[1]^3 x[2] + \frac{1}{6} bc[3, 0] x[1]^3 x[2] - \frac{1}{20} x[1]^2 x[2]^2 - \frac{1}{8} bc[0, 0] x[1]^2 x[2]^2 +
  \frac{1}{6} bc[0, 1] x[1]^2 x[2]^2 + \frac{1}{3} bc[1, 0] x[1]^2 x[2]^2 - \frac{1}{2} bc[1, 1] x[1]^2 x[2]^2 -
  \frac{1}{6} bc[0, 1] x[1] x[2]^3 - \frac{1}{4} bc[1, 0] x[1] x[2]^3 -
  \frac{1}{2} bc[1, 1] x[1] x[2]^3 + \frac{1}{2} bc[1, 2] x[1] x[2]^3 - \frac{x[2]^4}{120} - \frac{1}{24} bc[0, 0] x[2]^4 + \frac{1}{6} bc[0, 1] x[2]^4 -
  \frac{1}{4} bc[0, 2] x[2]^4 + \frac{1}{6} bc[0, 3] x[2]^4\right) z^4 + O[z]^5 \right) = AH\left(-\frac{e^{-x[1] x[2]} \left(-1 + e^{x[1]}\right)}{x[1]}\right]

sol = First[HSSolve[eq, unknowns]]

\{bc[0, 0] \rightarrow \frac{1}{2}, bc[1, 0] \rightarrow \frac{1}{12}, bc[0, 1] \rightarrow -\frac{1}{12}, bc[2, 0] \rightarrow 0, bc[1, 1] \rightarrow -\frac{1}{24},
bc[0, 2] \rightarrow 0, bc[3, 0] \rightarrow -\frac{1}{120}, bc[2, 1] \rightarrow -\frac{1}{90}, bc[1, 2] \rightarrow \frac{1}{90}, bc[0, 3] \rightarrow -\frac{1}{120}\}

bch = PH[bc] /. sol

PH[\left(\frac{x[1]}{12} - \frac{x[2]}{12}\right) z - \frac{1}{24} (x[1] x[2]) z^2 +
\left(\frac{1}{720} x[1]^3 - \frac{1}{180} x[1]^2 x[2] + \frac{1}{180} x[1] x[2]^2 + \frac{x[2]^3}{720}\right) z^3 + O[z]^5 \right)
Test[CanonicalForm[
    S[Exp[Ar[1, 3] + Ar[2, 3] + Y[1, 2, 3, bch]]] == ToPH[5, S[sigma[1, 3], sigma[2, 3]]]]
] == True

■ Compare with Kurlin

    PH[1 - e^-1 x + y x y z y + O[z]^4]]]
] == True

■ Testing Code

SetAttributes[{Test, Expect}, {HoldAll}];
Test[expr_] := If[TrueQ[Check[expr, False]], True,
    If[Head[$FailLog] == List, $FailLog = {}];
    AppendTo[$FailLog,
    "On " <> ToString[Date[]] <> " failed in " <> ToString[HoldForm[expr]]];
    Print[Last[$FailLog]]
];
Expect[val_, expr_] := If[TrueQ[Test[val == expr]], val];

SetDirectory["C:\drorbn\AcademicPensieve\Projects\ScatterAndGlow"]
<< ScatterAndGlow.m
C:\drorbn\AcademicPensieve\Projects\ScatterAndGlow

■ Test Test

Test[0 == 1]
On {2009, 1, 14, 11, 52, 12.9972000} failed in 0 == 1

■ Failed Tests

$FailLog
{On {2009, 1, 14, 11, 52, 6.3232000} failed in 0 == 1,
On {2009, 1, 14, 11, 52, 12.9972000} failed in 0 == 1}