

```
In[ ]:= SCountOUForms[n_, m_] := Module[{gens},
  gens = Flatten@Table[{σi,j, σ̄i,j}, {i, n}, {j, DeleteCases[Range@n, i]}];
  Length@
  Union@Flatten@Table[R2Reduce@UOFix@VPB[n, Sequence@@p], {p, Tuples[gens, m]}]]
```

```
In[ ]:= SCountOUForms[4, 2]
```

Out[]:= 505

```
In[ ]:= b42 = Flatten@{
  VPB[4],
  Table[VPB[4, st,h],
    {t, 4}, {h, Range[4] ~Complement~ {t}}, {s, {σ, σ̄}}
  ],
  Table[{t2, h2} = {t1, h1}[[o2]]; VPB[4, s1t1,h1, s2t2,h2],
    {t1, 4}, {h1, Range[4] ~Complement~ {t1}},
    {s1, {σ, σ̄}}, {o2, {{1, 2}, {2, 1}}}, {s2, {σ, σ̄}}
  ],
  Table[{t1, h1} = {t, h}[[o1]];
    {t2, h2} = {{t, h}[[w1], Complement[Range[4], {t, h}][[w2]]][[o2]]; VPB[4, s1t1,h1, s2t2,h2],
    {t, 4}, {h, Range[4] ~Complement~ {t}}, {s1, {σ, σ̄}},
    {o1, {{1, 2}, {2, 1}}}, {s2, {σ, σ̄}}, {o2, {{1, 2}, {2, 1}}}, {w1, 2}, {w2, 2}
  ],
  Table[{t1, h1} = {1, e1}[[o1]]; {t2, h2} = DeleteCases[{2, 3, 4}, e1][[o2]];
    VPB[4, s1t1,h1, s2t2,h2],
    {e1, 2, 4}, {s1, {σ, σ̄}}, {o1, {{1, 2}, {2, 1}}}, {s2, {σ, σ̄}}, {o2, {{1, 2}, {2, 1}}}
  ]
}
```

Out[]:=

```
{VPB[4], VPB[4, σ1,2], VPB[4, σ̄1,2], VPB[4, σ1,3], VPB[4, σ̄1,3], VPB[4, σ1,4],
  VPB[4, σ̄1,4], VPB[4, σ2,1], VPB[4, σ̄2,1], VPB[4, σ2,3], VPB[4, σ̄2,3],
  VPB[4, σ2,4], VPB[4, σ̄2,4], VPB[4, σ3,1], VPB[4, σ̄3,1], VPB[4, σ3,2],
  VPB[4, σ̄3,2], VPB[4, σ3,4], VPB[4, σ̄3,4], VPB[4, σ4,1], VPB[4, σ̄4,1],
  VPB[4, σ4,2], VPB[4, σ̄4,2], ... 892 ..., VPB[4, σ̄1,3, σ̄2,4], VPB[4, σ̄1,3, σ̄4,2],
  VPB[4, σ̄3,1, σ2,4], VPB[4, σ̄3,1, σ4,2], VPB[4, σ̄3,1, σ̄2,4], VPB[4, σ̄3,1, σ̄4,2],
  VPB[4, σ1,4, σ2,3], VPB[4, σ1,4, σ3,2], VPB[4, σ1,4, σ̄2,3], VPB[4, σ1,4, σ̄3,2],
  VPB[4, σ4,1, σ2,3], VPB[4, σ4,1, σ3,2], VPB[4, σ4,1, σ̄2,3], VPB[4, σ4,1, σ̄3,2],
  VPB[4, σ̄1,4, σ2,3], VPB[4, σ̄1,4, σ3,2], VPB[4, σ̄1,4, σ̄2,3], VPB[4, σ̄1,4, σ̄3,2],
  VPB[4, σ̄4,1, σ2,3], VPB[4, σ̄4,1, σ3,2], VPB[4, σ̄4,1, σ̄2,3], VPB[4, σ̄4,1, σ̄3,2]}
```

large output show less show more show all set size limit...

In[]:= **v42 = R2Reduce /@ UOFix /@ b42**

Out[]:=

```
{
  VD[EOS[1], EOS[2], EOS[3], EOS[4]], VD[EOS[2], EOS[4], EOS[5], EOS[6], X1[1, 3]],
  VD[EOS[2], EOS[4], EOS[5], EOS[6], X1[1, 3]],
  VD[EOS[2], EOS[3], EOS[5], EOS[6], X1[1, 4]],
  VD[EOS[2], EOS[3], EOS[5], EOS[6], X1[1, 4]],
  VD[EOS[2], EOS[3], EOS[4], EOS[6], X1[1, 5]],
  VD[EOS[2], EOS[3], EOS[4], EOS[6], X1[1, 5]], ... 923 ...
  VD[EOS[2], EOS[4], EOS[6], EOS[8], X1[1, 7], X1[5, 3]],
  VD[EOS[2], EOS[4], EOS[6], EOS[8], X1[1, 7], X1[3, 5]],
  VD[EOS[2], EOS[4], EOS[6], EOS[8], X1[1, 7], X1[5, 3]],
  VD[EOS[2], EOS[4], EOS[6], EOS[8], X1[7, 1], X1[3, 5]],
  VD[EOS[2], EOS[4], EOS[6], EOS[8], X1[7, 1], X1[5, 3]],
  VD[EOS[2], EOS[4], EOS[6], EOS[8], X1[3, 5], X1[7, 1]],
  VD[EOS[2], EOS[4], EOS[6], EOS[8], X1[5, 3], X1[7, 1]]
}
```

large output show less show more show all set size limit..

In[]:= **Length@Union@v42**

Out[]:= 529

In[]:= **With**[{m2 = 2, n = 4},
gens = Flatten@Table[{ $\sigma_{i,j}$, $\bar{\sigma}_{i,j}$ }, {i, n}, {j, DeleteCases[Range@n, i]}];
Table[{i, j, k, l} = ijkl[[perm]]; {
 T[VPB[n, Join[p, { $\sigma_{i,j}$, $\sigma_{k,1}$ }, q]], VPB[n, Join[p, { $\sigma_{k,1}$, $\sigma_{i,j}$ }, q]]],
 T[VPB[n, Join[p, { $\bar{\sigma}_{i,j}$, $\sigma_{k,1}$ }, q]], VPB[n, Join[p, { $\sigma_{k,1}$, $\bar{\sigma}_{i,j}$ }, q]]],
 T[VPB[n, Join[p, { $\sigma_{i,j}$, $\bar{\sigma}_{k,1}$ }, q]], VPB[n, Join[p, { $\bar{\sigma}_{k,1}$, $\sigma_{i,j}$ }, q]]],
 T[VPB[n, Join[p, { $\bar{\sigma}_{i,j}$, $\bar{\sigma}_{k,1}$ }, q]], VPB[n, Join[p, { $\bar{\sigma}_{k,1}$, $\bar{\sigma}_{i,j}$ }, q]]]
 },
 {s, 0, m2 - 2}, {t, 0, s}, {p, Tuples[gens, t]}, {q, Tuples[gens, s - t]},
 {ijkl, Subsets[Range[n], {4}]}, {perm, {{1, 2, 3, 4}, {1, 3, 2, 4}, {1, 4, 2, 3}}}
]]

Out[]:=

```
{
  {
    {
      {
        {
          T[VPB[4,  $\sigma_{1,2}$ ,  $\sigma_{3,4}$ ], VPB[4,  $\sigma_{3,4}$ ,  $\sigma_{1,2}$ ]], T[VPB[4,  $\bar{\sigma}_{1,2}$ ,  $\sigma_{3,4}$ ], VPB[4,  $\sigma_{3,4}$ ,  $\bar{\sigma}_{1,2}$ ]],
          T[VPB[4,  $\sigma_{1,2}$ ,  $\bar{\sigma}_{3,4}$ ], VPB[4,  $\bar{\sigma}_{3,4}$ ,  $\sigma_{1,2}$ ]], T[VPB[4,  $\bar{\sigma}_{1,2}$ ,  $\bar{\sigma}_{3,4}$ ], VPB[4,  $\bar{\sigma}_{3,4}$ ,  $\bar{\sigma}_{1,2}$ ]]
        },
        {
          T[VPB[4,  $\sigma_{1,3}$ ,  $\sigma_{2,4}$ ], VPB[4,  $\sigma_{2,4}$ ,  $\sigma_{1,3}$ ]], T[VPB[4,  $\bar{\sigma}_{1,3}$ ,  $\sigma_{2,4}$ ], VPB[4,  $\sigma_{2,4}$ ,  $\bar{\sigma}_{1,3}$ ]],
          T[VPB[4,  $\sigma_{1,3}$ ,  $\bar{\sigma}_{2,4}$ ], VPB[4,  $\bar{\sigma}_{2,4}$ ,  $\sigma_{1,3}$ ]], T[VPB[4,  $\bar{\sigma}_{1,3}$ ,  $\bar{\sigma}_{2,4}$ ], VPB[4,  $\bar{\sigma}_{2,4}$ ,  $\bar{\sigma}_{1,3}$ ]]
        },
        {
          T[VPB[4,  $\sigma_{1,4}$ ,  $\sigma_{2,3}$ ], VPB[4,  $\sigma_{2,3}$ ,  $\sigma_{1,4}$ ]], T[VPB[4,  $\bar{\sigma}_{1,4}$ ,  $\sigma_{2,3}$ ], VPB[4,  $\sigma_{2,3}$ ,  $\bar{\sigma}_{1,4}$ ]], T[
            VPB[4,  $\sigma_{1,4}$ ,  $\bar{\sigma}_{2,3}$ ], VPB[4,  $\bar{\sigma}_{2,3}$ ,  $\sigma_{1,4}$ ]], T[VPB[4,  $\bar{\sigma}_{1,4}$ ,  $\bar{\sigma}_{2,3}$ ], VPB[4,  $\bar{\sigma}_{2,3}$ ,  $\bar{\sigma}_{1,4}$ ]]
          }
        }
      }
    }
  }
}
```