

Reviewer 1:

“The paper, "OVER THEN UNDER TANGLES" by Bar-Natan/Dancso/van der Veen is a survey of ideas related to oriented tangles which admits an OU-tangle diagram; i.e., all over-crossings before any under-crossings. The authors prove that all OU-tangles are tangle-equivalent to braids (Corollary 3.10) in the classical case. But in the virtual case, they prove that not all OU-tangles are equivalent to virtual pure braids (Corollary 4.4), by using a theorem of Chterental (Theorem 4.1), for which they provide an independent proof. Much of the paper is in the form of interesting remarks (sections 5 and 7 in their entirety) and computations (section 6). Some notation/terminology is distracting (cinnamon rolls, tahini sauce) and should be addressed during detailed refereeing...”

Reviewer 2:

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A key result of the paper is a pictorial characterization of OU tangles (Thm. 3.3) : a tangle diagram is OU if it is « acyclic », that is if « it has no Escher waterfall — that is no closed cascade path can be drawn on it». (Def. 3.1).

The main consequence is that OU tangles necessarily are braids (Cor. 3.11). This is the main result of the paper. The core of the proof is an isomorphism from the braid group to a certain monoid of OU diagrams (Thm. 3.8). This isomorphism is summarized in Figure 6 where it is described as « Stirring a pool of tahini sauce garnished with parsley lines using a braid whisk ». The paper next proceeds with applying similar ideas for re-proving a result of Chterental (Thm. 4.1), which in turn tells us that virtual OU tangles are not necessarily virtual braids.

The authors then gather several comments on these two results in Section 5, give in Section 6 several Mathematica computations of various nature and with various motivations, and conclude in Section 7 with a brief presentation of some related works.

The paper certainly contains technicalities, and it is probably true that some parts (like the computational section 6) may rather appeal to a restricted audience within the ‘knot theory and its ramifications’ community. ..

In principle, I would say that the mathematical content of the paper certainly justifies a full refereeing process.

But I am afraid that I would rather not be a referee myself, mainly because of this ‘objection’ on the writing style (see examples above), which I personally find to be confusing and distracting.

Reviewer 3:

I know the paper and it seems interesting: there are clear perspectives, open questions and possible applications in Lie bialgebras.

My concern is more about the exposition: the paper starts with false theorems and false proofs ("Fheorems" and "Froofs" in the text) and it ends with a pictorial description of Mathematica implementations

and a quite speculative survey on further directions.

I think that to referee the paper is "philosophically" quite complicated: for instance I think that there are shorter "fast false" proofs and that there are simpler arguments to prove that proposed "Theorems" are false...

Reviewer 4:

My main concern is about the relationship with Chterental work. The most interesting in my opinion is how general the strategy is, and the fact that it reformulates nicely the classical case of braids is a very good sign.[The paper does not make the overlap of this work and the work of Chterental]

The writing is very particular and it clearly requires an effort for some readers and the fact that the authors do not make effort to cite for instance the work of Bellingeri, Cisneros de la Cruz and Paris (<https://arxiv.org/pdf/1506.05283.pdf>) who worked also on the word problem for the virtual braid group but definitively with a different language (and more generally the whole litterature on the word problem for braid groups) is a concern.