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In[1]:= Once[
  SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Invariant"];
  << SL2Invariant`]
]

Loading KnotTheory` version of January 20, 2015, 10:42:19.1122.
Read more at http://katlas.org/wiki/KnotTheory.
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.
This version: June 2018. Original version: July 1994.

In[2]:= AllKnots[{3, 7}]

Out[2]= {Knot[3, 1], Knot[4, 1], Knot[5, 1], Knot[5, 2], Knot[6, 1], Knot[6, 2], Knot[6, 3],
  Knot[7, 1], Knot[7, 2], Knot[7, 3], Knot[7, 4], Knot[7, 5], Knot[7, 6], Knot[7, 7]}

In[3]:= $k = 1; Zs = Timing[Z[#] & /@ AllKnots[{3, 7}]]

█ KnotTheory: Loading precomputed data in PD4Knots`.█

Out[3]= {7.10938,
  
$$\mathbb{E}_{\{\} \rightarrow \{\theta\}} [\theta, \theta, \frac{T}{1 - T + T^2} + \frac{T (-2 + 3 T - 2 T^2 + T^3 + 2 a (-1 + T - T^3 + T^4) - 2 x y - 2 T^3 x y) \in}{(1 - T + T^2)^3} + O[\epsilon]^2]$$
},
  {3.98438, 
$$\mathbb{E}_{\{\} \rightarrow \{\theta\}} [\theta, \theta, -\frac{T}{1 - 3 T + T^2} - \frac{T (1 + T) (-1 + 2 a (-1 + T) + T - 2 x y) \in}{(1 - 3 T + T^2)^2} + O[\epsilon]^2]$$
},
  {21.0938,
    
$$\mathbb{E}_{\{\} \rightarrow \{\theta\}} [\theta, \theta, \frac{T^2}{1 - T + T^2 - T^3 + T^4} + (T^2 (8 T^3 - 6 T^4 + a (-4 + 6 T - 6 T^2 + 4 T^3 - 4 T^5 + 6 T^6 - 6 T^7 + 4 T^8) + T^7 (1 - 4 x y) + T^5 (4 - 4 x y) + 2 T^6 (-1 + x y) - 4 (1 + x y) - 4 T^2 (2 + x y) + T (7 + 2 x y)) \in) / (1 - T + T^2 - T^3 + T^4)^3 + O[\epsilon]^2]$$
},
    {85.4063, 
$$\mathbb{E}_{\{\} \rightarrow \{\theta\}} [\theta, \theta, \frac{T}{2 - 3 T + 2 T^2} + (T (-9 - T^4 + 4 a (-2 + 3 T - 3 T^3 + 2 T^4) - 8 x y + T^3 (8 - 8 x y) + 4 T (5 + x y) + 2 T^2 (-9 + 2 x y)) \in) / (2 - 3 T + 2 T^2)^3 + O[\epsilon]^2]$$
},
    {103.234, 
$$\mathbb{E}_{\{\} \rightarrow \{\theta\}} [\theta, \theta, -\frac{T}{2 - 5 T + 2 T^2} + (T (5 - 3 T^4 - 4 a (-2 + 5 T - 5 T^3 + 2 T^4) + 8 x y - 4 T (4 + 3 x y) - 2 T^2 (-5 + 6 x y) + T^3 (4 + 8 x y)) \in) / (2 - 5 T + 2 T^2)^3 + O[\epsilon]^2]$$
},
    {74.4531, 
$$\mathbb{E}_{\{\} \rightarrow \{\theta\}} [\theta, \theta, -\frac{T^2}{1 - 3 T + 3 T^2 - 3 T^3 + T^4} - (T^2 (-3 + T^8 + 2 a (-2 + 9 T - 15 T^2 + 12 T^3 - 12 T^5 + 15 T^6 - 9 T^7 + 2 T^8) - 4 x y + T^5 (4 - 16 x y) + 8 T^4 (-2 + x y) + 4 T^3 (7 + 2 x y) - T^7 (3 + 4 x y) - 4 T^2 (7 + 4 x y) + 2 T^6 (1 + 7 x y) + T (15 + 14 x y)) \in) / (1 - 3 T + 3 T^2 - 3 T^3 + T^4)^3 + O[\epsilon]^2]$$
},
    {23.1719, 
$$\mathbb{E}_{\{\} \rightarrow \{\theta\}} [\theta, \theta, \frac{T^2}{1 - 3 T + 5 T^2 - 3 T^3 + T^4} + \frac{T^2 (2 - T - T^2 + 2 T^3) (-1 + 2 a (-1 + T) + T - 2 x y) \in}{(1 - 3 T + 5 T^2 - 3 T^3 + T^4)^2} + O[\epsilon]^2]$$
},
    {50.625, 
$$\mathbb{E}_{\{\} \rightarrow \{\theta\}} [\theta, \theta, \frac{T^3}{1 - T + T^2 - T^3 + T^4 - T^5 + T^6} + \frac{1}{(1 - T + T^2 - T^3 + T^4 - T^5 + T^6)^3}$$
},
    
$$\begin{aligned} & T^3 (15 T^5 - 12 T^6 + 2 a (-3 + 5 T - 6 T^2 + 6 T^3 - 5 T^4 + 3 T^5 - 3 T^7 + 5 T^8 - 6 T^9 + 6 T^{10} - 5 T^{11} + 3 T^{12}) + T^9 (4 - 8 x y) + T^{11} (1 - 6 x y) + T^7 (9 - 6 x y) - 6 (1 + x y) + 4 T^3 (4 + x y) - 2 T^4 (8 + 3 x y) + T^8 (-6 + 4 x y) + T^{10} (-2 + 4 x y) - 2 T^2 (7 + 4 x y) + T (11 + 4 x y)) \in + O[\epsilon]^2 \end{aligned}$$
},
    {195.469, 
$$\mathbb{E}_{\{\} \rightarrow \{\theta\}} [\theta, \theta, \frac{T}{3 - 5 T + 3 T^2} + (T (-23 - 5 T^4 + 6 a (-3 + 5 T - 5 T^3 + 3 T^4) - 18 x y +$$
 }

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$$\begin{aligned}
& \left. \frac{T^3 (29 - 18xy) + 12T^2 (-5 + xy) + T (59 + 12xy)}{(3 - 5T + 3T^2)^3 + O[\epsilon]^2} \right\}, \\
& \left. \frac{139.125, \mathbb{E}_{\{\}} \rightarrow \{\theta\}}{T^2} \left[\theta, \theta, \frac{T^2}{2 - 3T + 3T^2 - 3T^3 + 2T^4} + \right. \right. \\
& \quad \left(T^2 (1 + 17T^8 + 2a(-8 + 18T - 21T^2 + 15T^3 - 15T^5 + 21T^6 - 18T^7 + 8T^8) - 16xy + \right. \\
& \quad \left. T^2 (20 - 22xy) + 8T^4 (7 + xy) - 4T^7 (11 + 4xy) + 4T(-2 + 5xy) + T^3 (-37 + 8xy) + \right. \\
& \quad \left. T^6 (62 + 20xy) - T^5 (67 + 22xy)) \right) \left. \right] / (2 - 3T + 3T^2 - 3T^3 + 2T^4)^3 + O[\epsilon]^2 \right\}, \\
& \left. \frac{87.4531, \mathbb{E}_{\{\}} \rightarrow \{\theta\}}{T} \left[\theta, \theta, \frac{T}{4 - 7T + 4T^2} + \right. \right. \\
& \quad \left(4T (2 + 10T^4 + 2a(-4 + 7T - 7T^3 + 4T^4) - 8xy + \right. \\
& \quad \left. T(-13 + 6xy) + T^2 (28 + 6xy) - T^3 (27 + 8xy)) \right) \left. \right] / (4 - 7T + 4T^2)^3 + O[\epsilon]^2 \right\}, \\
& \left. \frac{201.922, \mathbb{E}_{\{\}} \rightarrow \{\theta\}}{T^2} \left[\theta, \theta, \frac{T^2}{2 - 4T + 5T^2 - 4T^3 + 2T^4} + \right. \right. \\
& \quad \left(T^2 (-17 - T^8 + 8a(-2 + 6T - 9T^2 + 7T^3 - 7T^5 + 9T^6 - 6T^7 + 2T^8) - 16xy + T^5 (74 - 40xy) + \right. \\
& \quad \left. 2T^4 (-57 + 8xy) - 2T^7 (-5 + 8xy) + 2T^3 (65 + 8xy) - 2T^2 (53 + 20xy) + \right. \\
& \quad \left. T^6 (-34 + 32xy) + T(58 + 32xy)) \right) \left. \right] / (2 - 4T + 5T^2 - 4T^3 + 2T^4)^3 + O[\epsilon]^2 \right\}, \\
& \left. \frac{335.141, \mathbb{E}_{\{\}} \rightarrow \{\theta\}}{T^2} \left[\theta, \theta, -\frac{T^2}{1 - 5T + 7T^2 - 5T^3 + T^4} - \right. \right. \\
& \quad \left(T^2 (-3 + T^8 + a(-4 + 30T - 78T^2 + 80T^3 - 80T^5 + 78T^6 - 30T^7 + 4T^8) - 4xy + \right. \\
& \quad \left. T^5 (26 - 52xy) - T^7 (5 + 4xy) + 2T^3 (53 + 14xy) + T^6 (3 + 26xy) + T(25 + 26xy) + \right. \\
& \quad \left. T^4 (-78 + 28xy) - T^2 (75 + 52xy)) \right) \left. \right] / (1 - 5T + 7T^2 - 5T^3 + T^4)^3 + O[\epsilon]^2 \right\}, \\
& \left. \frac{281.094, \mathbb{E}_{\{\}} \rightarrow \{\theta\}}{T^2} \left[\theta, \theta, \frac{T^2}{1 - 5T + 9T^2 - 5T^3 + T^4} + \right. \right. \\
& \quad \left(T^2 (-2 + 2T^8 + 2a(-2 + 15T - 43T^2 + 50T^3 - 50T^5 + 43T^6 - 15T^7 + 2T^8) - 4xy - \right. \\
& \quad \left. 20T^2 (2 + 3xy) - T^7 (15 + 4xy) + 4T^3 (9 + 10xy) - 4T^5 (16 + 15xy) + T(15 + 26xy) + \right. \\
& \quad \left. T^6 (46 + 26xy) + T^4 (22 + 40xy)) \right) \left. \right] / (1 - 5T + 9T^2 - 5T^3 + T^4)^3 + O[\epsilon]^2 \right\}
\end{aligned}$$

In[]:= **Zs[[All, 2, 3]] // Normal**

$$\begin{aligned}
& Out[]= \left\{ \frac{T}{1 - T + T^2} + \frac{T(-2 + 3T - 2T^2 + T^3 + 2a(-1 + T - T^3 + T^4) - 2xy - 2T^3xy)}{(1 - T + T^2)^3}, \right. \\
& \quad \left. - \frac{T}{1 - 3T + T^2} - \frac{T(1 + T)(-1 + 2a(-1 + T) + T - 2xy)}{(1 - 3T + T^2)^2}, \right. \\
& \quad \left. \frac{T^2}{1 - T + T^2 - T^3 + T^4} + \left(T^2 (8T^3 - 6T^4 + a(-4 + 6T - 6T^2 + 4T^3 - 4T^5 + 6T^6 - 6T^7 + 4T^8) + T^7 (1 - 4xy) + \right. \right. \\
& \quad \left. T^5 (4 - 4xy) + 2T^6 (-1 + xy) - 4(1 + xy) - 4T^2 (2 + xy) + T(7 + 2xy)) \right) \left. \right] / \\
& \quad \left. (1 - T + T^2 - T^3 + T^4)^3, \frac{T}{2 - 3T + 2T^2} + \frac{1}{(2 - 3T + 2T^2)^3} \right. \\
& \quad \left. T(-9 - T^4 + 4a(-2 + 3T - 3T^3 + 2T^4) - 8xy + T^3 (8 - 8xy) + 4T(5 + xy) + 2T^2 (-9 + 2xy)) \right) \in, \\
& \quad \left. - \frac{T}{2 - 5T + 2T^2} + \frac{1}{(2 - 5T + 2T^2)^3} \right. \\
& \quad \left. T(5 - 3T^4 - 4a(-2 + 5T - 5T^3 + 2T^4) + 8xy - 4T(4 + 3xy) - 2T^2 (-5 + 6xy) + T^3 (4 + 8xy)) \right) \in, \\
& \quad \left. - \frac{T^2}{1 - 3T + 3T^2 - 3T^3 + T^4} - \left(T^2 (-3 + T^8 + 2a(-2 + 9T - 15T^2 + 12T^3 - 12T^5 + 15T^6 - 9T^7 + 2T^8) - \right. \right. \\
& \quad \left. 4xy + T^5 (4 - 16xy) + 8T^4 (-2 + xy) + 4T^3 (7 + 2xy) - T^7 (3 + 4xy) - \right. \\
& \quad \left. 4T^2 (7 + 4xy) + 2T^6 (1 + 7xy) + T(15 + 14xy)) \right) \left. \right] / (1 - 3T + 3T^2 - 3T^3 + T^4)^3,
\end{aligned}$$

$$\begin{aligned}
& \frac{T^2}{1 - 3T + 5T^2 - 3T^3 + T^4} + \frac{T^2 (2 - T - T^2 + 2T^3) (-1 + 2a(-1 + T) + T - 2xy) \in}{(1 - 3T + 5T^2 - 3T^3 + T^4)^2}, \\
& \frac{T^3}{1 - T + T^2 - T^3 + T^4 - T^5 + T^6} + \frac{1}{(1 - T + T^2 - T^3 + T^4 - T^5 + T^6)^3} \\
& T^3 (15T^5 - 12T^6 + 2a(-3 + 5T - 6T^2 + 6T^3 - 5T^4 + 3T^5 - 3T^7 + 5T^8 - 6T^9 + 6T^{10} - 5T^{11} + 3T^{12}) + \\
& T^9 (4 - 8xy) + T^{11} (1 - 6xy) + T^7 (9 - 6xy) - 6(1 + xy) + 4T^3 (4 + xy) - \\
& 2T^4 (8 + 3xy) + T^8 (-6 + 4xy) + T^{10} (-2 + 4xy) - 2T^2 (7 + 4xy) + T (11 + 4xy)) \in, \\
& \frac{T}{3 - 5T + 3T^2} + \frac{1}{(3 - 5T + 3T^2)^3} T (-23 - 5T^4 + 6a(-3 + 5T - 5T^3 + 3T^4)) - 18xy + \\
& T^3 (29 - 18xy) + 12T^2 (-5 + xy) + T (59 + 12xy) \in, \frac{T^2}{2 - 3T + 3T^2 - 3T^3 + 2T^4} + \\
& (T^2 (1 + 17T^8 + 2a(-8 + 18T - 21T^2 + 15T^3 - 15T^5 + 21T^6 - 18T^7 + 8T^8) - 16xy + T^2 (20 - 22xy) + \\
& 8T^4 (7 + xy) - 4T^7 (11 + 4xy) + 4T (-2 + 5xy) + T^3 (-37 + 8xy) + T^6 (62 + 20xy) - \\
& T^5 (67 + 22xy)) \in) / (2 - 3T + 3T^2 - 3T^3 + 2T^4)^3, \frac{T}{4 - 7T + 4T^2} + \frac{1}{(4 - 7T + 4T^2)^3} \\
& 4T (2 + 10T^4 + 2a(-4 + 7T - 7T^3 + 4T^4)) - 8xy + T (-13 + 6xy) + T^2 (28 + 6xy) - T^3 (27 + 8xy) \in, \\
& \frac{T^2}{2 - 4T + 5T^2 - 4T^3 + 2T^4} + (T^2 (-17 - T^8 + 8a(-2 + 6T - 9T^2 + 7T^3 - 7T^5 + 9T^6 - 6T^7 + 2T^8) - \\
& 16xy + T^5 (74 - 40xy) + 2T^4 (-57 + 8xy) - 2T^7 (-5 + 8xy) + 2T^3 (65 + 8xy) - \\
& 2T^2 (53 + 20xy) + T^6 (-34 + 32xy) + T (58 + 32xy)) \in) / (2 - 4T + 5T^2 - 4T^3 + 2T^4)^3, \\
& - \frac{T^2}{1 - 5T + 7T^2 - 5T^3 + T^4} - (T^2 (-3 + T^8 + a(-4 + 30T - 78T^2 + 80T^3 - 80T^5 + 78T^6 - 30T^7 + 4T^8) - \\
& 4xy + T^5 (26 - 52xy) - T^7 (5 + 4xy) + 2T^3 (53 + 14xy) + T^6 (3 + 26xy) + \\
& T (25 + 26xy) + T^4 (-78 + 28xy) - T^2 (75 + 52xy)) \in) / (1 - 5T + 7T^2 - 5T^3 + T^4)^3, \\
& \frac{T^2}{1 - 5T + 9T^2 - 5T^3 + T^4} + (T^2 (-2 + 2T^8 + 2a(-2 + 15T - 43T^2 + 50T^3 - 50T^5 + 43T^6 - 15T^7 + 2T^8) - \\
& 4xy - 20T^2 (2 + 3xy) - T^7 (15 + 4xy) + 4T^3 (9 + 10xy) - 4T^5 (16 + 15xy) + \\
& T (15 + 26xy) + T^6 (46 + 26xy) + T^4 (22 + 40xy)) \in) / (1 - 5T + 9T^2 - 5T^3 + T^4)^3 \}
\end{aligned}$$