Pensieve header: Finding the braids for which the complexity of $\bar{\Gamma}$ grows in the fastest way.

(Alt) In[•]:=
```
SetDirectory@"C:\\drorbn\\AcademicPensieve\\Projects\\OU";
<< "OU-Programs.m"
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

(Alt) In[•]:=
```
A_ \ B_ := Complement[A, B];
VPBGens[n_] := VPBGens[n] = Flatten@Table[{σ_{i,j}, σ̄_{i,j}}, {i, n}, {j, Range[n] \ {i}}];
```

(Alt) In[•]:=
```
PF[n_, σ_{i_,j_}] := PF[n, σ_{i,j}] = Module[{p, q, s},
    Flatten@{σ_{i,j}, σ_{j,i}, σ̄_{j,i},
      Table[{σ_{p,q}, σ_{q,p}, σ̄_{p,q}, σ̄_{q,p}}, {p, {i, j}}, {q, Range[n] \ {i, j}}],
      Table[{σ_{p,q}, σ̄_{p,q}}, {p, Range[i + 1, n] \ {j}}, {q, Range[n] \ {i, j, p}}] }];
PF[n_, σ̄_{i_,j_}] := PF[n, σ̄_{i,j}] = PF[n, σ_{i,j}] /. σ_{i,j} → σ̄_{i,j}
```

(Alt) In[•]:=
```
PVPBDs[n_, 0] := {VPB[n]};
PVPBDs[n_, 1] := VPB[n, #] & /@ VPBGens[n];
PVPBDs[n_, m_] :=
 Flatten[PVPBDs[n, m - 1] /. VPB[n, σs___, σ_] :> (VPB[n, σs, σ, #] & /@ PF[n, σ])]
```

(Alt) In[•]:=
```
AllVPBs[n_, m_] := DeleteDuplicatesBy[Γ̄]@Flatten@Table[b, {k, 0, m}, {b, PVPBDs[n, k]}]
```

In[•]:=
```
Max[(Length[Γ̄[#]] - 3) & /@ AllBs[3, 3]]
```

Out[•]=
```
10
```

In[•]:=
```
ML[m_] := Max[(Length[Γ̄[#]] - 3) & /@ AllBs[3, m]]
```

In[•]:=
```
Echo[ML[#]] & /@ Range[8]
```

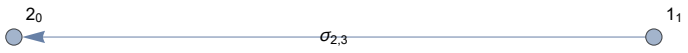
```
» 1
» 4
» 10
» 20
» 36
» 62
» 104
» 172
```

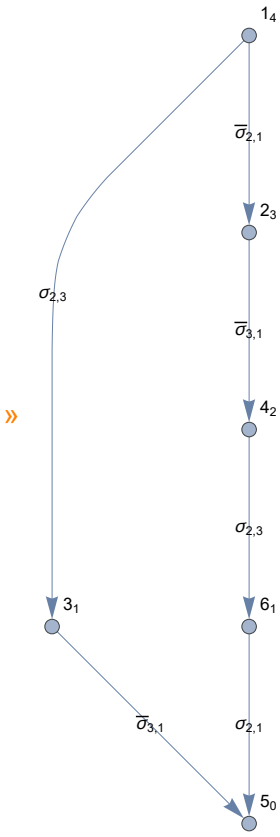
Out[•]=
```
{1, 4, 10, 20, 36, 62, 104, 172}
```

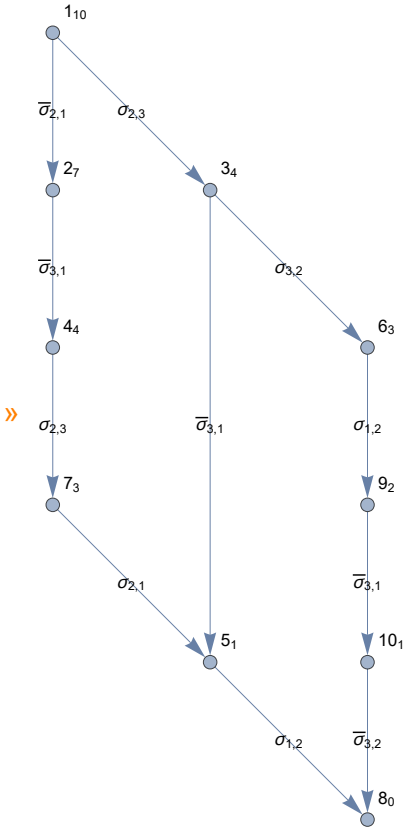
In[•]:=
```
MaximalBy[AllBs[3, 4], Length@*Γ̄]
```

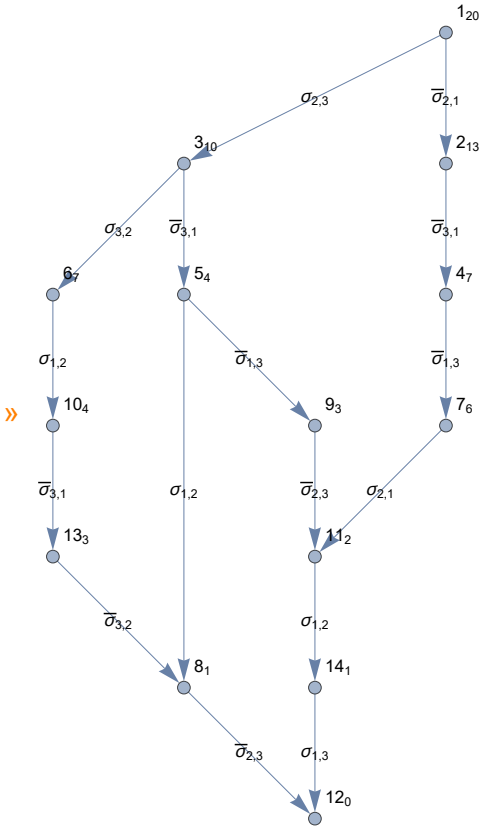
Out[•]=
```
{BR[3, {-1, 2, -1, 2}], BR[3, {2, -1, 2, -1}]}
```

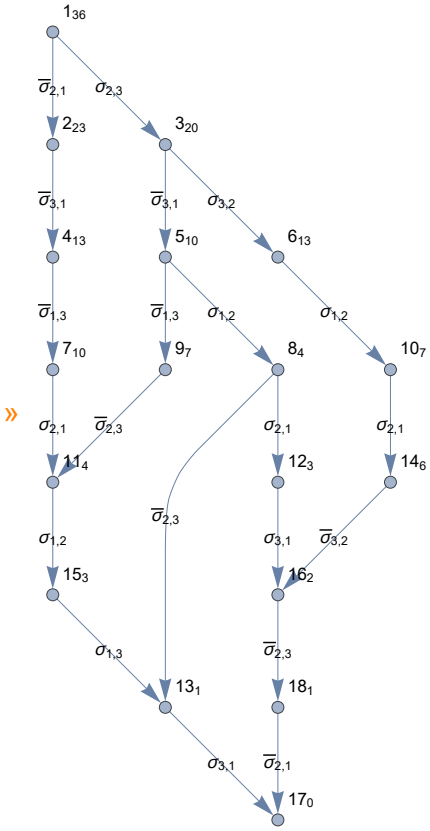
```
In[•]:= MaximalBy[AllBs[3, 5], Length@*Γ̄]
Out[•]= {BR[3, {-1, 2, -1, 2, -1}], BR[3, {2, -1, 2, -1, 2}]}

In[•]:= MaximalBy[AllBs[3, 6], Length@*Γ̄]
Out[•]= {BR[3, {-1, 2, -1, 2, -1, 2}], BR[3, {2, -1, 2, -1, 2, -1}]}

In[•]:= Table[If[OddQ[k], 2, -1], {k, 7}]
Out[•]= {2, -1, 2, -1, 2, -1, 2}

In[•]:= Table[Echo[Length[Γ̄[BR[3, Table[If[OddQ[k], 2, -1], {k, m}]]]] - 3], {m, 13}]
» 1
» 4
» 10
» 20
» 36
» 62
» 104
» 172
» 282
» 460
» 748
» 1214
» 1968
Out[•]= {1, 4, 10, 20, 36, 62, 104, 172, 282, 460, 748, 1214, 1968}

In[•]:= Table[Echo[ExtractionGraph[
   Γ̄[BR[3, Table[If[OddQ[k], 2, -1], {k, m}]]], GraphLayout → Automatic]], {m, 10}]
```

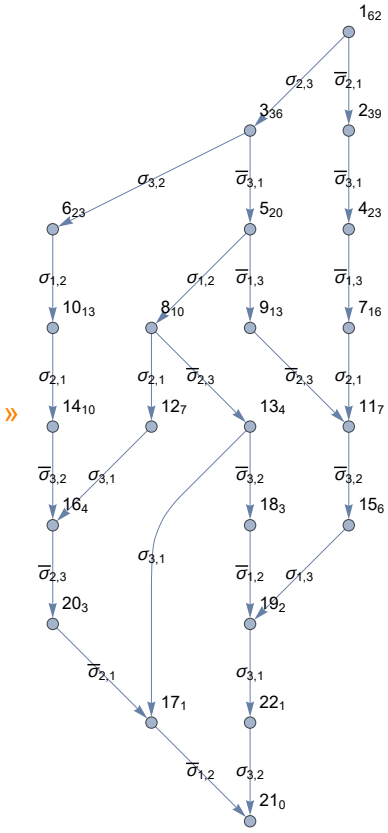
» 

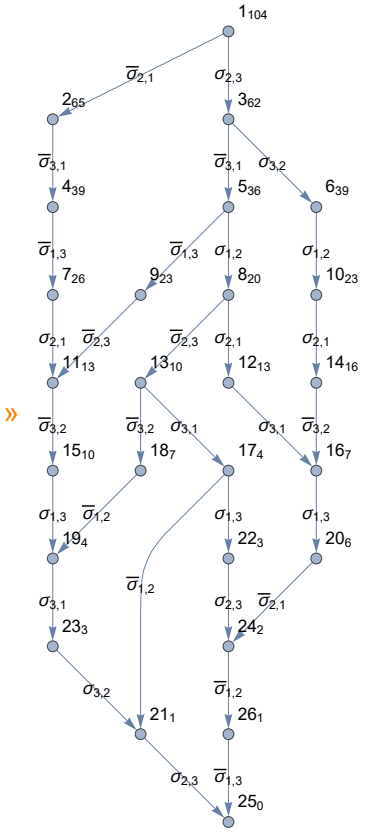

1 4
σ2,1
2 3
σ2,3
σ3,1
4 2
σ2,3
3 1
6 1
σ3,1
σ2,1
5 0

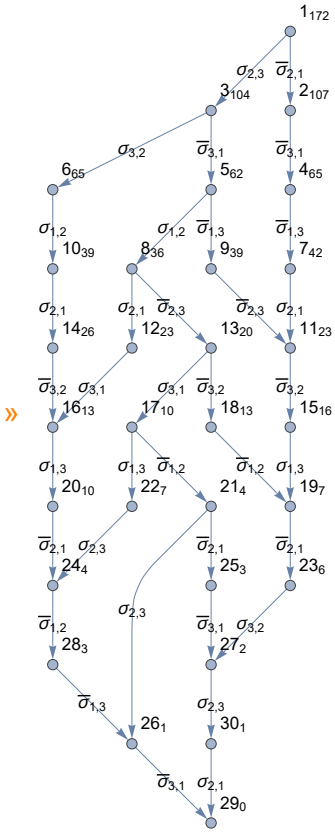

1 10
σ2,1
σ2,3
2 7
3 4
σ3,1
σ3,2
4 4
6 3
σ2,3
σ3,1
σ1,2
7 3
9 2
σ2,1
σ3,1
5 1
10 1
σ1,2
σ3,2
8 0

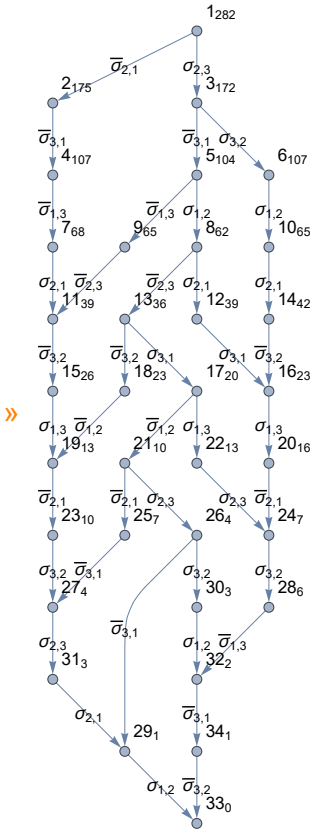

1 20
σ 2,3
σ̄ 2,1
3 10
2 13
σ 3,2
σ̄ 3,1
σ̄ 3,1
6 7
5 4
4 7
σ 1,2
σ̄ 1,3
σ̄ 1,3
10 4
9 3
7 6
σ̄ 3,1
σ 1,2
σ̄ 2,3
σ 2,1
13 3
11 2
σ̄ 3,2
σ 1,2
8 1
14 1
σ̄ 2,3
σ 1,3
12 0


1 36
σ̄ 2,1
σ 2,3
2 23
3 20
σ̄ 3,1
σ̄ 3,1
σ 3,2
4 13
5 10
6 13
σ̄ 1,3
σ̄ 1,3
σ 1,2
σ 1,2
7 10
9 7
8 4
10 7
σ 2,1
σ̄ 2,3
σ 2,1
σ 2,1
11 4
12 3
14 6
σ̄ 2,3
σ 1,2
σ 3,1
σ̄ 3,2
15 3
16 2
σ 1,3
σ̄ 2,3
13 1
18 1
σ 3,1
σ̄ 2,1
17 0

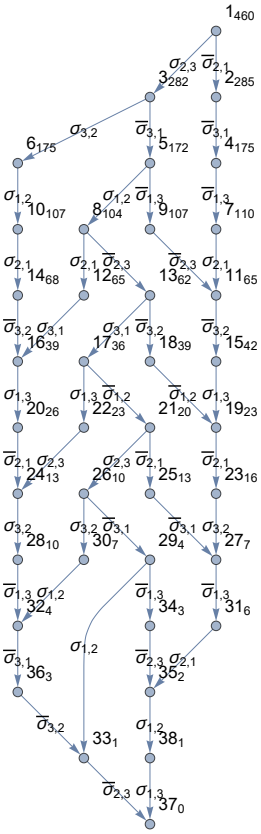

1_{62}
$\sigma_{2,3}$ $\overline{\sigma}_{2,1}$
3_{36} 2_{39}
$\sigma_{3,2}$ $\overline{\sigma}_{3,1}$ $\overline{\sigma}_{3,1}$
6_{23} 5_{20} 4_{23}
$\sigma_{1,2}$ $\sigma_{1,2}$ $\overline{\sigma}_{1,3}$ $\overline{\sigma}_{1,3}$
10_{13} 8_{10} 9_{13} 7_{16}
$\sigma_{2,1}$ $\sigma_{2,1}$ $\overline{\sigma}_{2,3}$ $\overline{\sigma}_{2,3}$ $\sigma_{2,1}$
14_{10} 12_7 13_4 11_7
$\overline{\sigma}_{3,2}$ $\sigma_{3,1}$ $\overline{\sigma}_{3,2}$ $\overline{\sigma}_{3,2}$
16_4 18_3 15_6
$\sigma_{3,1}$
$\overline{\sigma}_{2,3}$ $\overline{\sigma}_{1,2}$ $\sigma_{1,3}$
20_3 19_2
$\overline{\sigma}_{2,1}$ $\sigma_{3,1}$
17_1 22_1
$\overline{\sigma}_{1,2}$ $\sigma_{3,2}$
21_0


1_{104}
$\overline{\sigma}_{2,1}$ $\sigma_{2,3}$
2_{65} 3_{62}
$\overline{\sigma}_{3,1}$ $\overline{\sigma}_{3,1}$ $\sigma_{3,2}$
4_{39} 5_{36} 6_{39}
$\overline{\sigma}_{1,3}$ $\overline{\sigma}_{1,3}$ $\sigma_{1,2}$ $\sigma_{1,2}$
7_{26} 9_{23} 8_{20} 10_{23}
$\sigma_{2,1}$ $\overline{\sigma}_{2,3}$ $\overline{\sigma}_{2,3}$ $\sigma_{2,1}$ $\sigma_{2,1}$
11_{13} 13_{10} 12_{13} 14_{16}
$\overline{\sigma}_{3,2}$ $\overline{\sigma}_{3,2}$ $\sigma_{3,1}$ $\sigma_{3,1}$ $\overline{\sigma}_{3,2}$
15_{10} 18_7 17_4 16_7
$\sigma_{1,3}$ $\overline{\sigma}_{1,2}$ $\sigma_{1,3}$ $\sigma_{1,3}$
19_4 22_3 20_6
$\overline{\sigma}_{1,2}$
$\sigma_{3,1}$ $\sigma_{2,3}$ $\overline{\sigma}_{2,1}$
23_3 24_2
$\sigma_{3,2}$ $\overline{\sigma}_{1,2}$
21_1 26_1
$\sigma_{2,3}$ $\overline{\sigma}_{1,3}$
25_0

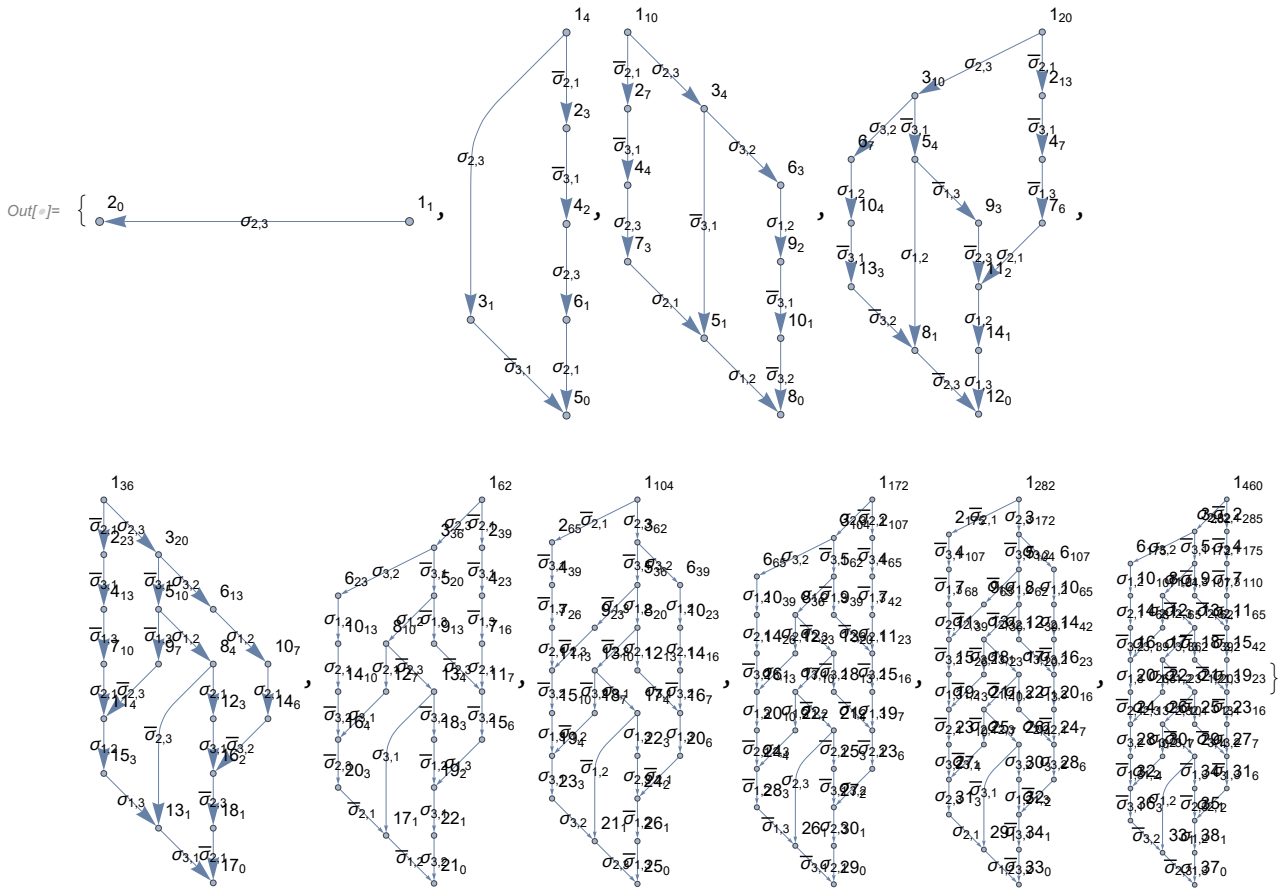

1172
σ2,3 σ̄2,1
3104 2107
σ3,2 σ̄3,1 σ̄3,1
6₆₅ 5₆₂ 4₆₅
σ1,2 σ1,2 σ̄1,3 σ̄1,3
10₃₉ 8₃₆ 9₃₉ 7₄₂
σ2,1 σ2,1 σ̄2,3 σ̄2,3 σ2,1
14₂₆ 12₂₃ 13₂₀ 11₂₃
σ̄3,2 σ3,1 σ3,1 σ̄3,2 σ̄3,2
16₁₃ 17₁₀ 18₁₃ 15₁₆
σ1,3 σ1,3 σ̄1,2 σ̄1,2 σ1,3
20₁₀ 22₇ 21₄ 19₇
σ̄2,1 σ2,3 σ̄2,1 σ̄2,1
24₄ 25₃ 23₆
σ2,3 σ̄1,2 σ̄3,1 σ3,2
28₃ 27₂
σ̄1,3 σ2,3
26₁ 30₁
σ̄3,1 σ2,1
29₀


1282
σ̄2,1 σ2,3
2175 3172
σ̄3,1 σ̄3,1 σ3,2
4107 5104 6107
σ̄1,3 σ̄1,3 σ1,2 σ1,2
7₆₈ 9₆₅ 8₆₂ 10₆₅
σ2,1 σ̄2,3 σ̄2,3 σ2,1 σ2,1
11₃₉ 13₃₆ 12₃₉ 14₄₂
σ̄3,2 σ̄3,2 σ3,1 σ3,1 σ̄3,2
15₂₆ 18₂₃ 17₂₀ 16₂₃
σ1,3 σ̄1,2 σ̄1,2 σ1,3 σ1,3
19₁₃ 21₁₀ 22₁₃ 20₁₆
σ̄2,1 σ̄2,1 σ2,3 σ2,3 σ̄2,1
23₁₀ 25₇ 26₄ 24₇
σ3,2 σ̄3,1 σ3,2 σ3,2
27₄ 30₃ 28₆
σ̄3,1
σ2,3 σ1,2 σ̄1,3
31₃ 32₂
σ2,1 σ̄3,1
29₁ 34₁
σ1,2 σ̄3,2
33₀

»


1_{460}
$\sigma_{2,3}$ $\overline{\sigma}_{2,1}$
3_{282} 2_{285}
$\sigma_{3,2}$ $\overline{\sigma}_{3,1}$ $\overline{\sigma}_{3,1}$
6_{175} 5_{172} 4_{175}
$\sigma_{1,2}$ $\sigma_{1,2}$ $\overline{\sigma}_{1,3}$ $\overline{\sigma}_{1,3}$
10_{107} 8_{104} 9_{107} 7_{110}
$\sigma_{2,1}$ $\sigma_{2,1}$ $\overline{\sigma}_{2,3}$ $\overline{\sigma}_{2,3}$ $\sigma_{2,1}$
14_{68} 12_{65} 13_{62} 11_{65}
$\overline{\sigma}_{3,2}$ $\sigma_{3,1}$ $\sigma_{3,1}$ $\overline{\sigma}_{3,2}$ $\overline{\sigma}_{3,2}$
16_{39} 17_{36} 18_{39} 15_{42}
$\sigma_{1,3}$ $\sigma_{1,3}$ $\overline{\sigma}_{1,2}$ $\overline{\sigma}_{1,2}$ $\sigma_{1,3}$
20_{26} 22_{23} 21_{20} 19_{23}
$\overline{\sigma}_{2,1}$ $\sigma_{2,3}$ $\sigma_{2,3}$ $\overline{\sigma}_{2,1}$ $\overline{\sigma}_{2,1}$
24_{13} 26_{10} 25_{13} 23_{16}
$\sigma_{3,2}$ $\sigma_{3,2}$ $\overline{\sigma}_{3,1}$ $\overline{\sigma}_{3,1}$ $\sigma_{3,2}$
28_{10} 30_{7} 29_{4} 27_{7}
$\overline{\sigma}_{1,3}$ $\sigma_{1,2}$ $\overline{\sigma}_{1,3}$ $\overline{\sigma}_{1,3}$
32_{4} 34_{3} 31_{6}
$\sigma_{1,2}$
$\overline{\sigma}_{3,1}$ $\overline{\sigma}_{2,3}$ $\sigma_{2,1}$
36_{3} 35_{2}
$\overline{\sigma}_{3,2}$ $\sigma_{1,2}$
33_{1} 38_{1}
$\overline{\sigma}_{2,3}$ $\sigma_{1,3}$
37_{0}

Out[•]=



In[•]:=
```
FindSequenceFunction[{1, 4, 10, 20, 36, 62, 104, 172, 282, 460}]
```

Out[•]=
```
DifferenceRoot[
 Function[{y, n}, {-6 - y[n] - y[1 + n] + y[2 + n] == 0, y[1] == 1, y[2] == 4, y[3] == 10}]]
```

In[•]:=
```
FindSequenceFunction[{4, 10, 20, 36, 62, 104, 172, 282, 460, 748, 1214, 1968}]
```

Out[•]=
```
-6 + 7 Fibonacci[#1] + 3 LucasL[#1] &
```

In[•]:=
```
ML[n_, m_] := Max[(Length[Γ[#]] - n) & /@ AllBs[n, m]]
```

In[•]:=
```
Echo[ML[4, #]] & /@ Range[5]
```

» 1

» 4

» 10

» 20

» 36

Out[•]=
```
{1, 4, 10, 20, 36}
```

In[•]:=
```
Echo[ML[2, #]] & /@ Range[10]
```

```
» 1
» 3
» 5
» 7
» 9
» 11
» 13
» 15
» 17
» 19
```

Out[•]= {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}

(Alt) In[•]:=
```
vML[n_, m_] := Max[(Length[Γ̄[#]] - n) & /@ AllVPBs[n, m]]
```

In[•]:=
```
Echo[vML[2, #]] & /@ Range[6]
```

```
» 1
» 4
» 11
» 28
» 69
» 168
```

Out[•]= {1, 4, 11, 28, 69, 168}

In[•]:=
```
MaximalBy[AllVPBs[2, 4], Length@*Γ̄]
```

Out[•]= $\{\mathrm{VPB}[2, \sigma_{1,2}, \bar{\sigma}_{2,1}, \sigma_{1,2}, \bar{\sigma}_{2,1}], \mathrm{VPB}[2, \bar{\sigma}_{1,2}, \sigma_{2,1}, \bar{\sigma}_{1,2}, \sigma_{2,1}],$
$\mathrm{VPB}[2, \sigma_{2,1}, \bar{\sigma}_{1,2}, \sigma_{2,1}, \bar{\sigma}_{1,2}], \mathrm{VPB}[2, \bar{\sigma}_{2,1}, \sigma_{1,2}, \bar{\sigma}_{2,1}, \sigma_{1,2}]\}$

In[•]:=
```
MaximalBy[AllVPBs[2, 5], Length@*Γ̄]
```

Out[•]= $\{\mathrm{VPB}[2, \sigma_{1,2}, \bar{\sigma}_{2,1}, \sigma_{1,2}, \bar{\sigma}_{2,1}, \sigma_{1,2}], \mathrm{VPB}[2, \bar{\sigma}_{1,2}, \sigma_{2,1}, \bar{\sigma}_{1,2}, \sigma_{2,1}, \bar{\sigma}_{1,2}],$
$\mathrm{VPB}[2, \sigma_{2,1}, \bar{\sigma}_{1,2}, \sigma_{2,1}, \bar{\sigma}_{1,2}, \sigma_{2,1}], \mathrm{VPB}[2, \bar{\sigma}_{2,1}, \sigma_{1,2}, \bar{\sigma}_{2,1}, \sigma_{1,2}, \bar{\sigma}_{2,1}]\}$

(Alt) In[•]:=
```
Table[
 Echo[Length[Γ̄[VPB[2, Sequence @@ Table[If[OddQ[k], σ_{1,2}, σ̄_{2,1}], {k, m}]]]] - 2], {m, 10}]
```

» 1

» 4

» 11

» 28

» 69

» 168

» 407

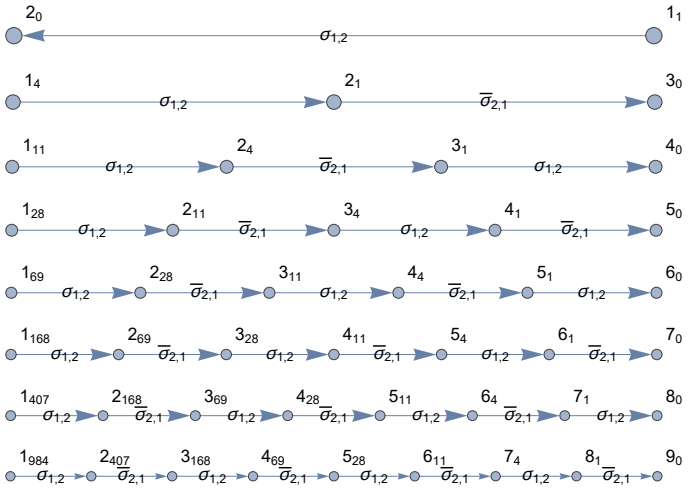
» 984

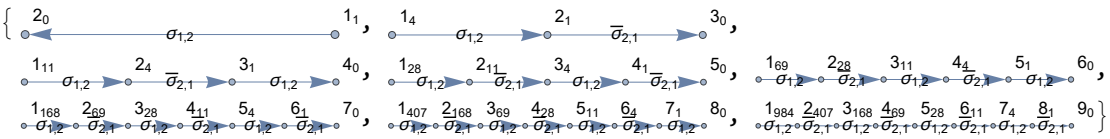
» 2377

» 5740

(Alt) Out[•]= {1, 4, 11, 28, 69, 168, 407, 984, 2377, 5740}

(Alt) In[•]:=
```
Table[Echo[ExtractionGraph[\[OverBar]\[Tau][VPB[2, Sequence @@ Table[If[OddQ[k], σ1,2, \[OverBar]σ2,1], {k, m}]]]],
  GraphLayout → Automatic]], {m, 8}]
```

» 

(Alt) Out[•]= 

(Alt) In[•]:=
```
FindLinearRecurrence[{1, 4, 11, 28, 69, 168, 407, 984, 2377, 5740}]
```

(Alt) Out[•]= {3, −1, −1}

(Alt) In[•]:=
```
Echo[vML[3, #]] & /@ Range[5]
```

» 1

» 4

» 11

» 28

» 69

(Alt) Out[•]= {1, 4, 11, 28, 69}