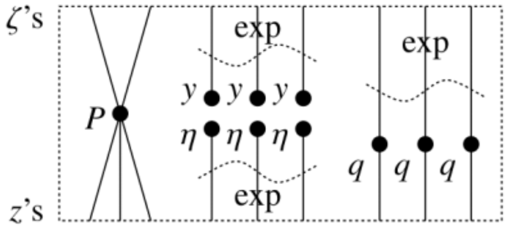
wZip

June 28, 2018 9:39 AM

(180629) **The Zipping Theorem.** If P has a finite ζ-degree and $\tilde{q}$ is the inverse matrix of $1-q$: $(\delta^i_j - q^i_j)\tilde{q}^j_k = \delta^i_k$, then



$$\left\langle P(z_i,\zeta^j)e^{c+\eta^i z_i + y_j\zeta^j + q^i_j z_i \zeta^j}\right\rangle_{(\zeta^j)}$$
$$= \det(\tilde{q})\left\langle P(z_i,\zeta^j)e^{c+\eta^i z_i}\Big|_{z_i\to\tilde{q}^k_i(z_k+y_k)}\right\rangle_{(\zeta^j)}.$$

I need a brainless proof?

$$\left(1-\frac{q}{w}\right)^{-1} = \left(w^{-1}(w-q)\right)^{-1} = w(w-q)^{-1} \qquad \tilde{q} = \frac{q}{w}$$

Plan. 1. In the RHS of the zipping formula, set $\tilde{q} = \frac{q}{w}$.
2. See where the w's go, and make it the definition of $\mathbb{E}(w,Q,P)$.

3. Write a zipping formula for $\mathbb{E}(w,Q,P)$.

$$\langle w^{-1}\exp(w^{-1}\tilde{Q})\cdot\tilde{P}(w^{-1}z_i, w^{-1}\zeta_i)\rangle =$$

```
𝔼 /: Zip_{ξs_List}@𝔼[Q_, P_] := Module[{ξ, z, zs, c, ys, ηs, qt, zrule, Q1, Q2},
  zs = Table[ξ*, {ξ, ξs}];
  c = Q /. Alternatives @@ (ξs ⋃ zs) → 0;
  ys = Table[∂_ξ (Q /. Alternatives @@ zs → 0), {ξ, ξs}];
  ηs = Table[∂_z (Q /. Alternatives @@ ξs → 0), {z, zs}];
  qt = ω0^-1 Inverse@Table[Kδ_{z,ξ*} - ∂_{z,ξ}Q, {ξ, ξs}, {z, zs}];
  zrule = Thread[zs → qt.(zs + ys)];
  Q1 = c + ηs.zs /. zrule;
  Q2 = Q1 /. Alternatives @@ zs → 0;
  Simplify /@ 𝔼[Q2, Det[qt] e^-Q2 ωZip_ξs[ω1, e^Q1 (P /. zrule)]] ];
```

```
Collect[Last@lhs, {ϵ, ξ3, ω0, ω1}, 1 &]
```

$$\frac{1}{\omega\theta^2} + \epsilon \left(\frac{\omega 1}{\omega\theta^3} + \left(\frac{1}{\omega\theta^2} + \frac{\omega 1}{\omega\theta^3}\right) \xi_3\right) + \epsilon^2 \left(\frac{\omega 1}{\omega\theta^3} + \frac{\omega 1^2}{\omega\theta^4} + \left(\frac{1}{\omega\theta^2} + \frac{\omega 1}{\omega\theta^3} + \frac{\omega 1^2}{\omega\theta^4}\right) \xi_3 + \left(\frac{1}{\omega\theta^2} + \frac{\omega 1}{\omega\theta^3} + \frac{\omega 1^2}{\omega\theta^4}\right) \xi_3^2\right)$$