Pensieve header: An attempt to "mine" all the diamonds within an Extraction Graph. Fails because the notion of a diamond is not entirely well-defined, as in ExtranctionGraph[BR[3,{-1,-2,-2,-2,-1}]].

In[•]:=
```
SetDirectory@"C:\\drorbn\\AcademicPensieve\\Projects\\OU";
<< "OU-Programs.m"
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

In[•]:=
```
MineDiamonds[∅_] := Module[{vd, n, gs, vs, p, m1, m2, g, q, k,
   c, cs (* children *), pds (* pure descendents *), ds, diamonds},
  gs = VPBGenerators[n = Count[vd = Γ̄[∅], _EOS]];
  vs = {vd}; cs = {{}}; p = 0;
  While[p < Length[vs],
   m1 = Count[vd = vs[[++p]], X_[_, _]];
   Do[
    m2 = Count[q = Γ̄[VD[VPB[n, g /. {σ → σ̄, σ̄ → σ}]] ** vd], X_[_, _]];
    If[m2 < m1,
     If[! MemberQ[vs, q], AppendTo[vs, q]; AppendTo[cs, {}]];
     k = Position[vs, q][[1, 1]];
     cs[[p]] = cs[[p]] ⋃ {g → k};
    ],
    {g, gs}
   ]
  ];
  pds = Table[{}, p]; diamonds = {};
  Do[
   ds = SortBy[Last]@Flatten@Table[
       {{c[[1]]} → (c[[2]])} ⋃ ((Prepend[#[[1]], c[[1]]] → (#[[2]])) & /@ pds[[c[[2]]]]),
       {c, cs[[k]]}];
   ds = GatherBy[ds, Last];
   ds = If[Length[#] == 1, #,
       diamonds = Join[diamonds, Prepend[k] /@ Diamond @@@ Subsets[First /@ #, {2}]]; {}
     ] & /@ ds;
   pds[[k]] = Join @@ ds;
   Print[{k, pds[[k]]}],
   {k, OrderingBy[vs, Length]}
  ];
  Sort@diamonds
 ]
```

In[•]:=
```
MineDiamonds[BR[3, {-1, -2, -2, -2, -1}]] // Column
```

```
{16, {}}
{11, {{σ̄3,2} → 16}}
{22, {{σ̄2,1} → 16}}
{7, {{σ̄3,1} → 11, {σ̄3,1, σ̄3,2} → 16}}
{19, {{σ̄3,1, σ̄2,1} → 16, {σ̄3,1} → 22}}
{20, {{σ̄2,3} → 11, {σ̄2,3, σ̄3,2} → 16}}
```

$$\{13,\ \{\{\overline{\sigma}_{2,1}\}\to 7,\ \{\overline{\sigma}_{2,1},\ \overline{\sigma}_{3,1}\}\to 11,\ \{\overline{\sigma}_{3,2}\}\to 19,\ \{\overline{\sigma}_{3,2},\ \overline{\sigma}_{3,1}\}\to 22\}\}$$

$$\{14,\ \{\{\overline{\sigma}_{2,1},\ \overline{\sigma}_{2,3}\}\to 11,\ \{\overline{\sigma}_{2,1},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2}\}\to 16,\ \{\overline{\sigma}_{2,1}\}\to 20\}\}$$

$$\{21,\ \{\{\overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{2,1}\}\to 16,\ \{\overline{\sigma}_{1,3}\}\to 19,\ \{\overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1}\}\to 22\}\}$$

$$\{9,\ \{\{\overline{\sigma}_{2,3},\ \overline{\sigma}_{2,1}\}\to 7,\ \{\overline{\sigma}_{2,3}\}\to 13,\ \{\overline{\sigma}_{3,1}\}\to 14,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{2,1},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2}\}\to 16,\ \{\overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2}\}\to 19,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{2,1}\}\to 20,\ \{\overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{3,1}\}\to 22\}\}$$

$$\{17,\ \{\{\overline{\sigma}_{1,2},\ \overline{\sigma}_{2,1}\}\to 7,\ \{\overline{\sigma}_{1,2},\ \overline{\sigma}_{2,1},\ \overline{\sigma}_{3,1}\}\to 11,\ \{\overline{\sigma}_{1,2}\}\to 13,\ \{\overline{\sigma}_{1,2},\ \overline{\sigma}_{3,2}\}\to 19,\ \{\overline{\sigma}_{1,2},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{3,1}\}\to 22\}\}$$

$$\{18,\ \{\{\overline{\sigma}_{1,2},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{2,1}\}\to 16,\ \{\overline{\sigma}_{1,2},\ \overline{\sigma}_{1,3}\}\to 19,\ \{\overline{\sigma}_{1,2}\}\to 21,\ \{\overline{\sigma}_{1,2},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1}\}\to 22\}\}$$

$$\{12,\ \{\{\overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{2,1}\}\to 7,\ \{\overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{2,1},\ \overline{\sigma}_{3,1}\}\to 11,\ \{\overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2}\}\to 13,\ \{\overline{\sigma}_{3,2},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{2,1}\}\to 16,\ \{\overline{\sigma}_{1,3}\}\to 17,\ \{\overline{\sigma}_{3,2}\}\to 18,\ \{\overline{\sigma}_{3,2},\ \overline{\sigma}_{1,2}\}\to 21\}\}$$

$$\{4,\ \{\{\overline{\sigma}_{1,3}\}\to 7,\ \{\overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1}\}\to 11,\ \{\overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2}\}\to 16\}\}$$

$$\{8,\ \{\{\overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{2,1}\}\to 7,\ \{\overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{2,1},\ \overline{\sigma}_{3,1}\}\to 11,\ \{\overline{\sigma}_{2,3}\}\to 12,\ \{\overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2}\}\to 13,\ \{\overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{2,1}\}\to 16,\ \{\overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3}\}\to 17,\ \{\overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2}\}\to 18,\ \{\overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{1,2}\}\to 21\}\}$$

$$\{5,\ \{\{\sigma_{1,2}\}\to 8,\ \{\overline{\sigma}_{1,3}\}\to 9,\ \{\sigma_{1,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{2,1},\ \overline{\sigma}_{3,1}\}\to 11,\ \{\sigma_{1,2},\ \overline{\sigma}_{2,3}\}\to 12,\ \{\overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1}\}\to 14,\ \{\sigma_{1,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3}\}\to 17,\ \{\sigma_{1,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2}\}\to 18,\ \{\overline{\sigma}_{1,3},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2}\}\to 19,\ \{\overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{2,1}\}\to 20,\ \{\sigma_{1,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{1,2}\}\to 21,\ \{\overline{\sigma}_{1,3},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{3,1}\}\to 22\}\}$$

$$\{15,\ \{\{\overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{2,1}\}\to 7,\ \{\overline{\sigma}_{3,2}\}\to 8,\ \{\overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{2,1},\ \overline{\sigma}_{3,1}\}\to 11,\ \{\overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3}\}\to 12,\ \{\overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2}\}\to 13,\ \{\overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{2,1}\}\to 16,\ \{\overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3}\}\to 17,\ \{\overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2}\}\to 18,\ \{\overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{1,2}\}\to 21\}\}$$

$$\{2,\ \{\{\overline{\sigma}_{3,1}\}\to 4,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{1,3}\}\to 7,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1}\}\to 11,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2}\}\to 16\}\}$$

$$\{10,\ \{\{\overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{2,1}\}\to 7,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2}\}\to 8,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{2,1},\ \overline{\sigma}_{3,1}\}\to 11,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3}\}\to 12,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2}\}\to 13,\ \{\overline{\sigma}_{3,1}\}\to 15,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{2,1}\}\to 16,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3}\}\to 17,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2}\}\to 18,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{1,2}\}\to 21\}\}$$

$$\{6,\ \{\{\sigma_{1,2},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{2,1}\}\to 7,\ \{\sigma_{1,2},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2}\}\to 8,\ \{\sigma_{1,2}\}\to 10,\ \{\sigma_{1,2},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{2,1},\ \overline{\sigma}_{3,1}\}\to 11,\ \{\sigma_{1,2},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3}\}\to 12,\ \{\sigma_{1,2},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2}\}\to 13,\ \{\sigma_{1,2},\ \overline{\sigma}_{3,1}\}\to 15,\ \{\sigma_{1,2},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{2,1}\}\to 16,\ \{\sigma_{1,2},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3}\}\to 17,\ \{\sigma_{1,2},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2}\}\to 18,\ \{\sigma_{1,2},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{1,2}\}\to 21\}\}$$

$$\{3,\ \{\{\overline{\sigma}_{3,1}\}\to 5,\ \{\sigma_{3,2}\}\to 6,\ \{\sigma_{3,2},\ \sigma_{1,2},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{2,1}\}\to 7,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{1,3}\}\to 9,\ \{\sigma_{3,2},\ \sigma_{1,2}\}\to 10,\ \{\sigma_{3,2},\ \sigma_{1,2},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2}\}\to 13,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1}\}\to 14,\ \{\sigma_{3,2},\ \sigma_{1,2},\ \overline{\sigma}_{3,1}\}\to 15,\ \{\sigma_{3,2},\ \sigma_{1,2},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{1,2},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{2,1}\}\to 16,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2}\}\to 19,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{2,1}\}\to 20,\ \{\overline{\sigma}_{3,1},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{3,1}\}\to 22\}\}$$

$$\{1,\ \{\{\overline{\sigma}_{2,1}\}\to 2,\ \{\sigma_{2,3}\}\to 3,\ \{\overline{\sigma}_{2,1},\ \overline{\sigma}_{3,1}\}\to 4,\ \{\sigma_{2,3},\ \overline{\sigma}_{3,1}\}\to 5,\ \{\sigma_{2,3},\ \sigma_{3,2}\}\to 6,\ \{\sigma_{2,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{1,3}\}\to 9,\ \{\sigma_{2,3},\ \sigma_{3,2},\ \sigma_{1,2}\}\to 10,\ \{\overline{\sigma}_{2,1},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1}\}\to 11,\ \{\sigma_{2,3},\ \sigma_{3,2},\ \sigma_{1,2},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{1,2}\}\to 13,\ \{\sigma_{2,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1}\}\to 14,\ \{\sigma_{2,3},\ \sigma_{3,2},\ \sigma_{1,2},\ \overline{\sigma}_{3,1}\}\to 15,\ \{\sigma_{2,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2}\}\to 19,\ \{\sigma_{2,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{2,1}\}\to 20,\ \{\sigma_{2,3},\ \overline{\sigma}_{3,1},\ \overline{\sigma}_{1,3},\ \overline{\sigma}_{2,3},\ \overline{\sigma}_{3,2},\ \overline{\sigma}_{3,1}\}\to 22\}\}$$

$1 \diamond \{\overline{\sigma}_{2,1}, \overline{\sigma}_{3,1}, \overline{\sigma}_{1,3}\} \diamond \{\sigma_{2,3}, \sigma_{3,2}, \sigma_{1,2}, \overline{\sigma}_{3,1}, \overline{\sigma}_{3,2}, \overline{\sigma}_{2,3}, \overline{\sigma}_{1,3}, \overline{\sigma}_{1,2}, \overline{\sigma}_{2,1}\}$

$1 \diamond \{\overline{\sigma}_{2,1}, \overline{\sigma}_{3,1}, \overline{\sigma}_{1,3}, \overline{\sigma}_{3,1}, \overline{\sigma}_{3,2}\} \diamond \{\sigma_{2,3}, \sigma_{3,2}, \sigma_{1,2}, \overline{\sigma}_{3,1}, \overline{\sigma}_{3,2}, \overline{\sigma}_{2,3}, \overline{\sigma}_{3,2}, \overline{\sigma}_{1,2}, \overline{\sigma}_{1,3}, \overline{\sigma}_{3,1}, \overline{\sigma}_{2,1}\}$

$3 \diamond \{\overline{\sigma}_{3,1}, \sigma_{1,2}\} \diamond \{\sigma_{3,2}, \sigma_{1,2}, \overline{\sigma}_{3,1}, \overline{\sigma}_{3,2}\}$

$3 \diamond \{\overline{\sigma}_{3,1}, \sigma_{1,2}, \overline{\sigma}_{2,3}\} \diamond \{\sigma_{3,2}, \sigma_{1,2}, \overline{\sigma}_{3,1}, \overline{\sigma}_{3,2}, \overline{\sigma}_{2,3}\}$

$3 \diamond \{\overline{\sigma}_{3,1}, \sigma_{1,2}, \overline{\sigma}_{2,3}, \overline{\sigma}_{1,3}\} \diamond \{\sigma_{3,2}, \sigma_{1,2}, \overline{\sigma}_{3,1}, \overline{\sigma}_{3,2}, \overline{\sigma}_{2,3}, \overline{\sigma}_{1,3}\}$

$3 \diamond \{\overline{\sigma}_{3,1}, \sigma_{1,2}, \overline{\sigma}_{2,3}, \overline{\sigma}_{3,2}\} \diamond \{\sigma_{3,2}, \sigma_{1,2}, \overline{\sigma}_{3,1}, \overline{\sigma}_{3,2}, \overline{\sigma}_{2,3}, \overline{\sigma}_{3,2}\}$

$3 \diamond \{\overline{\sigma}_{3,1}, \sigma_{1,2}, \overline{\sigma}_{2,3}, \overline{\sigma}_{3,2}, \overline{\sigma}_{1,2}\} \diamond \{\sigma_{3,2}, \sigma_{1,2}, \overline{\sigma}_{3,1}, \overline{\sigma}_{3,2}, \overline{\sigma}_{2,3}, \overline{\sigma}_{3,2}, \overline{\sigma}_{1,2}\}$

Out[•]= $3 \diamond \{\overline{\sigma}_{3,1}, \sigma_{1,2}, \overline{\sigma}_{2,3}, \overline{\sigma}_{1,3}, \overline{\sigma}_{1,2}, \overline{\sigma}_{2,1}, \overline{\sigma}_{3,1}\} \diamond \{\sigma_{3,2}, \sigma_{1,2}, \overline{\sigma}_{3,1}, \overline{\sigma}_{3,2}, \overline{\sigma}_{2,3}, \overline{\sigma}_{1,3}, \overline{\sigma}_{1,2}, \overline{\sigma}_{2,1}, \overline{\sigma}_{3,1}\}$

$5 \diamond \{\overline{\sigma}_{1,3}, \overline{\sigma}_{2,3}\} \diamond \{\sigma_{1,2}, \overline{\sigma}_{2,3}, \overline{\sigma}_{1,3}, \overline{\sigma}_{1,2}\}$

$5 \diamond \{\overline{\sigma}_{1,3}, \overline{\sigma}_{2,3}, \overline{\sigma}_{2,1}\} \diamond \{\sigma_{1,2}, \overline{\sigma}_{2,3}, \overline{\sigma}_{1,3}, \overline{\sigma}_{1,2}, \overline{\sigma}_{2,1}\}$

$5 \diamond \{\overline{\sigma}_{1,3}, \overline{\sigma}_{3,1}, \overline{\sigma}_{2,1}, \overline{\sigma}_{2,3}, \overline{\sigma}_{3,2}\} \diamond \{\sigma_{1,2}, \overline{\sigma}_{2,3}, \overline{\sigma}_{3,2}, \overline{\sigma}_{1,2}, \overline{\sigma}_{1,3}, \overline{\sigma}_{3,1}, \overline{\sigma}_{2,1}\}$

$9 \diamond \{\overline{\sigma}_{2,3}, \overline{\sigma}_{2,1}, \overline{\sigma}_{3,1}\} \diamond \{\overline{\sigma}_{3,1}, \overline{\sigma}_{2,1}, \overline{\sigma}_{2,3}\}$

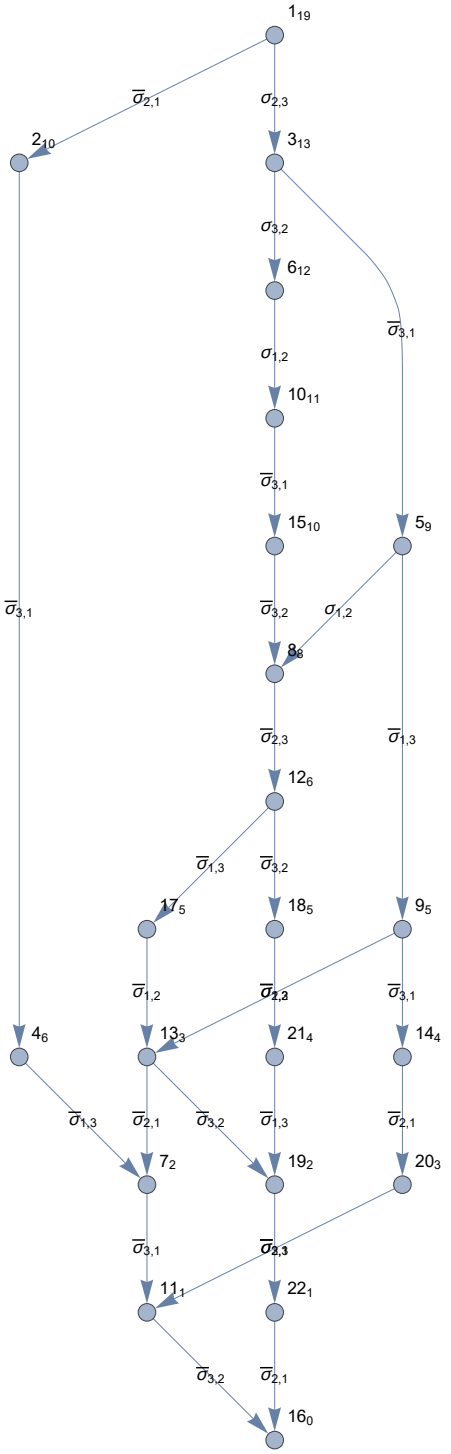
$12 \diamond \{\overline{\sigma}_{1,3}, \overline{\sigma}_{1,2}, \overline{\sigma}_{3,2}\} \diamond \{\overline{\sigma}_{3,2}, \overline{\sigma}_{1,2}, \overline{\sigma}_{1,3}\}$

$12 \diamond \{\overline{\sigma}_{1,3}, \overline{\sigma}_{1,2}, \overline{\sigma}_{3,2}, \overline{\sigma}_{3,1}\} \diamond \{\overline{\sigma}_{3,2}, \overline{\sigma}_{1,2}, \overline{\sigma}_{1,3}, \overline{\sigma}_{3,1}\}$

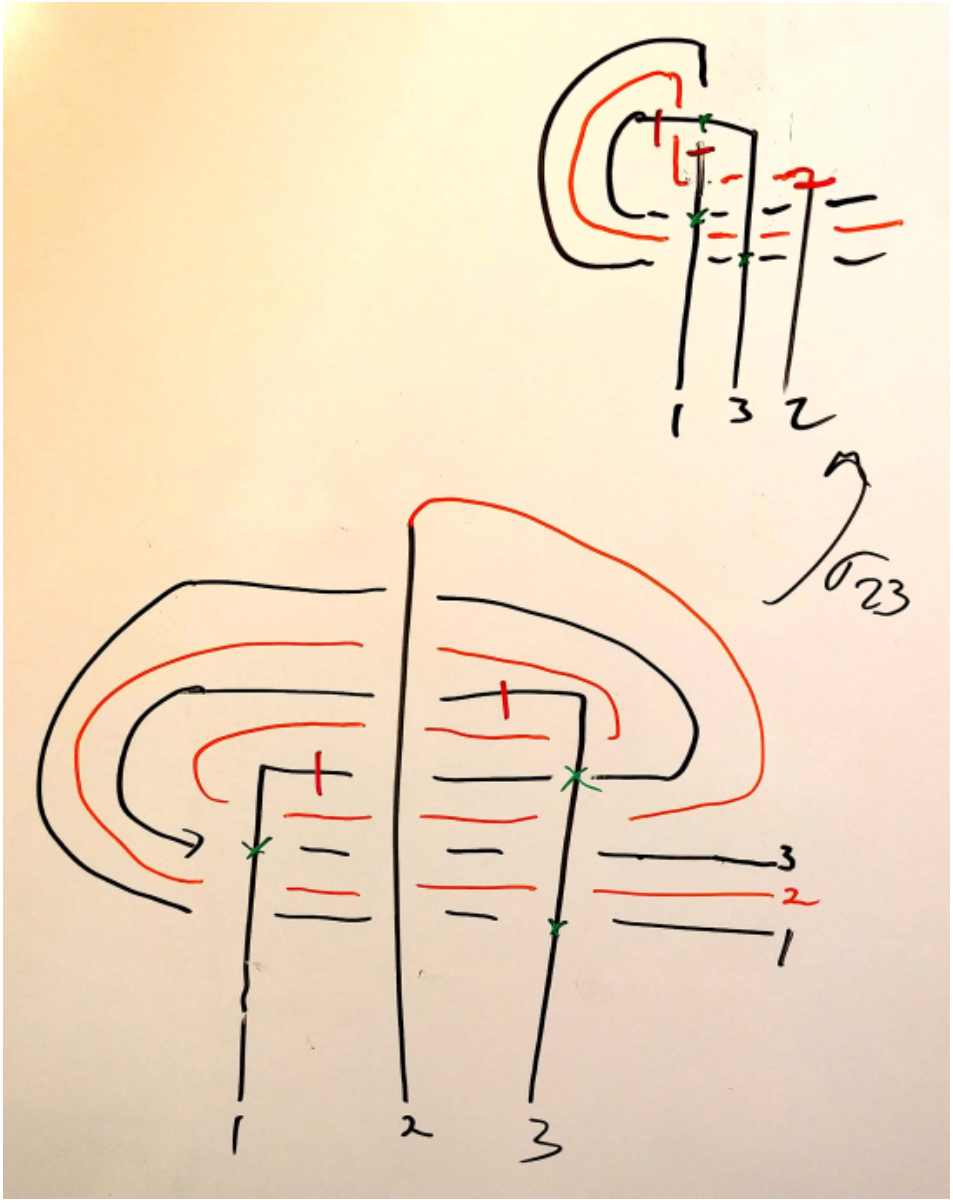
$13 \diamond \{\overline{\sigma}_{2,1}, \overline{\sigma}_{3,1}, \overline{\sigma}_{3,2}\} \diamond \{\overline{\sigma}_{3,2}, \overline{\sigma}_{3,1}, \overline{\sigma}_{2,1}\}$

In[•]:=
```
BR[3, {-1, -2, -2, -2, -1}] // ExtractionGraph
```

Out[•]=



The above OU tangle is probably:

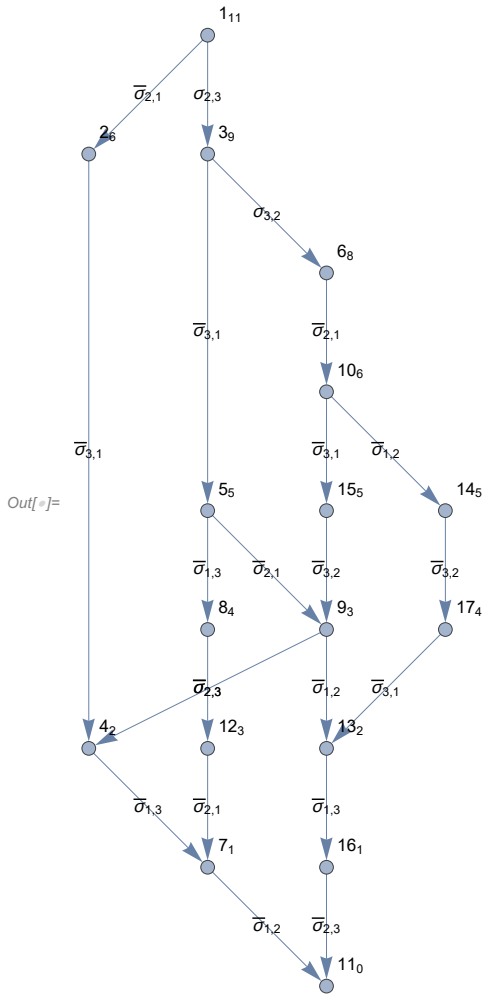


In[•]:= `BR[3, {-1, -2, -2, -2, -1}] // ExtractionGraph // PlanarGraphQ`

Out[•]= False

In[•]:=
```
BR[3, {-1, -2, -2, -1}] // ExtractionGraph
```

Out[•]=

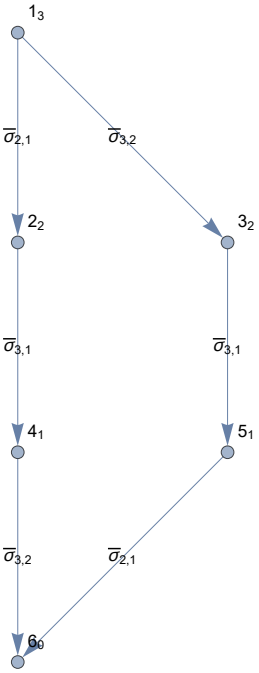


In[•]:=
```
BR[3, {-1, -2, -2, -1}] // ExtractionGraph // PlanarGraphQ
```

Out[•]=
```
False
```

In[•]:=
```
BR[3, {-1, -2, -1}] // ExtractionGraph
```

Out[•]=

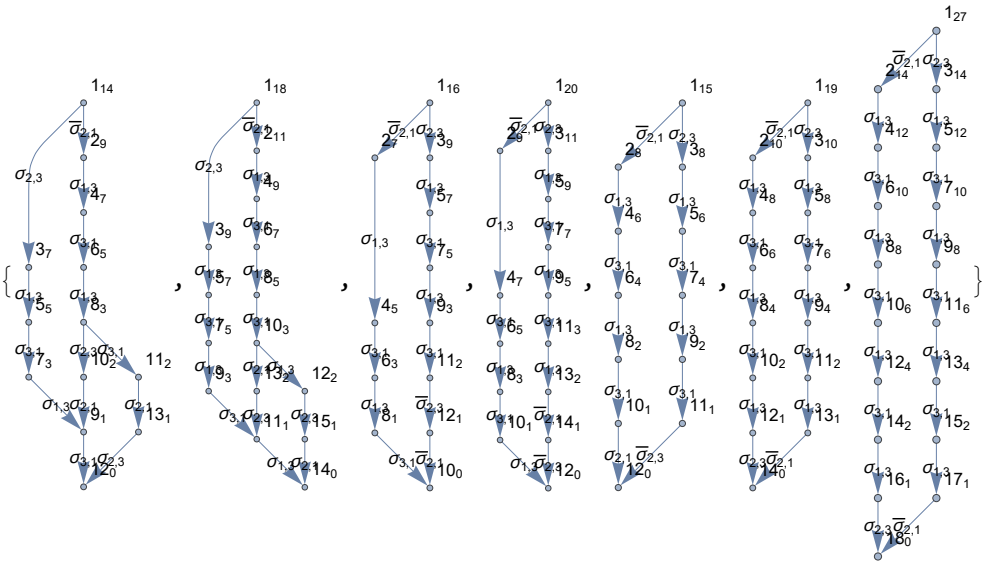


In[•]:=
```
BR[3, {-1, -2, -1}] // ExtractionGraph // PlanarGraphQ
```

Out[•]=
```
True
```

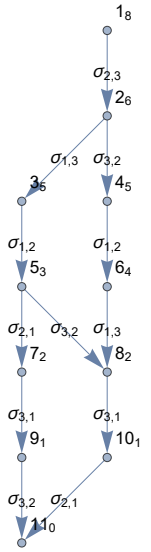
In[•]:=
```
ExtractionGraph /@ {BR[3, {2, 1, 1, 1, 1}], BR[3, {2, 1, 1, 1, 1, 1}],
  BR[3, {-1, 2, 2, 2, 2}], BR[3, {-1, 2, 2, 2, 2, 2}], BR[3, {-1, 2, 2, 2, 2, 1}],
  BR[3, {-1, 2, 2, 2, 2, 2, 1}], BR[3, {2, 1, 1, 1, 1, 1, 1, 1, -2}]}
```

Out[•]=



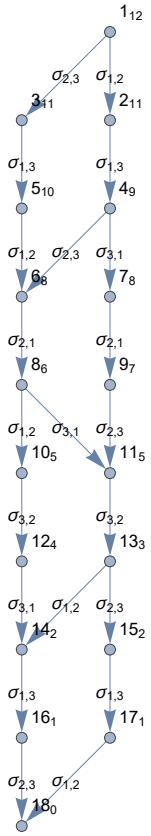
In[•]:= **BR[3, {1, 2, 1, 1, 2, 1}] // ExtractionGraph**

Out[•]=



In[•]:= **BR[3, {1, 2, 1, 1, 2, 1, 1, 2, 1}] // ExtractionGraph**

Out[•]=

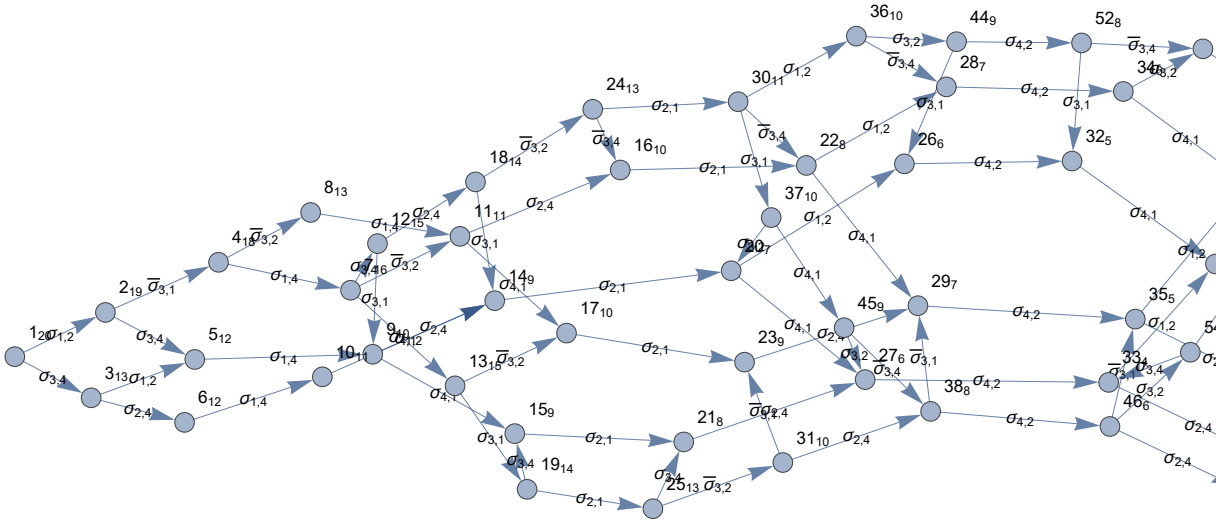


In[•]:= **BR[4, {3, 1, 2, 1, 1, 2, 1, 1, 2, 1, -3}] // ExtractionGraph // PlanarGraphQ**

Out[•]= False

In[•]:= **BR[4, {3, 1, 2, 1, 1, 2, 1, 1, 2, 1, -3}] // ExtractionGraph**

Out[•]=

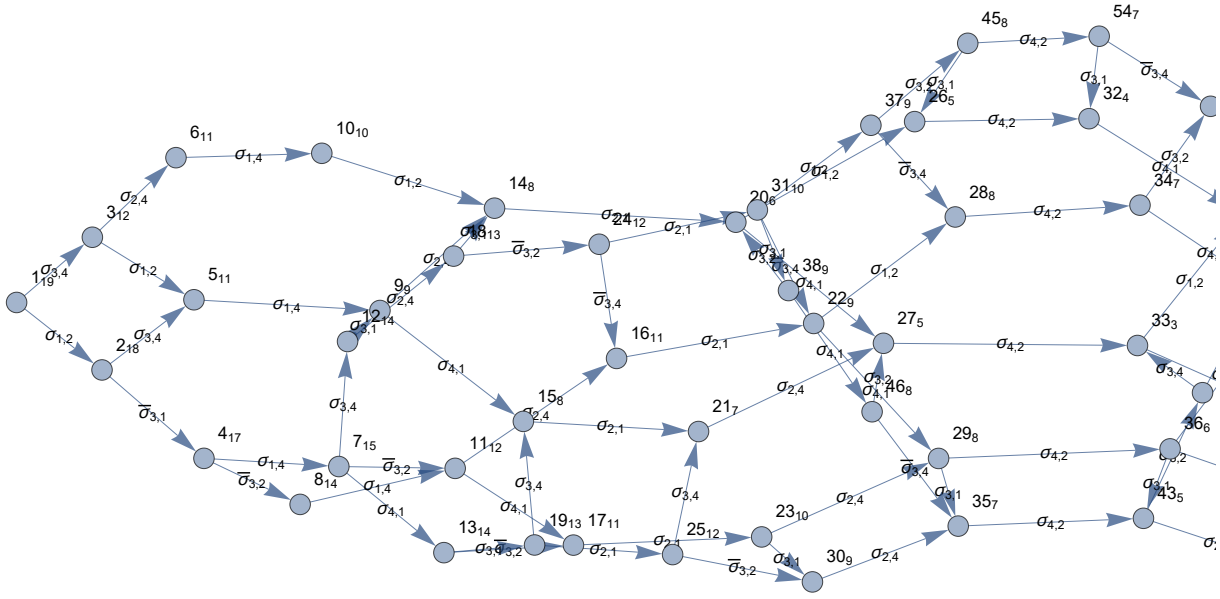


In[•]:= **BR[4, {3, 1, 2, 1, 1, 2, 1, 1, 2, 1}] // ExtractionGraph // PlanarGraphQ**

Out[•]= False

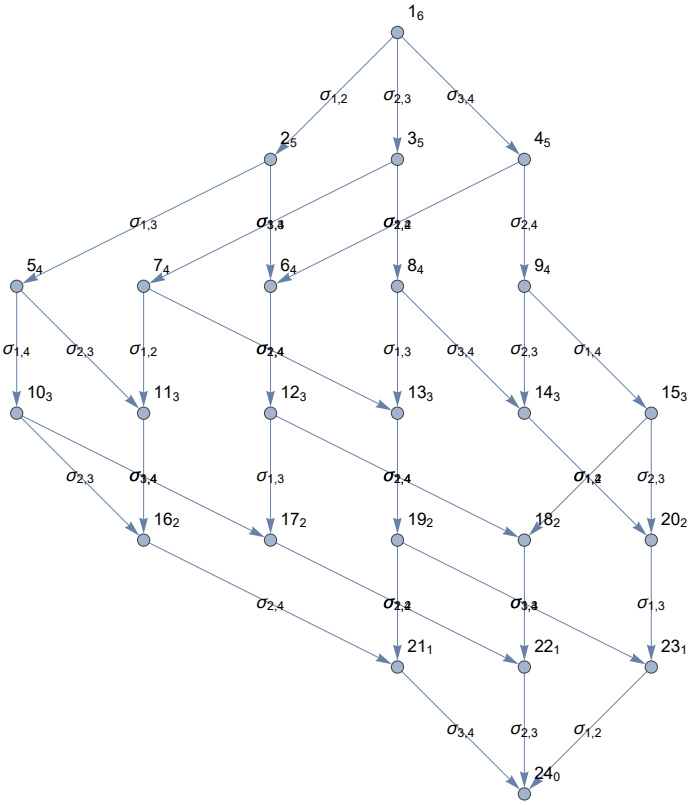
In[•]:= **BR[4, {3, 1, 2, 1, 1, 2, 1, 1, 2, 1}] // ExtractionGraph**

Out[•]=



In[•]:=
```
BR[4, {1, 2, 3, 1, 2, 1}] // ExtractionGraph
```

Out[•]=

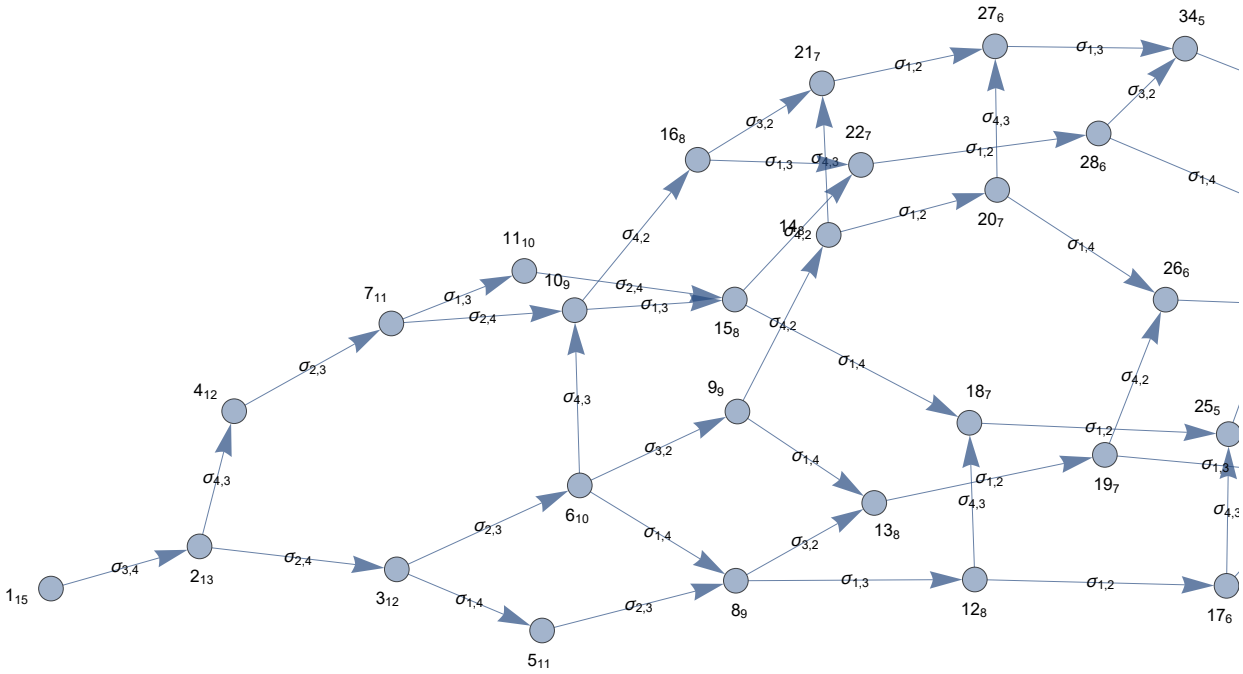


In[•]:=
```
ExtractionGraph[BR[4, {1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1}]] // PlanarGraphQ
```

Out[•]= True

In[•]:=
```
ExtractionGraph[BR[4, {1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1}],
 GraphLayout → "SpringElectricalEmbedding"]
```

Out[•]=

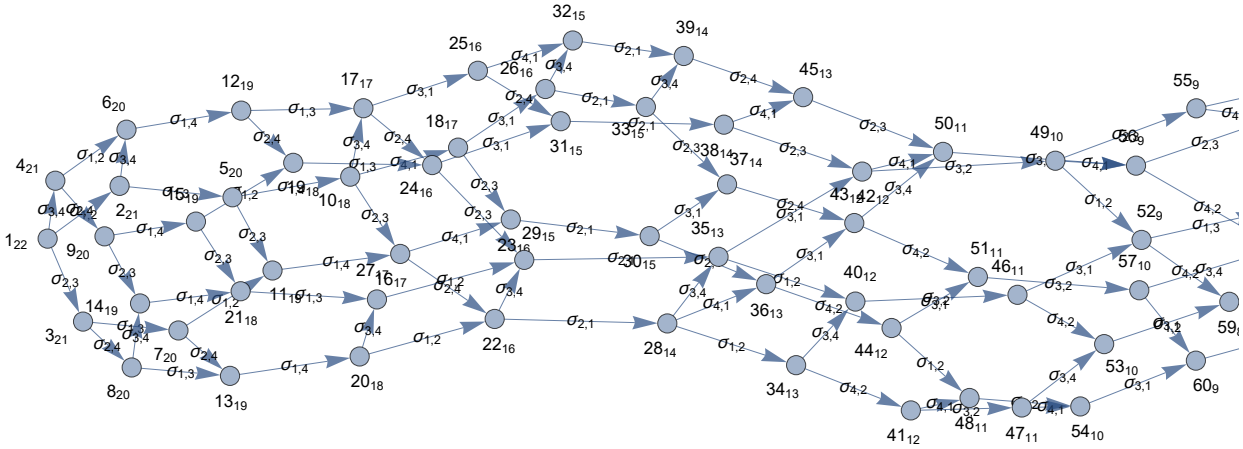


In[•]:=
```
ExtractionGraph[BR[4, {1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1}]] // PlanarGraphQ
```

Out[•]= True

In[•]:=
```
ExtractionGraph[BR[4, {1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1}],
 GraphLayout → "SpringElectricalEmbedding"]
```

Out[•]=

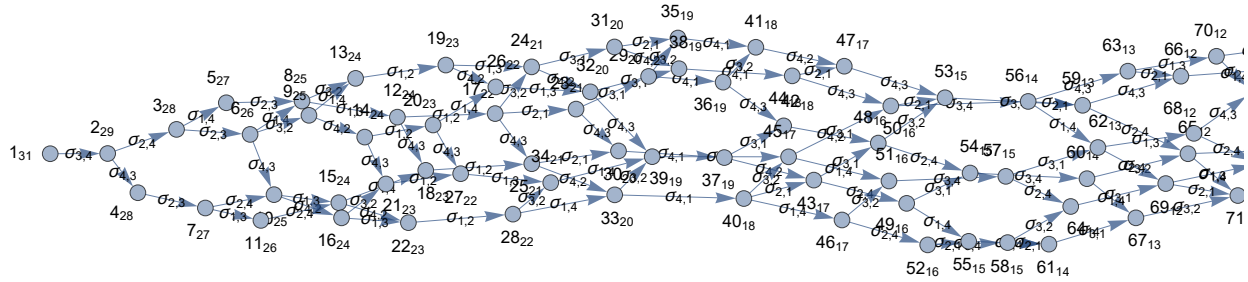


In[•]:=
```
ExtractionGraph[BR[4, {1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1}]] //
 PlanarGraphQ
```

Out[•]= True

```
In[•]:= ExtractionGraph[BR[4, {1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1}],
         GraphLayout → "SpringElectricalEmbedding"]
```

Out[•]=



```
In[•]:= ExtractionGraph[BR[4, {1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2,
            1, 1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1}]] // PlanarGraphQ
```

Out[•]= True

```
In[•]:= ExtractionGraph[
         BR[4, {1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1, 1, 2, 3, 1, 2, 1}],
         GraphLayout → "SpringElectricalEmbedding"]
```

Out[•]=