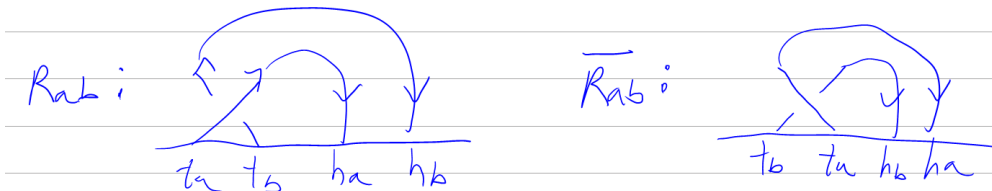


Pensieve header: Unitarity for Γ -calculus.

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\MetaCalculi"];
Once[<< KnotTheory`]
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at <http://katlas.org/wiki/KnotTheory>.



```
In[*]:= R_{a,b}_ := \Gamma @ < | \varsigma \to \{t_a, t_b, h_a, h_b\}, \omega \to 1, \sigma \to h_a + h_b T_a, \lambda \to \{t_a, t_b\} \cdot \begin{pmatrix} T_a & 1 - T_a \\ 0 & 1 \end{pmatrix} \cdot \{h_a, h_b\} | >;
\bar{R}_{a,b}_ := \Gamma @ < | \varsigma \to \{t_b, t_a, h_b, h_a\}, \omega \to 1, \sigma \to h_a + h_b T_a^{-1}, \lambda \to \{t_a, t_b\} \cdot \begin{pmatrix} T_a^{-1} & 1 - T_a^{-1} \\ 0 & 1 \end{pmatrix} \cdot \{h_a, h_b\} | >;
```

```
In[*]:= \bar{R}_{1,2}
Out[*]=
```

$$\Gamma \left[\left\langle \left| \varsigma \to \{t_2, t_1, h_2, h_1\}, \omega \to 1, \sigma \to h_1 + \frac{h_2}{T_1}, \lambda \to h_2 \left(t_2 + t_1 \left(1 - \frac{1}{T_1} \right) \right) + \frac{h_1 t_1}{T_1} \right| \right\rangle \right]$$

```
In[*]:= \Gamma[\alpha_][x_] := \alpha[x];
\Gamma[\alpha_][S] := Union@Cases[\alpha[\varsigma], t_a_ \to a, \infty];
\Gamma[\alpha_][n] := Length[\Gamma[\alpha_][S]];
\Gamma[\alpha_][\Sigma] := (\partial_{h_#} \alpha[\sigma]) & /@ \Gamma[\alpha_][S];
\Gamma[\alpha_][A] := Outer[Factor[\partial_{#1, #2} \Gamma[\alpha_][\lambda]] &, Cases[\alpha[\varsigma], t_], Cases[\alpha[\varsigma], h_]]];
```

```
In[*]:= Table[\{\gamma @ \varsigma, \gamma @ \omega, \gamma @ \sigma, \gamma @ \lambda, \gamma @ S, \gamma @ \Sigma, \gamma @ A // MatrixForm\}, \{\gamma, \{R_{1,2}, \bar{R}_{1,2}\}\} // Transpose // MatrixForm
```

```
Out[*]//MatrixForm=
```

$$\begin{pmatrix} \begin{matrix} \{t_1, t_2, h_1, h_2\} \\ 1 \end{matrix} & \begin{matrix} \{t_2, t_1, h_2, h_1\} \\ 1 \end{matrix} \\ h_1 + h_2 T_1 & h_1 + \frac{h_2}{T_1} \\ h_2 (t_2 + t_1 (1 - T_1)) + h_1 t_1 T_1 & h_2 \left(t_2 + t_1 \left(1 - \frac{1}{T_1} \right) \right) + \frac{h_1 t_1}{T_1} \\ \{1, 2\} & \{1, 2\} \\ \{1, T_1\} & \left\{1, \frac{1}{T_1}\right\} \\ \begin{pmatrix} T_1 & 1 - T_1 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ -\frac{1+T_1}{T_1} & \frac{1}{T_1} \end{pmatrix} \end{pmatrix}$$

```
In[*]:= \GammaCollect[\gamma_I] := \Gamma @ < | \varsigma \to \gamma @ \varsigma, \omega \to Factor @ \gamma @ \omega, \sigma \to Expand @ \gamma @ \sigma,
\lambda \to Total[CoefficientRules[\gamma @ \lambda, \gamma @ \varsigma] /. (ps_ \to c_) \to Factor[c] Times @@ (\gamma @ \varsigma)^{ps}] | >
```

In[*]:= **RCollect**[$\bar{R}_{1,2}$]

Out[*]=

$$\Gamma \left[\left\langle \left| \zeta \rightarrow \{t_2, t_1, h_2, h_1\}, \omega \rightarrow 1, \sigma \rightarrow h_1 + \frac{h_2}{T_1}, \lambda \rightarrow h_2 t_2 + \frac{h_1 t_1}{T_1} + \frac{h_2 t_1 (-1 + T_1)}{T_1} \right| \right\rangle \right]$$

In[*]:= **ConservativeQ**[γ_I] := **Simplify**[**Expand**[$\gamma @ \lambda /. h_ \rightarrow 1$] == **Sum**[t_a , { a , $\gamma @ S$ }]]

In[*]:= **ConservativeQ**[$R_{1,2}$]

Out[*]=

True

In[*]:= $T^* \wedge := T^{-1}$;

vpn $_{\zeta}$ [$c_$, $d_$] := **Expand@Module**[{ e , f }, (* Vortex Pairing North *)

Expand[c ($d /. \{T \rightarrow T^*, t_{i_} \Rightarrow t_{i^*}, h_{i_} \Rightarrow h_{i^*}\}$)] /.

{ $t_{i^*} t_{i_} \rightarrow 0$, $h_{i^*} h_{i_} \rightarrow T - T^*$, ($f : t | h$) $_{j_}^* (e : t | h)_{i_} \Rightarrow$

If[**Position**[ζ , e_i][[1, 1]] < **Position**[ζ , f_j][[1, 1], $T - 1$, $1 - T^*$]]

];

vps $_{\zeta}$ [$c_$, $d_$] :=

Expand@Module[{ e , f }, (* Vortex Pairing South (rel. to north, $T \rightarrow T^*$ and $t \leftrightarrow h$) *)

Expand[c ($d /. \{T \rightarrow T^*, t_{i_} \Rightarrow t_{i^*}, h_{i_} \Rightarrow h_{i^*}\}$)] /.

{ $t_{i^*} t_{i_} \rightarrow T^* - T$, $h_{i^*} h_{i_} \rightarrow 0$, ($f : t | h$) $_{j_}^* (e : t | h)_{i_} \Rightarrow$

If[**Position**[ζ , e_i][[1, 1]] < **Position**[ζ , f_j][[1, 1], $T^* - 1$, $1 - T$]]

];

UnitaryQ[γ_I] := **Module**[{ vs },

$vs =$ **Table**[- $t_i + \partial_{t_i} \gamma @ \lambda /. T_ \rightarrow T$, { i , $\gamma @ S$ }];

And@@Flatten@Table[

Simplify[**vpn** $_{\gamma @ \zeta}$ [$vs[[i]]$, $vs[[j]]$] == 0 == **vps** $_{\gamma @ \zeta}$ [$vs[[i]]$, $vs[[j]]$]], { i , $\gamma @ n$ }, { j , $\gamma @ n$ }]

];

In[*]:= **UnitaryQ** /@ { $R_{1,2}$, $\bar{R}_{1,2}$ }

Out[*]=

{True, True}

```
In[*]:=  $\Gamma$ Form[ $\gamma_{-}\Gamma$ ] := Module[{M},
  M =  $\gamma$ [A] // Transpose;
  PrependTo[M, t_# & /@  $\gamma$ [S]];
  M = Join[
    {Prepend[h_# & /@  $\gamma$ [S],  $\gamma$ [ $\omega$ ]},
    Transpose[M],
    {Prepend[ $\gamma$ [ $\Sigma$ ], If[TrueQ[ConservativeQ@ $\gamma$  & UnitaryQ@ $\gamma$ ],  $\blacksquare$ ,  $\color{red}\blacksquare$ ]}
  ];
  Column[{ $\gamma$ [c], MatrixForm[M]}]
];
 $\Gamma$ Form[else_] := else /.  $\gamma_{-}\Gamma$  :->  $\Gamma$ Form[ $\gamma$ ];
Format[ $\gamma_{-}\Gamma$ , StandardForm] :=  $\Gamma$ Form[ $\gamma$ ];
```

```
In[*]:= {R1,2,  $\bar{R}$ 1,2}
```

Out[*]=

$$\left\{ \begin{matrix} \{t_1, t_2, h_1, h_2\} \\ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & T_1 & 1 - T_1 \\ t_2 & 0 & 1 \\ \blacksquare & 1 & T_1 \end{pmatrix} \end{matrix}, \begin{matrix} \{t_2, t_1, h_2, h_1\} \\ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & 1 & 0 \\ t_2 & \frac{-1+T_1}{T_1} & \frac{1}{T_1} \\ \blacksquare & 1 & \frac{1}{T_1} \end{pmatrix} \end{matrix} \right\}$$

```
In[*]:= dmij→kj[ $\gamma_{-}\Gamma$ ] := Module[{a, b, c, d,  $\theta$ ,  $\epsilon$ ,  $\phi$ ,  $\psi$ ,  $\Sigma$ ,  $\mu$ },
   $\begin{pmatrix} a & b & \theta \\ c & d & \epsilon \\ \phi & \psi & \Sigma \end{pmatrix} = \begin{pmatrix} \partial_{t_i, h_i} \gamma @ \lambda & \partial_{t_i, h_j} \gamma @ \lambda & \partial_{t_i} \gamma @ \lambda \\ \partial_{t_j, h_i} \gamma @ \lambda & \partial_{t_j, h_j} \gamma @ \lambda & \partial_{t_j} \gamma @ \lambda \\ \partial_{h_i} \gamma @ \lambda & \partial_{h_j} \gamma @ \lambda & \gamma @ \lambda \end{pmatrix} /. (t | h)_{i|j} \rightarrow \theta$ ;
   $\Gamma$ Collect[ $\Gamma$ [<|
    c → DeleteCases[ $\gamma$ @c, hi | tj] /. {ti → tk, hj → hk},
     $\omega$  → ( $\mu = 1 - c$ )  $\gamma$ @ $\omega$ ,
     $\sigma$  → hk ( $\partial_{h_i} \sigma$ ) ( $\partial_{h_j} \sigma$ ) + ( $\sigma$  /. hi|j →  $\theta$ ),
     $\lambda$  → {tk, 1} .  $\begin{pmatrix} b + a d / \mu & \theta + a \epsilon / \mu \\ \psi + d \phi / \mu & \Sigma + \epsilon \phi / \mu \end{pmatrix}$  . {hk, 1}
  ]> /. {Ti → Tk, Tj → Tk}
];
```

```
In[*]:= {R1,2 // dm1,2→1, R1,2 // dm2,1→1,  $\bar{R}$ 1,2 // dm1,2→1,  $\bar{R}$ 1,2 // dm2,1→1}
```

Out[*]=

$$\left\{ \begin{matrix} \{t_1, h_1\} \\ \begin{pmatrix} 1 & h_1 \\ t_1 & 1 \\ \blacksquare & 0 \end{pmatrix} \end{matrix}, \begin{matrix} \{t_1, h_1\} \\ \begin{pmatrix} T_1 & h_1 \\ t_1 & 1 \\ \blacksquare & 0 \end{pmatrix} \end{matrix}, \begin{matrix} \{t_1, h_1\} \\ \begin{pmatrix} 1 & h_1 \\ t_1 & 1 \\ \blacksquare & 0 \end{pmatrix} \end{matrix}, \begin{matrix} \{t_1, h_1\} \\ \begin{pmatrix} \frac{1}{T_1} & h_1 \\ t_1 & 1 \\ \blacksquare & 0 \end{pmatrix} \end{matrix} \right\}$$

In[]:= With[{n = 3},

$$\gamma\theta = \Gamma@<|\zeta \rightarrow \text{Flatten@Table}[\{\mathbf{t}_a, \mathbf{h}_a\}, \{\mathbf{a}, \mathbf{n}\}], \omega \rightarrow \omega, \sigma \rightarrow \sum_{a=1}^n \mathbf{h}_a \sigma_a, \lambda \rightarrow \sum_{a=1}^n \sum_{b=1}^n \mathbf{t}_a \mathbf{h}_b \alpha_{ab} |>$$

$\gamma\theta$ // dm_{1,2→1}

Out[]:=

$$\{\mathbf{t}_1, \mathbf{h}_1, \mathbf{t}_2, \mathbf{h}_2, \mathbf{t}_3, \mathbf{h}_3\}$$

$$\begin{pmatrix} \omega & \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 \\ \mathbf{t}_1 & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \mathbf{t}_2 & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \mathbf{t}_3 & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \\ \blacksquare & \sigma_1 & \sigma_2 & \sigma_3 \end{pmatrix}$$

Out[]:=

$$\{\mathbf{t}_1, \mathbf{h}_1, \mathbf{t}_3, \mathbf{h}_3\}$$

$$\begin{pmatrix} -\omega (-1 + \alpha_{2,1}) & \mathbf{h}_1 & \mathbf{h}_3 \\ \mathbf{t}_1 & \frac{-\alpha_{1,2} + \alpha_{1,2} \alpha_{2,1} - \alpha_{1,1} \alpha_{2,2}}{-1 + \alpha_{2,1}} & \frac{-\alpha_{1,3} + \alpha_{1,3} \alpha_{2,1} - \alpha_{1,1} \alpha_{2,3}}{-1 + \alpha_{2,1}} \\ \mathbf{t}_3 & \frac{-\alpha_{2,2} \alpha_{3,1} - \alpha_{3,2} + \alpha_{2,1} \alpha_{3,2}}{-1 + \alpha_{2,1}} & \frac{-\alpha_{2,3} \alpha_{3,1} - \alpha_{3,3} + \alpha_{2,1} \alpha_{3,3}}{-1 + \alpha_{2,1}} \\ \blacksquare & \emptyset & \emptyset \end{pmatrix}$$

In[]:= With[{n = 4},

$$\gamma\theta = \Gamma@<|\zeta \rightarrow \text{Flatten@Table}[\{\mathbf{t}_a, \mathbf{h}_a\}, \{\mathbf{a}, \mathbf{n}\}], \omega \rightarrow \omega, \sigma \rightarrow \sum_{a=1}^n \mathbf{h}_a \sigma_a, \lambda \rightarrow \sum_{a=1}^n \sum_{b=1}^n \mathbf{t}_a \mathbf{h}_b \alpha_{ab} |>$$

Out[]:=

$$\{\mathbf{t}_1, \mathbf{h}_1, \mathbf{t}_2, \mathbf{h}_2, \mathbf{t}_3, \mathbf{h}_3, \mathbf{t}_4, \mathbf{h}_4\}$$

$$\begin{pmatrix} \omega & \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 & \mathbf{h}_4 \\ \mathbf{t}_1 & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{1,4} \\ \mathbf{t}_2 & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} & \alpha_{2,4} \\ \mathbf{t}_3 & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} & \alpha_{3,4} \\ \mathbf{t}_4 & \alpha_{4,1} & \alpha_{4,2} & \alpha_{4,3} & \alpha_{4,4} \\ \blacksquare & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 \end{pmatrix}$$

In[]:= lhs = $\gamma\theta$ // dm_{1,2→1} // dm_{1,3→1} // Γ Collect

Out[]:=

$$\{\mathbf{t}_1, \mathbf{h}_1, \mathbf{t}_4, \mathbf{h}_4\}$$

$$\begin{pmatrix} \omega (1 - \alpha_{2,1} - \alpha_{2,2} \alpha_{3,1} - \alpha_{3,2} + \alpha_{2,1} \alpha_{3,2}) & \mathbf{h}_1 \\ \mathbf{t}_1 & \frac{\alpha_{1,3} - \alpha_{1,3} \alpha_{2,1} + \alpha_{1,1} \alpha_{2,3} - \alpha_{1,3} \alpha_{2,2} \alpha_{3,1} + \alpha_{1,2} \alpha_{2,3} \alpha_{3,1} - \alpha_{1,3} \alpha_{3,2} + \alpha_{1,3} \alpha_{2,1} \alpha_{3,2} - \alpha_{1,1} \alpha_{2,3} \alpha_{3,2} + \alpha_{1,2} \alpha_{3,2}}{1 - \alpha_{2,1} - \alpha_{2,2} \alpha_{3,1} - \alpha_{3,2} + \alpha_{2,1} \alpha_{3,2}} \\ \mathbf{t}_4 & \frac{\alpha_{2,3} \alpha_{4,1} - \alpha_{2,3} \alpha_{3,2} \alpha_{4,1} + \alpha_{2,2} \alpha_{3,3} \alpha_{4,1} + \alpha_{2,3} \alpha_{3,1} \alpha_{4,2} + \alpha_{3,3} \alpha_{4,2} - \alpha_{2,1} \alpha_{3,3} \alpha_{4,2} + \alpha_{4,3} - \alpha_{2,1} \alpha_{4,3} - \alpha_{2,2} \alpha_{3,3} \alpha_{4,3}}{1 - \alpha_{2,1} - \alpha_{2,2} \alpha_{3,1} - \alpha_{3,2} + \alpha_{2,1} \alpha_{3,2}} \\ \blacksquare & \emptyset \end{pmatrix}$$

In[*]:= rhs = $\gamma\theta$ // dm_{2,3→2} // dm_{1,2→1} // Γ Collect

Out[*]=

$$\begin{pmatrix} \{t_1, h_1, t_4, h_4\} \\ \omega (1 - \alpha_{2,1} - \alpha_{2,2} \alpha_{3,1} - \alpha_{3,2} + \alpha_{2,1} \alpha_{3,2}) \\ t_1 \\ t_4 \\ \blacksquare \end{pmatrix} \begin{matrix} h_1 \\ \frac{\alpha_{1,3} - \alpha_{1,3} \alpha_{2,1} + \alpha_{1,1} \alpha_{2,3} - \alpha_{1,3} \alpha_{2,2} \alpha_{3,1} + \alpha_{1,2} \alpha_{2,3} \alpha_{3,1} - \alpha_{1,3} \alpha_{3,2} + \alpha_{1,1} \alpha_{2,1} \alpha_{3,2} - \alpha_{1,1} \alpha_{2,3} \alpha_{3,2} + \alpha_{1,1} \alpha_{2,3} \alpha_{3,2}}{1 - \alpha_{2,1} - \alpha_{2,2} \alpha_{3,1} - \alpha_{3,2} + \alpha_{2,1} \alpha_{3,2}} \\ \frac{\alpha_{2,3} \alpha_{4,1} - \alpha_{2,3} \alpha_{3,2} \alpha_{4,1} + \alpha_{2,2} \alpha_{3,3} \alpha_{4,1} + \alpha_{2,3} \alpha_{4,1} + \alpha_{2,3} \alpha_{3,1} \alpha_{4,2} + \alpha_{3,3} \alpha_{4,2} - \alpha_{2,1} \alpha_{3,3} \alpha_{4,2} + \alpha_{4,3} - \alpha_{2,1} \alpha_{4,3} - \alpha_{2,1} \alpha_{4,3}}{1 - \alpha_{2,1} - \alpha_{2,2} \alpha_{3,1} - \alpha_{3,2} + \alpha_{2,1} \alpha_{3,2}} \\ 0 \end{matrix}$$

In[*]:= lhs@ λ == rhs@ λ

Out[*]=

True

In[*]:= Γ /: RotateLeft[Γ [α], n___] := Γ @ReplacePart[α , Key@ ζ → RotateLeft[α @ ζ , n]]

In[*]:= {Table[RotateLeft[R_{1,2}, k], {k, 0, 3}], Table[RotateLeft[R̄_{1,2}, k], {k, 0, 3}]}

Out[*]=

$$\left\{ \left\{ \begin{pmatrix} t_1 & t_2 & h_1 & h_2 \\ 1 & h_1 & h_2 \\ t_1 & T_1 & 1 - T_1 \\ t_2 & 0 & 1 \\ \blacksquare & 1 & T_1 \end{pmatrix}, \begin{pmatrix} t_2 & h_1 & h_2 & t_1 \\ 1 & h_1 & h_2 \\ t_1 & 0 & 1 \\ t_2 & T_1 & 1 - T_1 \\ \blacksquare & 1 & T_1 \end{pmatrix}, \begin{pmatrix} h_1 & h_2 & t_1 & t_2 \\ 1 & h_1 & h_2 \\ t_1 & T_1 & 1 - T_1 \\ t_2 & 0 & 1 \\ \blacksquare & 1 & T_1 \end{pmatrix}, \begin{pmatrix} h_2 & t_1 & t_2 & h_1 \\ 1 & h_1 & h_2 \\ t_1 & 1 - T_1 & T_1 \\ t_2 & 1 & 0 \\ \blacksquare & 1 & T_1 \end{pmatrix} \right\}, \left\{ \begin{pmatrix} t_2 & t_1 & h_2 & h_1 \\ 1 & h_1 & h_2 \\ t_1 & 1 & 0 \\ t_2 & \frac{-1+T_1}{T_1} & \frac{1}{T_1} \\ \blacksquare & 1 & \frac{1}{T_1} \end{pmatrix}, \begin{pmatrix} t_1 & h_2 & h_1 & t_2 \\ 1 & h_1 & h_2 \\ t_1 & \frac{-1+T_1}{T_1} & \frac{1}{T_1} \\ t_2 & 1 & 0 \\ \blacksquare & 1 & \frac{1}{T_1} \end{pmatrix}, \begin{pmatrix} h_2 & h_1 & t_2 & t_1 \\ 1 & h_1 & h_2 \\ t_1 & 1 & 0 \\ t_2 & \frac{-1+T_1}{T_1} & \frac{1}{T_1} \\ \blacksquare & 1 & \frac{1}{T_1} \end{pmatrix}, \begin{pmatrix} h_1 & t_2 & t_1 & h_2 \\ 1 & h_1 & h_2 \\ t_1 & 0 & 1 \\ t_2 & \frac{1}{T_1} & \frac{-1+T_1}{T_1} \\ \blacksquare & 1 & \frac{1}{T_1} \end{pmatrix} \right\} \right\}$$

In[*]:= Γ /: Insert[γ_1 _ Γ , γ_2 _ Γ , k_] := Γ @<|
 ζ → Flatten[Insert[γ_1 @ ζ , γ_2 @ ζ , k]],
 ω → γ_1 @ ω γ_2 @ ω ,
 σ → γ_1 @ σ + γ_2 @ σ ,
 λ → γ_1 @ λ + γ_2 @ λ
 |>
 Γ /: Insert[γ_2 _ Γ , k_] [γ_1 _ Γ] := Insert[γ_1 , γ_2 , k]

In[*]:= R_{1,2} // Insert[R_{3,4}, 2]

Out[*]=

$$\begin{pmatrix} \{t_1, t_3, t_4, h_3, h_4, t_2, h_1, h_2\} \\ 1 & h_1 & h_2 & h_3 & h_4 \\ t_1 & 0 & 0 & T_1 & 1 - T_1 \\ t_2 & T_3 & 1 - T_3 & 0 & 0 \\ t_3 & 0 & 1 & 0 & 0 \\ t_4 & 0 & 0 & 0 & 1 \\ \blacksquare & 1 & T_1 & 1 & T_3 \end{pmatrix}$$

```
In[*]:= ComposeList[{Insert[R3,4, 5], Insert[R5,6, 9], dm2,3→2, dm1,4→1, dm1,5→1, dm2,6→2, dm2,1→2},  
R1,2]
```

Out[]=

$$\left\{ \begin{matrix} \{t_1, t_2, h_1, h_2\} \\ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & T_1 & 1 - T_1 \\ t_2 & 0 & 1 \\ \blacksquare & 1 & T_1 \end{pmatrix} \end{matrix} \right\}, \left\{ \begin{matrix} \{t_1, t_2, h_1, h_2, t_3, t_4, h_3, h_4\} \\ \begin{pmatrix} 1 & h_1 & h_2 & h_3 & h_4 \\ t_1 & T_1 & 1 - T_1 & 0 & 0 \\ t_2 & 0 & 1 & 0 & 0 \\ t_3 & 0 & 0 & T_3 & 1 - T_3 \\ t_4 & 0 & 0 & 0 & 1 \\ \blacksquare & 1 & T_1 & 1 & T_3 \end{pmatrix} \end{matrix} \right\},$$

$$\left\{ \begin{matrix} \{t_1, t_2, h_1, h_2, t_3, t_4, h_3, h_4, t_5, t_6, h_5, h_6\} \\ \begin{pmatrix} 1 & h_1 & h_2 & h_3 & h_4 & h_5 & h_6 \\ t_1 & T_1 & 1 - T_1 & 0 & 0 & 0 & 0 \\ t_2 & 0 & 1 & 0 & 0 & 0 & 0 \\ t_3 & 0 & 0 & T_3 & 1 - T_3 & 0 & 0 \\ t_4 & 0 & 0 & 0 & 1 & 0 & 0 \\ t_5 & 0 & 0 & 0 & 0 & T_5 & 1 - T_5 \\ t_6 & 0 & 0 & 0 & 0 & 0 & 1 \\ \blacksquare & 1 & T_1 & 1 & T_3 & 1 & T_5 \end{pmatrix} \end{matrix} \right\},$$

$$\left\{ \begin{matrix} \{t_1, t_2, h_1, t_4, h_2, h_4, t_5, t_6, h_5, h_6\} \\ \begin{pmatrix} 1 & h_1 & h_2 & h_4 & h_5 & h_6 \\ t_1 & T_1 & -((-1 + T_1) T_2) & (-1 + T_1) & (-1 + T_2) & 0 & 0 \\ t_2 & 0 & T_2 & 1 - T_2 & 0 & 0 & 0 \\ t_4 & 0 & 0 & 1 & 0 & 0 & 0 \\ t_5 & 0 & 0 & 0 & T_5 & 1 - T_5 & 0 \\ t_6 & 0 & 0 & 0 & 0 & 0 & 1 \\ \blacksquare & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix} \right\},$$

$$\left\{ \begin{matrix} \{t_1, t_2, h_2, h_1, t_5, t_6, h_5, h_6\} \\ \begin{pmatrix} 1 & h_1 & h_2 & h_5 & h_6 \\ t_1 & -((-1 + T_1) T_2) & 1 - T_2 + T_1 T_2 & 0 & 0 \\ t_2 & T_2 & 1 - T_2 & 0 & 0 \\ t_5 & 0 & 0 & T_5 & 1 - T_5 \\ t_6 & 0 & 0 & 0 & 1 \\ \blacksquare & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix} \right\},$$

$$\left\{ \begin{matrix} \{t_1, t_2, h_2, t_6, h_1, h_6\} \\ \begin{pmatrix} 1 & h_1 & h_2 & h_6 \\ t_1 & -((-1 + T_1) T_2) & T_1 (1 - T_2 + T_1 T_2) & -((-1 + T_1) (1 - T_2 + T_1 T_2)) \\ t_2 & T_2 & -T_1 (-1 + T_2) & (-1 + T_1) (-1 + T_2) \\ t_6 & 0 & 0 & 1 \\ \blacksquare & 0 & 0 & 0 \end{pmatrix} \end{matrix} \right\},$$

$$\left\{ \begin{matrix} \{t_1, t_2, h_1, h_2\} \\ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & T_1 (1 - T_2 + T_1 T_2) & -((-1 + T_1) (1 + T_1 T_2)) \\ t_2 & -T_1 (-1 + T_2) & 1 - T_1 + T_1 T_2 \\ \blacksquare & 0 & 0 \end{pmatrix} \end{matrix} \right\}, \left\{ \begin{matrix} \{t_2, h_2\} \\ \begin{pmatrix} T_2 (1 - T_2 + T_2^2) & h_2 \\ t_2 & 1 \\ \blacksquare & 0 \end{pmatrix} \end{matrix} \right\}$$

In[*]:= {Insert[RotateLeft[$\bar{R}_{4,11}$, 1], 4], dm_{1,4→1}, dm_{11,12→11}, , Insert[R_{10,15}, 5]}

Out[*]=

$$\left\{ \text{Insert} \left[\begin{pmatrix} t_4 & h_{11} & h_4 & t_{11} \\ 1 & h_4 & h_{11} \\ t_4 & \frac{-1+T_4}{T_4} & \frac{1}{T_4} \\ t_{11} & 1 & 0 \\ \blacksquare & 1 & \frac{1}{T_4} \end{pmatrix}, 4 \right], dm_{1,4 \rightarrow 1}, dm_{11,12 \rightarrow 11}, \text{Null}, \text{Insert} \left[\begin{pmatrix} t_{10} & t_{15} & h_{10} & h_{15} \\ 1 & h_{10} & h_{15} \\ t_{10} & T_{10} & 1 - T_{10} \\ t_{15} & 0 & 1 \\ \blacksquare & 1 & T_{10} \end{pmatrix}, 5 \right] \right\}$$

In[*]:= Column@Reverse@

ComposeList[{Insert[RotateLeft[$\bar{R}_{2,7}$, 1], 4], dm_{1,2→1}, Insert[RotateLeft[R_{6,13}, 1], 7],
 dm_{12,13→12}, dm_{6,7→6}, Insert[$\bar{R}_{8,3}$, 5], dm_{1,3→1}, dm_{6,8→6}, Insert[R_{14,9}, 6], dm_{12,14→12},
 dm_{6,9→6}, Insert[RotateLeft[$\bar{R}_{4,11}$, 1], 4], dm_{1,4→1}, dm_{11,12→11}, Insert[R_{10,15}, 6],
 dm_{6,10→6}, dm_{11,15→11}, dm_{6,11→6}, Insert[R_{16,5}, 4], dm_{6,16→6}, dm_{1,5→1}, dm_{1,6→1}},
 $\bar{R}_{12,1}$]

Out[*]=

$$\begin{pmatrix} t_1 & h_1 \\ -\frac{1-4T_1+8T_1^2-11T_1^3+8T_1^4-4T_1^5+T_1^6}{T_1^2} & h_1 \\ t_1 & 1 \\ \blacksquare & 0 \end{pmatrix}$$

$$\begin{pmatrix} t_1 & h_6 & h_1 & t_6 \\ \frac{-1+2T_1-T_1^2+T_6-4T_1T_6+3T_1^2T_6+2T_1T_6^2-2T_1^2T_6^2+T_1^3T_6^3}{T_1^2T_6^2} & h_1 \\ t_1 & \frac{(-1+T_1)T_6(1-T_1-T_6+3T_1T_6+T_6^2-2T_1T_6^2+T_1T_6^3)}{-1+2T_1-T_1^2+T_6-4T_1T_6+3T_1^2T_6+2T_1T_6^2-2T_1^2T_6^2+T_1^3T_6^3} \\ t_6 & -\frac{T_6(1-2T_1+T_1^2-2T_6+5T_1T_6-3T_1^2T_6+2T_1^2T_6^2-6T_1T_6^2+2T_1^2T_6^2-2T_6^3+5T_1T_6^3-T_1^3T_6^3+T_6^4-3T_1T_6^4+T_1T_6^5)}{-1+2T_1-T_1^2+T_6-4T_1T_6+3T_1^2T_6+2T_1T_6^2-2T_1^2T_6^2+T_1^3T_6^3} \\ \blacksquare & 0 \end{pmatrix}$$

$$\begin{pmatrix} t_1 & h_1 & t_5 & h_6 & h_5 & t_6 \\ \frac{-1+2T_1-T_1^2+T_6-4T_1T_6+3T_1^2T_6+2T_1T_6^2-2T_1^2T_6^2+T_1^3T_6^3}{T_1^2T_6^2} & h_1 \\ t_1 & \frac{T_6^2(1-T_1+T_1T_6)}{-1+2T_1-T_1^2+T_6-4T_1T_6+3T_1^2T_6+2T_1T_6^2-2T_1^2T_6^2+T_1^3T_6^3} & \frac{(-1+T_1)T_6(1-T_1-T_6+3T_1T_6+T_6^2-2T_1T_6^2+T_1T_6^3)}{-1+2T_1-T_1^2+T_6-4T_1T_6+3T_1^2T_6+2T_1T_6^2-2T_1^2T_6^2+T_1^3T_6^3} \\ t_5 & 0 & 0 \\ t_6 & \frac{(-1+T_6)T_6(1-T_1-T_6+3T_1T_6+T_6^2-2T_1T_6^2+T_1T_6^3)}{-1+2T_1-T_1^2+T_6-4T_1T_6+3T_1^2T_6+2T_1T_6^2-2T_1^2T_6^2+T_1^3T_6^3} & -\frac{T_6(1-2T_1+T_1^2-2T_6+5T_1T_6-3T_1^2T_6+2T_1^2T_6^2-6T_1T_6^2+2T_1^2T_6^2-2T_6^3+5T_1T_6^3-T_1^3T_6^3+T_6^4-3T_1T_6^4+T_1T_6^5)}{-1+2T_1-T_1^2+T_6-4T_1T_6+3T_1^2T_6+2T_1T_6^2-2T_1^2T_6^2+T_1^3T_6^3} \\ \blacksquare & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} t_1 & h_1 & h_6 & t_{16} & t_5 & h_{16} & h_5 & t_6 \\ \frac{-1+2T_1-T_1^2+T_6-4T_1T_6+3T_1^2T_6+2T_1T_6^2-2T_1^2T_6^2+T_1^3T_6^3}{T_1^2T_6^2} & h_1 \\ t_1 & \frac{T_6^2(1-T_1+T_1T_6)}{-1+2T_1-T_1^2+T_6-4T_1T_6+3T_1^2T_6+2T_1T_6^2-2T_1^2T_6^2+T_1^3T_6^3} & \frac{(-1+T_1)(1-T_1-T_6+3T_1T_6+T_6^2-2T_1T_6^2+T_1T_6^3)}{-1+2T_1-T_1^2+T_6-4T_1T_6+3T_1^2T_6+2T_1T_6^2-2T_1^2T_6^2+T_1^3T_6^3} \\ t_5 & 0 & 0 \\ t_6 & \frac{(-1+T_6)T_6(1-T_1-T_6+3T_1T_6+T_6^2-2T_1T_6^2+T_1T_6^3)}{-1+2T_1-T_1^2+T_6-4T_1T_6+3T_1^2T_6+2T_1T_6^2-2T_1^2T_6^2+T_1^3T_6^3} & -\frac{T_6(1-2T_1+T_1^2-2T_6+5T_1T_6-3T_1^2T_6+2T_1^2T_6^2-6T_1T_6^2+2T_1^2T_6^2-2T_6^3+5T_1T_6^3-T_1^3T_6^3+T_6^4-3T_1T_6^4+T_1T_6^5)}{-1+2T_1-T_1^2+T_6-4T_1T_6+3T_1^2T_6+2T_1T_6^2-2T_1^2T_6^2+T_1^3T_6^3} \\ \blacksquare & 0 & 0 \end{pmatrix}$$

$\left\{ \begin{array}{l} \mathbf{t}_1, \mathbf{h}_1, \mathbf{h}_6, \mathbf{t}_6 \\ \frac{-1+2 T_1 - T_1^2 + T_6 - 4 T_1 T_6 + 3 T_1^2 T_6 + 2 T_1 T_6^2 - 2 T_1^2 T_6^2 + T_1^3 T_6^3}{T_1^2 T_6^2} \\ \mathbf{t}_1 \\ \mathbf{t}_6 \\ \blacksquare \end{array} \right.$	$\begin{array}{l} \mathbf{h}_1 \\ \frac{T_6^2 (1 - T_1 + T_1 T_6)}{-1+2 T_1 - T_1^2 + T_6 - 4 T_1 T_6 + 3 T_1^2 T_6 + 2 T_1 T_6^2 - 2 T_1^2 T_6^2 + T_1^3 T_6^3} \\ \frac{(-1+T_6) T_6 (1 - T_1 - T_6 + 3 T_1 T_6 + T_6^2 - 2 T_1 T_6^2 + T_1 T_6^3)}{-1+2 T_1 - T_1^2 + T_6 - 4 T_1 T_6 + 3 T_1^2 T_6 + 2 T_1 T_6^2 - 2 T_1^2 T_6^2 + T_1^3 T_6^3} \\ \emptyset \end{array}$	$\frac{(-1+T_1) (1 - T_1 - T_6)}{-1+2 T_1 - T_1^2 + T_6 - 4 T_1 T_6} \\ \frac{-1+2 T_1 - T_1^2 + 2 T_6 - 5 T_1 T_6 + 3 T_1^2 T_6 - 2 T_1^2 T_6^2 + T_1^3 T_6^3}{-1+2 T_1 - T_1^2 + T_6 - 4 T_1 T_6}$
$\left\{ \begin{array}{l} \mathbf{t}_1, \mathbf{h}_1, \mathbf{t}_{11}, \mathbf{h}_6, \mathbf{h}_{11}, \mathbf{t}_6 \\ \frac{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}}{T_1^2 T_6 T_{11}} \\ \mathbf{t}_1 \\ \mathbf{t}_6 \\ \mathbf{t}_{11} \\ \blacksquare \end{array} \right.$	$\begin{array}{l} \mathbf{h}_1 \\ \frac{(1 - T_1 + T_1 T_6) T_{11}}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \frac{(1 - T_1 + T_1 T_6) (-1 + T_{11})}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \frac{T_1 (-1 + T_6) T_6 T_{11}}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \emptyset \end{array}$	$\frac{T_6 (-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + 2 T_{11} - 4 T_1 T_{11})}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}}$
$\left\{ \begin{array}{l} \mathbf{t}_1, \mathbf{h}_1, \mathbf{t}_{11}, \mathbf{h}_{11}, \mathbf{t}_{15}, \mathbf{h}_6, \mathbf{h}_{15}, \mathbf{t}_6 \\ \frac{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}}{T_1^2 T_6 T_{11}} \\ \mathbf{t}_1 \\ \mathbf{t}_6 \\ \mathbf{t}_{11} \\ \mathbf{t}_{15} \\ \blacksquare \end{array} \right.$	$\begin{array}{l} \mathbf{h}_1 \\ \frac{(1 - T_1 + T_1 T_6) T_{11}}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \frac{(1 - T_1 + T_1 T_6) (-1 + T_{11})}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \emptyset \\ \frac{T_1 (-1 + T_6) T_6 T_{11}}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \emptyset \end{array}$	$\mathbf{h}_6 \\ \frac{(-1+T_1) (1 - T_1 + T_1 T_6)}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \frac{T_1^2 T_6 T_{11}}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \emptyset \\ \frac{(-1+T_6) T_{11} (-1+2 T_1 - T_1^2 + T_{11} - 2 T_1 T_{11})}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \emptyset$
$\left\{ \begin{array}{l} \mathbf{t}_1, \mathbf{h}_1, \mathbf{t}_{11}, \mathbf{h}_{11}, \mathbf{h}_6, \mathbf{t}_{10}, \mathbf{t}_{15}, \mathbf{h}_{10}, \mathbf{h}_{15}, \mathbf{t}_6 \\ \frac{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}}{T_1^2 T_6 T_{11}} \\ \mathbf{t}_1 \\ \mathbf{t}_6 \\ \mathbf{t}_{10} \\ \mathbf{t}_{11} \\ \mathbf{t}_{15} \\ \blacksquare \end{array} \right.$	$\begin{array}{l} \mathbf{h}_1 \\ \frac{(1 - T_1 + T_1 T_6) T_{11}}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \frac{(1 - T_1 + T_1 T_6) (-1 + T_{11})}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \emptyset \\ \emptyset \\ \frac{T_1 (-1 + T_6) T_6 T_{11}}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \emptyset \end{array}$	$\mathbf{h}_6 \\ \frac{(-1+T_1) (1 - T_1 + T_1 T_6)}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \frac{T_1^2 T_6 T_{11}}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \emptyset \\ \emptyset \\ \frac{(-1+T_6) T_{11} (-1+2 T_1 - T_1^2 + T_{11} - 2 T_1 T_{11})}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \emptyset$
$\left\{ \begin{array}{l} \mathbf{t}_1, \mathbf{h}_1, \mathbf{t}_{11}, \mathbf{h}_{11}, \mathbf{h}_6, \mathbf{t}_6 \\ \frac{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}}{T_1^2 T_6 T_{11}} \\ \mathbf{t}_1 \\ \mathbf{t}_6 \\ \mathbf{t}_{11} \\ \blacksquare \end{array} \right.$	$\begin{array}{l} \mathbf{h}_1 \\ \frac{(1 - T_1 + T_1 T_6) T_{11}}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \frac{(1 - T_1 + T_1 T_6) (-1 + T_{11})}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \frac{T_1 (-1 + T_6) T_6 T_{11}}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \emptyset \end{array}$	$\mathbf{h}_6 \\ \frac{(-1+T_1) (1 - T_1 + T_1 T_6)}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \frac{T_1^2 T_6 T_{11}}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \frac{(-1+T_6) T_{11} (-1+2 T_1 - T_1^2 + T_{11} - 2 T_1 T_{11})}{-1+2 T_1 - T_1^2 - T_1 T_6 + T_1^2 T_6 + T_{11} - 2 T_1 T_{11} + T_1^2 T_{11} + T_1 T_6 T_{11}} \\ \emptyset$

$$\left\{ t_1, t_{12}, h_{11}, h_1, t_{11}, h_{12}, h_6, t_6 \right\}$$

$$\begin{pmatrix} 1 & h_1 & h_6 & h_{11} & h_{12} \\ t_1 & \frac{(-1+T_1)(1-T_1+T_1 T_6)}{T_1^2 T_6} & \frac{1-T_1+T_1 T_6}{T_1^2 T_6} & 0 & \frac{-1+T_1}{T_1 T_6} \\ t_6 & \frac{(-1+T_1)(1-T_1+T_1 T_6)(-1+T_{12})}{T_1^2 T_6 T_{12}} & \frac{(1-T_1+T_1 T_6)(-1+T_{12})}{T_1^2 T_6 T_{12}} & 1 & -\frac{(1-T_1+T_1 T_6)(-1+T_{12})}{T_1 T_6 T_{12}} \\ t_{11} & 1 & 0 & 0 & 0 \\ t_{12} & \frac{(-1+T_1)(-1+T_6)}{T_1} & \frac{-1+T_6}{T_1} & -((-1 + T_6) T_{12}) & 2 - T_6 - T_{12} + T_6 T_{12} \\ \blacksquare & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\left\{ t_1, t_{12}, h_1, t_4, h_{11}, h_4, t_{11}, h_{12}, h_6, t_6 \right\}$$

$$\begin{pmatrix} 1 & h_1 & h_4 & h_6 & h_{11} & h_{12} \\ t_1 & \frac{1-T_1+T_1 T_6}{T_1 T_6} & 0 & 0 & 0 & \frac{-1+T_1}{T_1 T_6} \\ t_4 & \frac{(1-T_1+T_1 T_6)(-1+T_{12})}{T_1 T_6 T_{12}} & 0 & 0 & 1 & -\frac{(1-T_1+T_1 T_6)(-1+T_{12})}{T_1 T_6 T_{12}} \\ t_6 & 0 & \frac{-1+T_4}{T_4} & \frac{1}{T_4} & 0 & 0 \\ t_{11} & 0 & 1 & 0 & 0 & 0 \\ t_{12} & -1 + T_6 & 0 & 0 & -((-1 + T_6) T_{12}) & 2 - T_6 - T_{12} + T_6 T_{12} \\ \blacksquare & 0 & 1 & 0 & \frac{1}{T_4} & 0 \end{pmatrix}$$

$$\left\{ t_1, t_{12}, h_1, h_{12}, h_6, t_6 \right\}$$

$$\begin{pmatrix} 1 & h_1 & h_6 & h_{12} \\ t_1 & \frac{1-T_1+T_1 T_6}{T_1 T_6} & 0 & \frac{-1+T_1}{T_1 T_6} \\ t_6 & \frac{(1-T_1+T_1 T_6)(-1+T_{12})}{T_1 T_6 T_{12}} & 1 & -\frac{(1-T_1+T_1 T_6)(-1+T_{12})}{T_1 T_6 T_{12}} \\ t_{12} & -1 + T_6 & -((-1 + T_6) T_{12}) & 2 - T_6 - T_{12} + T_6 T_{12} \\ \blacksquare & 0 & 0 & 0 \end{pmatrix}$$

$$\left\{ t_1, t_{12}, h_1, h_6, t_9, h_{12}, h_9, t_6 \right\}$$

$$\begin{pmatrix} 1 & h_1 & h_6 & h_9 & h_{12} \\ t_1 & \frac{1-T_1+T_1 T_6}{T_1 T_6} & \frac{-1+T_1}{T_1 T_6} & 0 & 0 \\ t_6 & \frac{(1-T_1+T_1 T_6)(-1+T_{12})}{T_1 T_6 T_{12}} & \frac{(-1+T_1)(-1+T_{12})}{T_1 T_6 T_{12}} & 1 & -\frac{-1+T_{12}}{T_{12}} \\ t_9 & 0 & 0 & 0 & 1 \\ t_{12} & -1 + T_6 & 1 & -((-1 + T_6) T_{12}) & (-1 + T_6)(-1 + T_{12}) \\ \blacksquare & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\left\{ t_1, t_{12}, h_1, h_6, h_{12}, t_{14}, t_9, h_{14}, h_9, t_6 \right\}$$

$$\begin{pmatrix} 1 & h_1 & h_6 & h_9 & h_{12} & h_{14} \\ t_1 & \frac{1-T_1+T_1 T_6}{T_1 T_6} & \frac{-1+T_1}{T_1 T_6} & 0 & 0 & 0 \\ t_6 & \frac{(1-T_1+T_1 T_6)(-1+T_{12})}{T_1 T_6 T_{12}} & \frac{(-1+T_1)(-1+T_{12})}{T_1 T_6 T_{12}} & \frac{1}{T_{12}} & 0 & 0 \\ t_9 & 0 & 0 & 0 & T_{14} & 1 - T_{14} \\ t_{12} & 0 & 0 & 0 & 0 & 1 \\ t_{14} & -1 + T_6 & 1 & 1 - T_6 & 0 & 0 \\ \blacksquare & 0 & 0 & T_{14} & 0 & 1 \end{pmatrix}$$

$$\left\{ t_1, t_{12}, h_1, h_6, h_{12}, t_6 \right\}$$

$$\begin{pmatrix} 1 & h_1 & h_6 & h_{12} \\ t_1 & \frac{1-T_1+T_1 T_6}{T_1 T_6} & \frac{-1+T_1}{T_1 T_6} & 0 \\ t_6 & \frac{(1-T_1+T_1 T_6)(-1+T_{12})}{T_1 T_6 T_{12}} & \frac{(-1+T_1)(-1+T_{12})}{T_1 T_6 T_{12}} & \frac{1}{T_{12}} \\ t_{12} & -1 + T_6 & 1 & 1 - T_6 \\ \blacksquare & 0 & 0 & 0 \end{pmatrix}$$

$$\{t_1, t_{12}, h_6, t_8, h_1, h_8, h_{12}, t_6\}$$

$$\begin{pmatrix} 1 & h_1 & h_6 & h_8 & h_{12} \\ t_1 & \frac{-1+T_1}{T_1} & \frac{1}{T_1} & 0 & 0 \\ t_6 & \frac{(-1+T_1)(-1+T_{12})}{T_1 T_{12}} & \frac{-1+T_{12}}{T_1 T_{12}} & 0 & \frac{1}{T_{12}} \\ t_8 & 0 & \frac{-1+T_8}{T_8} & \frac{1}{T_8} & 0 \\ t_{12} & T_6 & 0 & 0 & 1 - T_6 \\ \blacksquare & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\{t_1, t_{12}, h_6, h_1, t_3, t_8, h_3, h_8, h_{12}, t_6\}$$

$$\begin{pmatrix} 1 & h_1 & h_3 & h_6 & h_8 & h_{12} \\ t_1 & \frac{-1+T_1}{T_1} & \frac{1}{T_1} & 0 & 0 & 0 \\ t_3 & \frac{(-1+T_1)(-1+T_{12})}{T_1 T_{12}} & \frac{-1+T_{12}}{T_1 T_{12}} & 0 & 0 & \frac{1}{T_{12}} \\ t_6 & 0 & 0 & 1 & 0 & 0 \\ t_8 & 0 & 0 & \frac{-1+T_8}{T_8} & \frac{1}{T_8} & 0 \\ t_{12} & T_6 & 0 & 0 & 0 & 1 - T_6 \\ \blacksquare & 0 & \frac{1}{T_8} & 0 & 1 & 0 \end{pmatrix}$$

$$\{t_1, t_{12}, h_6, h_1, h_{12}, t_6\}$$

$$\begin{pmatrix} 1 & h_1 & h_6 & h_{12} \\ t_1 & \frac{-1+T_1}{T_1} & \frac{1}{T_1} & 0 \\ t_6 & \frac{(-1+T_1)(-1+T_{12})}{T_1 T_{12}} & \frac{-1+T_{12}}{T_1 T_{12}} & \frac{1}{T_{12}} \\ t_{12} & T_6 & 0 & 1 - T_6 \\ \blacksquare & 0 & 0 & 0 \end{pmatrix}$$

$$\{t_1, t_{12}, h_7, h_1, t_7, h_6, h_{12}, t_6\}$$

$$\begin{pmatrix} 1 & h_1 & h_6 & h_7 & h_{12} \\ t_1 & \frac{-1+T_1}{T_1} & \frac{1}{T_1} & 0 & 0 \\ t_6 & \frac{(-1+T_1)(-1+T_{12})}{T_1 T_{12}} & \frac{-1+T_{12}}{T_1 T_{12}} & 0 & \frac{1}{T_{12}} \\ t_7 & 1 & 0 & 0 & 0 \\ t_{12} & 0 & 0 & T_6 & 1 - T_6 \\ \blacksquare & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\{t_1, t_{12}, h_7, h_1, t_7, h_{12}, t_{13}, h_6, h_{13}, t_6\}$$

$$\begin{pmatrix} 1 & h_1 & h_6 & h_7 & h_{12} & h_{13} \\ t_1 & \frac{-1+T_1}{T_1} & \frac{1}{T_1} & 0 & 0 & 0 \\ t_6 & \frac{(-1+T_1)(-1+T_{12})}{T_1 T_{12}} & \frac{-1+T_{12}}{T_1 T_{12}} & \frac{1}{T_{12}} & 0 & 0 \\ t_7 & 1 & 0 & 0 & 0 & 0 \\ t_{12} & 0 & 0 & 0 & 0 & 1 \\ t_{13} & 0 & 0 & 0 & T_6 & 1 - T_6 \\ \blacksquare & 0 & 1 & 0 & 0 & T_6 \end{pmatrix}$$

$$\{t_1, t_{12}, h_7, h_1, t_7, h_{12}\}$$

$$\begin{pmatrix} 1 & h_1 & h_7 & h_{12} \\ t_1 & \frac{-1+T_1}{T_1} & \frac{1}{T_1} & 0 \\ t_7 & \frac{(-1+T_1)(-1+T_{12})}{T_1 T_{12}} & \frac{-1+T_{12}}{T_1 T_{12}} & \frac{1}{T_{12}} \\ t_{12} & 1 & 0 & 0 \\ \blacksquare & 0 & 0 & 0 \end{pmatrix}$$

{t₁, t₁₂, h₁, t₂, h₇, h₂, t₇, h₁₂}

$$\begin{pmatrix} 1 & h_1 & h_2 & h_7 & h_{12} \\ t_1 & 1 & 0 & 0 & 0 \\ t_2 & \frac{-1+T_{12}}{T_{12}} & 0 & 0 & \frac{1}{T_{12}} \\ t_7 & 0 & \frac{-1+T_2}{T_2} & \frac{1}{T_2} & 0 \\ t_{12} & 0 & 1 & 0 & 0 \\ \blacksquare & \frac{1}{T_{12}} & 1 & \frac{1}{T_2} & 1 \end{pmatrix}$$

{t₁, t₁₂, h₁, h₁₂}

$$\begin{pmatrix} 1 & h_1 & h_{12} \\ t_1 & 1 & 0 \\ t_{12} & \frac{-1+T_{12}}{T_{12}} & \frac{1}{T_{12}} \\ \blacksquare & \frac{1}{T_{12}} & 1 \end{pmatrix}$$

```
In[*]:= seq = Factor [
  Reverse@ComposeList [ { Insert [ RotateLeft [ R2,7, 1 ], 4 ], dm1,2→1, Insert [ RotateLeft [ R6,13, 1 ],
    7 ], dm12,13→12, dm6,7→6, Insert [ R8,3, 5 ], dm1,3→1, dm6,8→6, Insert [ R14,9, 6 ], dm12,14→12,
    dm6,9→6, Insert [ RotateLeft [ R4,11, 1 ], 4 ], dm1,4→1, dm11,12→11, Insert [ R10,15, 6 ],
    dm6,10→6, dm11,15→11, dm6,11→6, Insert [ R16,5, 4 ], dm6,16→6, dm1,5→1, dm1,6→1
  },
  R12,1 ] /. T_ -> T
];
```

```
In[*]:= MatrixForm@Transpose [
  Factor [ { Det [ #@A ], (#@w), Det [ #@A ] (#@w), Det [ #@A ]2 (#@w), Det [ #@A ]-2 (#@w),
    Det [ #@A ] (#@w)2, Det [ #@A ] (#@w)-2 } ] & /@ seq ]
```

Out[*]//MatrixForm=

$$\begin{pmatrix} 1 & \frac{T^2 (-1+2 T-5 T^2+5 T^3-3 T^4+T^5)}{-1+3 T-5 T^2+5 T^3-2 T^4+T^5} & \frac{T^2 (-1+2 T-5 T^2+5 T^3-3 T^4+T^5)}{-1+3 T-5 T^2+5 T^3-2 T^4+T^5} \\ -\frac{1-4 T+8 T^2-11 T^3+8 T^4-4 T^5+T^6}{T^3} & \frac{-1+3 T-5 T^2+5 T^3-2 T^4+T^5}{T^4} & \frac{-1+3 T-5 T^2+5 T^3-2 T^4+T^5}{T^4} \\ -\frac{1-4 T+8 T^2-11 T^3+8 T^4-4 T^5+T^6}{T^3} & \frac{-1+2 T-5 T^2+5 T^3-3 T^4+T^5}{T^2} & \frac{-1+2 T-5 T^2+5 T^3-3 T^4+T^5}{T^2} \\ -\frac{1-4 T+8 T^2-11 T^3+8 T^4-4 T^5+T^6}{T^3} & \frac{(-1+2 T-5 T^2+5 T^3-3 T^4+T^5)^2}{-1+3 T-5 T^2+5 T^3-2 T^4+T^5} & \frac{(-1+2 T-5 T^2+5 T^3-3 T^4+T^5)^2}{-1+3 T-5 T^2+5 T^3-2 T^4+T^5} \\ -\frac{1-4 T+8 T^2-11 T^3+8 T^4-4 T^5+T^6}{T^3} & \frac{(-1+3 T-5 T^2+5 T^3-2 T^4+T^5)^3}{T^8 (-1+2 T-5 T^2+5 T^3-3 T^4+T^5)^2} & \frac{(-1+3 T-5 T^2+5 T^3-2 T^4+T^5)^3}{T^8 (-1+2 T-5 T^2+5 T^3-3 T^4+T^5)^2} \\ \frac{(1-4 T+8 T^2-11 T^3+8 T^4-4 T^5+T^6)^2}{T^6} & \frac{(-1+2 T-5 T^2+5 T^3-3 T^4+T^5) (-1+3 T-5 T^2+5 T^3-2 T^4+T^5)}{T^6} & \frac{(-1+2 T-5 T^2+5 T^3-3 T^4+T^5) (-1+3 T-5 T^2+5 T^3-2 T^4+T^5)}{T^6} \\ \frac{(1-4 T+8 T^2-11 T^3+8 T^4-4 T^5+T^6)^2}{T^6} & \frac{T^{10} (-1+2 T-5 T^2+5 T^3-3 T^4+T^5)}{(-1+3 T-5 T^2+5 T^3-2 T^4+T^5)^3} & \frac{T^{10} (-1+2 T-5 T^2+5 T^3-3 T^4+T^5)}{(-1+3 T-5 T^2+5 T^3-2 T^4+T^5)^3} \end{pmatrix}$$

In[*]:= $\gamma\theta = \text{seq}[[6]]$

Out[*]=

$$\left\{ \begin{array}{c} \mathbf{t_1, h_1, t_{11}, h_6, h_{11}, t_6} \\ \frac{-1+3T-4T^2+3T^3}{T^4} \quad \mathbf{h_1} \quad \mathbf{h_6} \quad \mathbf{h_{11}} \\ \mathbf{t_1} \quad \frac{T(1-T+T^2)}{-1+3T-4T^2+3T^3} \quad - \frac{(-1+T)T(-1+2T-3T^2+T^3)}{-1+3T-4T^2+3T^3} \quad \frac{(-1+T)(1-2T+4T^2-3T^3+T^4)}{-1+3T-4T^2+3T^3} \\ \mathbf{t_6} \quad \frac{(-1+T)(1-T+T^2)}{-1+3T-4T^2+3T^3} \quad - \frac{(-1+T)T^2(1-T+T^2)}{-1+3T-4T^2+3T^3} \quad \frac{T(1-3T+4T^2-2T^3+T^4)}{-1+3T-4T^2+3T^3} \\ \mathbf{t_{11}} \quad \frac{(-1+T)T^3}{-1+3T-4T^2+3T^3} \quad \frac{T(-1+4T-8T^2+10T^3-6T^4+2T^5)}{-1+3T-4T^2+3T^3} \quad - \frac{(-1+T)(-1+3T-5T^2+7T^3-4T^4+2T^5)}{-1+3T-4T^2+3T^3} \\ \mathbf{\theta} \quad \mathbf{\theta} \quad \mathbf{\theta} \end{array} \right.$$



In[*]:= $\{\zeta\theta = \gamma\theta[\zeta], \lambda\theta = \gamma\theta[\lambda]\}$

Out[*]=

$$\left\{ \begin{array}{c} \mathbf{t_1, h_1, t_{11}, h_6, h_{11}, t_6}, \frac{T(1-T+T^2)h_1t_1}{-1+3T-4T^2+3T^3} - \frac{(-1+T)T(-1+2T-3T^2+T^3)h_6t_1}{-1+3T-4T^2+3T^3} + \\ \frac{(-1+T)(1-2T+4T^2-3T^3+T^4)h_{11}t_1}{-1+3T-4T^2+3T^3} + \frac{(-1+T)T^3h_1t_6}{-1+3T-4T^2+3T^3} + \\ \frac{T(-1+4T-8T^2+10T^3-6T^4+2T^5)h_6t_6}{-1+3T-4T^2+3T^3} - \frac{(-1+T)(-1+3T-5T^2+7T^3-4T^4+2T^5)h_{11}t_6}{-1+3T-4T^2+3T^3} + \\ \frac{(-1+T)(1-T+T^2)h_1t_{11}}{-1+3T-4T^2+3T^3} - \frac{(-1+T)T^2(1-T+T^2)h_6t_{11}}{-1+3T-4T^2+3T^3} + \frac{T(1-3T+4T^2-2T^3+T^4)h_{11}t_{11}}{-1+3T-4T^2+3T^3} \end{array} \right\}$$

In[*]:= $\text{If}[\text{MatchQ}[\#, h_], 1, \theta] \& /@ \zeta\theta$

Out[*]=

$$\{\theta, 1, \theta, 1, 1, \theta\}$$

In[*]:= $hc = \text{Thread}[\zeta\theta \rightarrow \text{Rest}@\text{FoldList}[\text{Plus}, \theta, \text{If}[\text{MatchQ}[\#, h_], 1, \theta] \& /@ \zeta\theta]]$

Out[*]=

$$\{\mathbf{t_1} \rightarrow \theta, \mathbf{h_1} \rightarrow 1, \mathbf{t_{11}} \rightarrow 1, \mathbf{h_6} \rightarrow 2, \mathbf{h_{11}} \rightarrow 3, \mathbf{t_6} \rightarrow 3\}$$

```
In[*]:= λ0 /. Cases[ϕ0, t_i_ => (t_i -> T_i t_i)]
```

```
Out[*]=
```

$$\frac{T(1-T+T^2)h_1t_1T_1}{-1+3T-4T^2+3T^3} - \frac{(-1+T)T(-1+2T-3T^2+T^3)h_6t_1T_1}{-1+3T-4T^2+3T^3} +$$

$$\frac{(-1+T)(1-2T+4T^2-3T^3+T^4)h_{11}t_1T_1}{-1+3T-4T^2+3T^3} + \frac{(-1+T)T^3h_1t_6T_6}{-1+3T-4T^2+3T^3} +$$

$$\frac{T(-1+4T-8T^2+10T^3-6T^4+2T^5)h_6t_6T_6}{-1+3T-4T^2+3T^3} -$$

$$\frac{(-1+T)(-1+3T-5T^2+7T^3-4T^4+2T^5)h_{11}t_6T_6}{-1+3T-4T^2+3T^3} + \frac{(-1+T)(1-T+T^2)h_1t_{11}T_{11}}{-1+3T-4T^2+3T^3} -$$

$$\frac{(-1+T)T^2(1-T+T^2)h_6t_{11}T_{11}}{-1+3T-4T^2+3T^3} + \frac{T(1-3T+4T^2-2T^3+T^4)h_{11}t_{11}T_{11}}{-1+3T-4T^2+3T^3}$$

ϕ0

```
Out[*]=
```

```
{t1, h1, t11, h6, h11, t6}
```

```
In[*]:= ReplaceList[ϕ0, {____, h_i_, ____ , t_j_, ____} => {i, j}]
```

```
Out[*]=
```

```
{{1, 11}, {1, 6}, {6, 6}, {11, 6}}
```

```
In[*]:= ReplaceList[ϕ0, {____, h_i_, ____ , t_j_, ____} => T_j^{h_i/.hc} (1 - T_j) h_i t_j]
```

```
Out[*]=
```

```
{h1 t11 (1 - T11) T11, h1 t6 (1 - T6) T6, h6 t6 (1 - T6) T6^2, h11 t6 (1 - T6) T6^3}
```

```
In[*]:= Γ[α_] [NOλ] := Module[{ϕ0 = α[ϕ], hc},
  hc = Thread[ϕ0 -> Most@FoldList[Plus, 0, If[MatchQ[#, h_], 1, 0] & /@ ϕ0]];
  Plus[
    α[λ] /. Cases[ϕ0, t_i_ => (t_i -> T_i^{-t_i/.hc} t_i)],
    Total@ReplaceList[ϕ0, {____, h_i_, ____ , t_j_, ____} => T_j^{(h_i/.hc)} (1 - T_j^{-1}) h_i t_j]
  ];
  Γ[α_] [NOA] := Outer[Factor[0_#1, #2 Γ[α] [NOλ]] &, Cases[α[ϕ], t_], Cases[α[ϕ], h_]];
```

In[*]:= $\gamma\theta$ [NO λ]

Out[*]=

$$\frac{T(1-T+T^2)h_1t_1}{-1+3T-4T^2+3T^3} - \frac{(-1+T)T(-1+2T-3T^2+T^3)h_6t_1}{-1+3T-4T^2+3T^3} + \frac{(-1+T)(1-2T+4T^2-3T^3+T^4)h_{11}t_1}{-1+3T-4T^2+3T^3} +$$

$$h_1t_6\left(1-\frac{1}{T_6}\right) + \frac{(-1+T)T^3h_1t_6}{(-1+3T-4T^2+3T^3)T_6^3} + \frac{T(-1+4T-8T^2+10T^3-6T^4+2T^5)h_6t_6}{(-1+3T-4T^2+3T^3)T_6^3} -$$

$$\frac{(-1+T)(-1+3T-5T^2+7T^3-4T^4+2T^5)h_{11}t_6}{(-1+3T-4T^2+3T^3)T_6^3} + \frac{h_{11}t_6\left(1-\frac{1}{T_6}\right)}{T_6^3} + \frac{h_6t_6\left(1-\frac{1}{T_6}\right)}{T_6} + h_1t_{11}\left(1-\frac{1}{T_{11}}\right) +$$

$$\frac{(-1+T)(1-T+T^2)h_1t_{11}}{(-1+3T-4T^2+3T^3)T_{11}} - \frac{(-1+T)T^2(1-T+T^2)h_6t_{11}}{(-1+3T-4T^2+3T^3)T_{11}} + \frac{T(1-3T+4T^2-2T^3+T^4)h_{11}t_{11}}{(-1+3T-4T^2+3T^3)T_{11}}$$

In[*]:= $\gamma\theta$ [A] // MatrixForm

Out[*]//MatrixForm=

$$\left(\begin{array}{ccc} \frac{T(1-T+T^2)}{-1+3T-4T^2+3T^3} & -\frac{(-1+T)T(-1+2T-3T^2+T^3)}{-1+3T-4T^2+3T^3} & \frac{(-1+T)(1-2T+4T^2-3T^3+T^4)}{-1+3T-4T^2+3T^3} \\ \frac{(-1+T)(1-T+T^2)}{-1+3T-4T^2+3T^3} & -\frac{(-1+T)T^2(1-T+T^2)}{-1+3T-4T^2+3T^3} & \frac{T(1-3T+4T^2-2T^3+T^4)}{-1+3T-4T^2+3T^3} \\ \frac{(-1+T)T^3}{-1+3T-4T^2+3T^3} & \frac{T(-1+4T-8T^2+10T^3-6T^4+2T^5)}{-1+3T-4T^2+3T^3} & -\frac{(-1+T)(-1+3T-5T^2+7T^3-4T^4+2T^5)}{-1+3T-4T^2+3T^3} \end{array} \right)$$

In[*]:= $\gamma\theta$ [NOA] /. T_ -> T // MatrixForm

Out[*]//MatrixForm=

$$\left(\begin{array}{ccc} \frac{T(1-T+T^2)}{-1+3T-4T^2+3T^3} & -\frac{(-1+T)T(-1+2T-3T^2+T^3)}{-1+3T-4T^2+3T^3} & \frac{(-1+T)(1-2T+4T^2-3T^3+T^4)}{-1+3T-4T^2+3T^3} \\ \frac{-2T+5T^2-6T^3+3T^4}{T(-1+3T-4T^2+3T^3)} & -\frac{(-1+T)T(1-T+T^2)}{-1+3T-4T^2+3T^3} & \frac{1-3T+4T^2-2T^3+T^4}{-1+3T-4T^2+3T^3} \\ \frac{T^2-5T^3+8T^4-7T^5+3T^6}{T^3(-1+3T-4T^2+3T^3)} & \frac{-T^3+3T^4-3T^5+2T^6}{T^3(-1+3T-4T^2+3T^3)} & -\frac{T^2-5T^3+8T^4-6T^5+2T^6}{T^3(-1+3T-4T^2+3T^3)} \end{array} \right)$$

In[*]:= Factor[[Det[# [NOA] /. T_ -> T] * $\frac{\# [\omega]^*}{\# [\omega]}$]] & /@ {R_{1,2}, $\bar{R}_{1,2}$ }

Out[*]=

$$\left\{ T, \frac{1}{T} \right\}$$

In[*]:= (Simplify[# [NO λ] /. T_ -> T /. h_ -> 1]) & /@ seq

Out[*]=

$$\{t_1, t_1 + t_6, t_1 + t_5 + t_6, t_1 + t_5 + t_6 + t_{16}, t_1 + t_6, t_1 + t_6 + t_{11}, t_1 + t_6 + t_{11} + t_{15},$$

$$t_1 + t_6 + t_{10} + t_{11} + t_{15}, t_1 + t_6 + t_{11}, t_1 + t_6 + t_{11} + t_{12}, t_1 + t_4 + t_6 + t_{11} + t_{12}, t_1 + t_6 + t_{12},$$

$$t_1 + t_6 + t_9 + t_{12}, t_1 + t_6 + t_9 + t_{12} + t_{14}, t_1 + t_6 + t_{12}, t_1 + t_6 + t_8 + t_{12}, t_1 + t_3 + t_6 + t_8 + t_{12},$$

$$t_1 + t_6 + t_{12}, t_1 + t_6 + t_7 + t_{12}, t_1 + t_6 + t_7 + t_{12} + t_{13}, t_1 + t_7 + t_{12}, t_1 + t_2 + t_7 + t_{12}, t_1 + t_{12}\}$$

In[*]:= Factor[{ Det[#[NOA] /. T_ -> T] * $\frac{\#[\omega]}{\#[\omega]^*}$ }] & /@ seq

Out[*]=

$$\left\{ \{1\}, \left\{-\frac{1}{T^2}\right\}, \left\{-\frac{1}{T^4}\right\}, \left\{\frac{1}{T^8}\right\}, \left\{\frac{1}{T^3}\right\}, \left\{-\frac{1}{T^4}\right\}, \left\{-\frac{1}{T^7}\right\}, \left\{\frac{1}{T^{12}}\right\}, \left\{\frac{1}{T^5}\right\}, \left\{\frac{1}{T^6}\right\}, \left\{-\frac{1}{T^9}\right\}, \left\{\frac{1}{T^3}\right\}, \right. \\ \left. \left\{\frac{1}{T^6}\right\}, \left\{-\frac{1}{T^{11}}\right\}, \left\{-\frac{1}{T^4}\right\}, \left\{\frac{1}{T^6}\right\}, \left\{\frac{1}{T^{10}}\right\}, \left\{\frac{1}{T^3}\right\}, \left\{-\frac{1}{T^5}\right\}, \left\{-\frac{1}{T^9}\right\}, \left\{\frac{1}{T^3}\right\}, \left\{-\frac{1}{T^5}\right\}, \left\{\frac{1}{T}\right\} \right\}$$