Pensieve header: Testing the global picture.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1604"];
<< Global.m
```

```
In the U(T)⊗U(H) conventions. Internal use symbols: {rr, pp}
```

Verifying Ea

```
{CF@Ea[t, 1, 2]₁, CF@Ea[t, 1, 2]₂} /. t → 0
```

```
{UU[a[1, 1, h∞]], UU[a[1, 2, h∞]]}
```

```
{D[Ea[t, 1, 2]₁, t], D[Ea[t, 1, 2]₂, t]}
```

$$\left\{UU\left[\delta aa[1, ç, h\infty, 1, 2] + \delta aa\left[-e^{-t\,b_1}, ç, 2, 1, h\infty\right] + \delta aa\left[\frac{-1+e^{-t\,b_1}}{b_1}, 1, 2, 1, h\infty\right]\right],\right.$$
$$UU\left[a\left[e^{t\,b_1}\,b_1, 2, h\infty\right] + a\left[-e^{t\,b_1}\,b_2, 1, h\infty\right] + \delta a\left[-e^{t\,b_1}\,t\,b_2, 1, h\infty\right] +\right.$$
$$\delta a\left[e^{t\,b_1}\,t\,b_1\,b_2, ç, h\infty\right] + \delta aa\left[e^{t\,b_1}\,t, 1, 2, 2, h\infty\right] + \delta aa\left[-e^{t\,b_1}\,t\,b_1, ç, 2, 2, h\infty\right] +$$
$$\delta aa\left[-\frac{e^{t\,b_1}\,b_1 + \left(-1+e^{t\,b_1}\right)b_2}{b_1}, ç, h\infty, 1, 2\right] + \delta aa\left[e^{-t\,b_1} + e^{t\,b_1}\,t\,b_2, ç, 2, 1, h\infty\right] +$$
$$\left.\left.\delta aa\left[-\frac{-\left(-1+e^{t\,b_1}\right)b_2 + b_1\left(e^{-t\,b_1} - e^{t\,b_1} + e^{t\,b_1}\,t\,b_2\right)}{b_1^2}, 1, 2, 1, h\infty\right]\right]\right\}$$

```
{D[Ea[t, 1, 2]₁, t] - bb[1, 2][UU[a[1, 1, 2]], Ea[t, 1, 2]₁],
 D[Ea[t, 1, 2]₂, t] - bb[1, 2][UU[a[1, 1, 2]], Ea[t, 1, 2]₂]}
```

```
{UU[0], UU[0]}
```

Testing ct

```
? ct
```

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ct[h,t][L,R] contracts the head h in L with the tail t in R.

ct[s][L,R] takes h=t=s, and ct[][L,R] takes s=0. When ambiguous, L is placeed below R.

```
ct[h∞, 0][Ea[t, 1, 2]₁, UU[β[b₀]]]
```

$$UU\left[\beta[b_1] + \delta a\left[-1+e^{-t\,b_1}, ç, 2\right] + \delta a\left[\frac{1-e^{-t\,b_1}}{b_1}, 1, 2\right]\right]$$

```
ct[h∞, 0][Ea[t, 1, 2]₂, UU[β[b₀]]]
```

$$UU\left[\beta[b_2] + \delta a\left[1-e^{-t\,b_1}, ç, 2\right] + \delta a\left[\frac{-1+e^{-t\,b_1}}{b_1}, 1, 2\right]\right]$$

Testing β[] Scattering

```
β[f[b1, b2]] // Ea[t, 1, 2]
```

$$\text{UU}\left[\beta[f[b_1, b_2]] + \delta a\left[e^{-t\,b_1}\left(-1+e^{t\,b_1}\right)\left(f^{(0,1)}[b_1, b_2] - f^{(1,0)}[b_1, b_2]\right), ç, 2\right] + \delta a\left[-\frac{e^{-t\,b_1}\left(-1+e^{t\,b_1}\right)\left(f^{(0,1)}[b_1, b_2] - f^{(1,0)}[b_1, b_2]\right)}{b_1}, 1, 2\right]\right]$$

Testing a[] Scattering

```
Table[a[f[b1, b2], k, h∞] // Ea[t, 1, 2], {k, 2}]
```

$$\Big\{\text{UU}\Big[a[f[b_1, b_2], 1, h\infty] + \delta aa[t\,f[b_1, b_2], ç, h\infty, 1, 2] + \delta aa\Big[-\frac{1}{b_1^2}\left(f[b_1, b_2]\left(-1+e^{-t\,b_1}+t\,b_1\right)+e^{-t\,b_1}\left(-1+e^{t\,b_1}\right)b_1\left(f^{(0,1)}[b_1, b_2] - f^{(1,0)}[b_1, b_2]\right)\right), 1, 2, 1, h\infty\Big] + \delta aa\Big[-\frac{1}{b_1}e^{-t\,b_1}\left(-1+e^{t\,b_1}\right)\left(f[b_1, b_2] + b_1\left(-f^{(0,1)}[b_1, b_2] + f^{(1,0)}[b_1, b_2]\right)\right), ç, 2, 1, h\infty\Big]\Big],$$

$$\text{UU}\Big[a\left[e^{t\,b_1} f[b_1, b_2], 2, h\infty\right] + a\left[-\frac{\left(-1+e^{t\,b_1}\right)f[b_1, b_2]\,b_2}{b_1}, 1, h\infty\right] + \delta a\Big[b_2\left(\frac{f[b_1, b_2]\left(1-e^{t\,b_1}+e^{t\,b_1}\,t\,b_1\right)}{b_1} - \left(-1+e^{t\,b_1}\right)\left(f^{(0,1)}[b_1, b_2] - f^{(1,0)}[b_1, b_2]\right)\right), ç, h\infty\Big] + \delta a\Big[-\frac{1}{b_1^2}b_2\left(f[b_1, b_2]\left(1-e^{t\,b_1}+e^{t\,b_1}\,t\,b_1\right) - \left(-1+e^{t\,b_1}\right)b_1\left(f^{(0,1)}[b_1, b_2] - f^{(1,0)}[b_1, b_2]\right)\right), 1, h\infty\Big] + \delta aa\Big[-\frac{f[b_1, b_2]\left(\left(-1+e^{t\,b_1}\right)b_2 + b_1\left(-1+e^{t\,b_1}-t\,b_2\right)\right)}{b_1^2}, ç, h\infty, 1, 2\Big] + \delta aa\Big[-\frac{f[b_1, b_2]\left(1-e^{t\,b_1}+e^{t\,b_1}\,t\,b_1\right)}{b_1} + \left(-1+e^{t\,b_1}\right)\left(f^{(0,1)}[b_1, b_2] - f^{(1,0)}[b_1, b_2]\right), ç, 2, 2, h\infty\Big] + \delta aa\Big[\frac{1}{b_1^2}\left(f[b_1, b_2]\left(1-e^{t\,b_1}+e^{t\,b_1}\,t\,b_1\right) - \left(-1+e^{t\,b_1}\right)b_1\left(f^{(0,1)}[b_1, b_2] - f^{(1,0)}[b_1, b_2]\right)\right), 1, 2, 2, h\infty\Big] + \delta aa\Big[\frac{1}{b_1^3}e^{-t\,b_1}\left(f[b_1, b_2]\left(2\,e^{t\,b_1}\left(-1+e^{t\,b_1}\right)b_2 + b_1\left(\left(-1+e^{t\,b_1}\right)^2 - e^{t\,b_1}\left(1+e^{t\,b_1}\right)t\,b_2\right)\right) + \left(-1+e^{t\,b_1}\right)^2 b_1\,b_2\left(f^{(0,1)}[b_1, b_2] - f^{(1,0)}[b_1, b_2]\right)\right), 1, 2, 1, h\infty\Big] + \delta aa\Big[\frac{1}{b_1^2}\left(f[b_1, b_2]\left(-\left(-1+e^{t\,b_1}\right)b_2 + b_1\left(1-e^{-t\,b_1}+e^{t\,b_1}\,t\,b_2\right)\right) - e^{-t\,b_1}\left(-1+e^{t\,b_1}\right)^2 b_1\,b_2\left(f^{(0,1)}[b_1, b_2] - f^{(1,0)}[b_1, b_2]\right)\right), ç, 2, 1, h\infty\Big]\Big]\Big\}$$

Testing δa[] Scattering

```
TSD0[Ea[t, 1, 2]]
```

$$\text{TSD0}\left[\left\{1 \to \text{UU}[a[1, 1, h\infty]],\ 2 \to \text{UU}\left[a\left[e^{t\,b_1}, 2, h\infty\right] + a\left[-\frac{\left(-1+e^{t\,b_1}\right)b_2}{b_1}, 1, h\infty\right]\right]\right\}\right]$$

```
Table[δa[f[b1, b2], k, h∞] // Ea[t, 1, 2], {k, 2}]
```

$$\left\{UU[\delta a[f[b_1, b_2], 1, h\infty]],\ UU\left[\delta a\left[e^{t\,b_1} f[b_1, b_2], 2, h\infty\right] + \delta a\left[-\frac{\left(-1+e^{t\,b_1}\right) f[b_1, b_2]\, b_2}{b_1}, 1, h\infty\right]\right]\right\}$$

```
Table[δa[f[b1, b2], t∞, k] // Ea[t, 1, 2], {k, 2}]
```

$$\left\{UU\left[\delta a[f[b_1, b_2], t\infty, 1] + \delta a\left[\left(1-e^{-t\,b_1}\right) f[b_1, b_2], t\infty, 2\right] + \delta a\left[-\frac{\left(1-e^{-t\,b_1}\right) f[b_1, b_2]\, b_{t\infty}}{b_1}, 1, 2\right]\right],\ UU\left[\delta a\left[e^{-t\,b_1} f[b_1, b_2], t\infty, 2\right] + \delta a\left[\frac{e^{-t\,b_1}\left(-1+e^{t\,b_1}\right) f[b_1, b_2]\, b_{t\infty}}{b_1}, 1, 2\right]\right]\right\}$$

```
Table[δa[f, j, k] // Ea[t, 1, 2], {j, 2}, {k, 2}]
```

$$\left\{\{UU[\delta a[f, 1, 1]], UU[\delta a[f, 1, 2]]\},\ \left\{UU\left[\delta a\left[e^{t\,b_1} f, 2, 1\right] + \delta a\left[\left(-1+e^{t\,b_1}\right) f, 2, 2\right] + \delta a\left[-\frac{\left(-1+e^{t\,b_1}\right) f\, b_2}{b_1}, 1, 1\right] + \delta a\left[-\frac{\left(-1+e^{t\,b_1}\right) f\, b_2}{b_1}, 1, 2\right]\right], UU[\delta a[f, 2, 2]]\right\}\right\}$$

Compositions

```
Ea[t, 1, 2] ** Ea[s, 1, 2]
```

$$\begin{aligned}
&\mathrm{TSD}\Big[\big\langle\, 1 \to UU\big[a[1, 1, h\infty] + \delta aa[s+t, ç, h\infty, 1, 2] + \delta aa\Big[\frac{-1+e^{-(s+t)\,b_1}}{b_1}, ç, 2, 1, h\infty\Big] + \delta aa\Big[-\frac{-1+e^{-(s+t)\,b_1}+(s+t)\,b_1}{b_1^2}, 1, 2, 1, h\infty\Big]\big],\\
&\quad 2 \to UU\Big[a\left[e^{(s+t)\,b_1}, 2, h\infty\right] + a\Big[-\frac{\left(-1+e^{(s+t)\,b_1}\right) b_2}{b_1}, 1, h\infty\Big] + \delta a\Big[-\frac{\left(1-e^{(s+t)\,b_1}+e^{(s+t)\,b_1}\,(s+t)\,b_1\right) b_2}{b_1^2}, 1, h\infty\Big] + \\
&\qquad \delta a\Big[\frac{\left(1-e^{(s+t)\,b_1}+e^{(s+t)\,b_1}\,(s+t)\,b_1\right) b_2}{b_1}, ç, h\infty\Big] + \delta aa\Big[\frac{-1+e^{(s+t)\,b_1}-e^{(s+t)\,b_1}\,(s+t)\,b_1}{b_1}, ç, 2, 2, h\infty\Big] + \delta aa\Big[\frac{1-e^{(s+t)\,b_1}+e^{(s+t)\,b_1}\,(s+t)\,b_1}{b_1^2}, 1, 2, 2, h\infty\Big] + \\
&\qquad \delta aa\Big[\frac{-\left(-1+e^{(s+t)\,b_1}\right) b_2 + b_1\left(1-e^{(s+t)\,b_1}+(s+t)\,b_2\right)}{b_1^2}, ç, h\infty, 1, 2\Big] + \\
&\qquad \delta aa\Big[\frac{1}{b_1^2} e^{-(s+t)\,b_1}\left(-e^{(s+t)\,b_1}\left(-1+e^{(s+t)\,b_1}\right) b_2 + b_1\left(-1+e^{(s+t)\,b_1}+e^{2\,(s+t)\,b_1}\,(s+t)\,b_2\right)\right), ç, 2, 1, h\infty\Big] + \delta aa\Big[\frac{1}{b_1^3} e^{-(s+t)\,b_1}\left(2\,e^{(s+t)\,b_1}\left(-1+e^{(s+t)\,b_1}\right) b_2 - b_1\left(-\left(-1+e^{(s+t)\,b_1}\right)^2 + e^{(s+t)\,b_1}\left(1+e^{(s+t)\,b_1}\right)(s+t)\,b_2\right)\right), 1, 2, 1, h\infty\Big]\big]\,\big|\,\big\rangle\Big]
\end{aligned}$$

```
Ea[s + t, 1, 2]
```

$$\text{TSD}\Big[\langle\Big|\, 1 \to \text{UU}\Big[\text{a}[1,\,1,\,\text{h}\infty] + \delta\text{aa}[s+t,\,\text{ç},\,\text{h}\infty,\,1,\,2] + \delta\text{aa}\Big[\frac{-1+e^{-(s+t)\,b_1}}{b_1},\,\text{ç},\,2,\,1,\,\text{h}\infty\Big] + \delta\text{aa}\Big[-\frac{-1+e^{-(s+t)\,b_1}+(s+t)\,b_1}{b_1^2},\,1,\,2,\,1,\,\text{h}\infty\Big]\Big],$$

$$2 \to \text{UU}\Big[\text{a}\Big[e^{(s+t)\,b_1},\,2,\,\text{h}\infty\Big] + \text{a}\Big[-\frac{\left(-1+e^{(s+t)\,b_1}\right)b_2}{b_1},\,1,\,\text{h}\infty\Big] + \delta\text{a}\Big[-\frac{e^{(s+t)\,b_1}\left(-1+e^{-(s+t)\,b_1}+(s+t)\,b_1\right)b_2}{b_1^2},\,1,\,\text{h}\infty\Big] + \delta\text{a}\Big[\frac{e^{(s+t)\,b_1}\left(-1+e^{-(s+t)\,b_1}+(s+t)\,b_1\right)b_2}{b_1},\,\text{ç},\,\text{h}\infty\Big] + \delta\text{aa}\Big[\frac{e^{(s+t)\,b_1}\left(-1+e^{-(s+t)\,b_1}+(s+t)\,b_1\right)}{b_1^2},\,1,\,2,\,2,\,\text{h}\infty\Big] + \delta\text{aa}\Big[-\frac{e^{(s+t)\,b_1}\left(-1+e^{-(s+t)\,b_1}+(s+t)\,b_1\right)}{b_1},\,\text{ç},\,2,\,2,\,\text{h}\infty\Big] + \delta\text{aa}\Big[-\frac{-1+e^{(s+t)\,b_1}}{b_1}-\frac{\left(-1+e^{(s+t)\,b_1}-(s+t)\,b_1\right)b_2}{b_1^2},\,\text{ç},\,\text{h}\infty,\,1,\,2\Big] + \delta\text{aa}\Big[-\frac{-1+e^{-(s+t)\,b_1}}{b_1}+\frac{e^{(s+t)\,b_1}\left(-1+e^{-(s+t)\,b_1}+(s+t)\,b_1\right)b_2}{b_1^2},\,\text{ç},\,2,\,1,\,\text{h}\infty\Big] + \delta\text{aa}\Big[\frac{(s+t)\left(\frac{-1+e^{-(s+t)\,b_1}}{(s+t)\,b_1}+\frac{-1+e^{(s+t)\,b_1}}{(s+t)\,b_1}\right)}{b_1}-\frac{\left(2-2\,e^{(s+t)\,b_1}+\left(1+e^{(s+t)\,b_1}\right)(s+t)\,b_1\right)b_2}{b_1^3},\,1,\,2,\,1,\,\text{h}\infty\Big]\Big]\,\Big|\rangle\Big]$$

```
Table[(Ea[t, 1, 2] ** Ea[s, 1, 2])_k - Ea[s + t, 1, 2]_k, {k, 2}]
```

```
{UU[0], UU[0]}
```

```
hωBasis[{1, 2}, f]
```

```
{UU[a[f1[b1, b2], 1, hω]], UU[a[f2[b1, b2], 2, hω]],
 UU[δa[f3[b1, b2], 1, hω]], UU[δa[f4[b1, b2], 2, hω]], UU[δa[f5[b1, b2], ç, hω]],
 UU[δaa[f6[b1, b2], ç, 1, 1, hω]], UU[δaa[f7[b1, b2], ç, 2, 1, hω]],
 UU[δaa[f8[b1, b2], ç, hω, 1, 1]], UU[δaa[f9[b1, b2], ç, hω, 1, 2]],
 UU[δaa[f10[b1, b2], ç, 1, 2, hω]], UU[δaa[f11[b1, b2], ç, 2, 2, hω]],
 UU[δaa[f12[b1, b2], ç, hω, 2, 1]], UU[δaa[f13[b1, b2], ç, hω, 2, 2]],
 UU[δaa[f14[b1, b2], 1, 1, 1, hω]], UU[δaa[f15[b1, b2], 1, 2, 1, hω]],
 UU[δaa[f16[b1, b2], 1, 1, 2, hω]], UU[δaa[f17[b1, b2], 1, 2, 2, hω]],
 UU[δaa[f18[b1, b2], 2, 1, 2, hω]], UU[δaa[f19[b1, b2], 2, 2, 2, hω]]}
```

```
Table[u → (u // Ea[s + t, 1, 2]) - (u // Ea[t, 1, 2] // Ea[s, 1, 2]),
 {u, h∞Basis[{1, 2}, f]}]
```

$$\{UU[a[f_1[b_1, b_2], 1, h\infty]] \to UU[0],$$
$$UU[a[f_2[b_1, b_2], 2, h\infty]] \to UU[0], UU[\delta a[f_3[b_1, b_2], 1, h\infty]] \to UU[0],$$
$$UU[\delta a[f_4[b_1, b_2], 2, h\infty]] \to UU[0], UU[\delta a[f_5[b_1, b_2], ç, h\infty]] \to UU[0],$$
$$UU[\delta aa[f_6[b_1, b_2], ç, 1, 1, h\infty]] \to UU[0], UU[\delta aa[f_7[b_1, b_2], ç, 2, 1, h\infty]] \to UU[0],$$
$$UU[\delta aa[f_8[b_1, b_2], ç, h\infty, 1, 1]] \to UU[0], UU[\delta aa[f_9[b_1, b_2], ç, h\infty, 1, 2]] \to UU[0],$$
$$UU[\delta aa[f_{10}[b_1, b_2], ç, 1, 2, h\infty]] \to UU[0], UU[\delta aa[f_{11}[b_1, b_2], ç, 2, 2, h\infty]] \to UU[0],$$
$$UU[\delta aa[f_{12}[b_1, b_2], ç, h\infty, 2, 1]] \to UU[0], UU[\delta aa[f_{13}[b_1, b_2], ç, h\infty, 2, 2]] \to UU[0],$$
$$UU[\delta aa[f_{14}[b_1, b_2], 1, 1, 1, h\infty]] \to UU[0], UU[\delta aa[f_{15}[b_1, b_2], 1, 2, 1, h\infty]] \to UU[0],$$
$$UU[\delta aa[f_{16}[b_1, b_2], 1, 1, 2, h\infty]] \to UU[0], UU[\delta aa[f_{17}[b_1, b_2], 1, 2, 2, h\infty]] \to UU[0],$$
$$UU[\delta aa[f_{18}[b_1, b_2], 2, 1, 2, h\infty]] \to UU[0], UU[\delta aa[f_{19}[b_1, b_2], 2, 2, 2, h\infty]] \to UU[0]\}$$