Pensieve header: The trace in α-calculus.

```
dir = SetDirectory["C:/drorbn/AcademicPensieve/Projects/MetaCalculi/"];
<< KnotTheory`
<< MetaCalculi-Program.m
```

```
Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.
Read more at http://katlas.org/wiki/KnotTheory.
```

n = 3; γ0 = $\Gamma\left[\omega, \sum_{a=0}^{n} h_a \sigma_a, \sum_{a=1}^{n}\sum_{b=1}^{n} t_a h_b \alpha_{10\,a+b}\right]$

α0 = $A\left[\omega, \{t[a], t[b], t[S]\}.\begin{pmatrix}\alpha & \beta & \theta \\ \gamma & \delta & \epsilon \\ \phi & \psi & \Xi\end{pmatrix}.\{h[a], h[b], h[S]\}\right]$

ω	h[a]	h[b]	h[S]
t[a]	α	β	θ
t[b]	γ	δ	ϵ
t[S]	ϕ	ψ	Ξ

```
α0 // tha[a, b]
```

$\omega + \beta\,\omega\, c_a$	h[a]	h[b]	h[S]
t[a]	$\frac{\alpha+\alpha\beta c_a+\alpha\delta c_b+\alpha\psi c_S}{1+\beta c_a}$	$\frac{\beta+\beta^2 c_a+\beta\delta c_b+\beta\psi c_S}{1+\beta c_a}$	$\frac{\theta+\beta\theta c_a+\delta\theta c_b+\theta\psi c_S}{1+\beta c_a}$
t[b]	$\frac{\gamma+\beta\gamma c_a-\alpha\delta c_a}{1+\beta c_a}$	$\frac{\delta}{1+\beta c_a}$	$\frac{\epsilon+\beta\epsilon c_a-\delta\theta c_a}{1+\beta c_a}$
t[S]	$\frac{\phi+\beta\phi c_a-\alpha\psi c_a}{1+\beta c_a}$	$\frac{\psi}{1+\beta c_a}$	$\frac{\Xi+\beta\Xi c_a-\theta\psi c_a}{1+\beta c_a}$

```
α0 // hm[a, b, c]
```

ω	h[c]	h[S]
t[a]	$\alpha+\beta+\alpha\beta c_a+\beta\gamma c_b+\beta\phi c_S$	θ
t[b]	$\gamma+\delta+\alpha\delta c_a+\gamma\delta c_b+\delta\phi c_S$	ϵ
t[S]	$\phi+\psi+\alpha\psi c_a+\gamma\psi c_b+\phi\psi c_S$	Ξ

```
α0 // tm[a, b, c]
```

ω	h[a]	h[b]	h[S]
t[c]	$\alpha+\gamma$	$\beta+\delta$	$\epsilon+\theta$
t[S]	ϕ	ψ	Ξ

```
αSimplify[expr_] := expr // Simplify;
```

```
α0 // dm[a, b, c] // αSimplify
```

$\omega + \beta\,\omega\, c_c$	h[c]	h[S]
t[c]	$\frac{\gamma+\delta+\beta\gamma c_c+\gamma\delta c_c+\delta\phi c_S+(\alpha+\beta+\beta(\alpha+\gamma)c_c+\beta\phi c_S)(1+(\beta+\delta)c_c+\psi c_S)}{1+\beta c_c}$	$\frac{\epsilon+\theta+\beta(\epsilon+\theta)c_c+\theta\psi c_S}{1+\beta c_c}$
t[S]	$\frac{\phi+\psi+(\beta\phi+\gamma\psi)c_c+\phi\psi c_S}{1+\beta c_c}$	$\frac{\Xi+(\beta\Xi-\theta\psi)c_c}{1+\beta c_c}$

α1 = $A\left[\omega, \{t[a], t[S]\}.\begin{pmatrix}\alpha & \theta \\ \phi & \Xi\end{pmatrix}.\{h[a], h[S]\}\right]$

ω	h[a]	h[S]
t[a]	α	θ
t[S]	ϕ	Ξ

```
(α1 // Γ // tr[a] // A) /. Log[e^x_] :> x
```

$$\begin{pmatrix} -\phi\,\omega\, c_S & h[S] \\ t[a] & 0 \\ t[S] & \Xi + \frac{\theta\, c_a}{c_S} \end{pmatrix}$$

```
{α0 // dσ[S -> c],
 (α0 // dσ[S -> c] // Γ // tr[c] // A) /. Log[e^x_] :> x}
```

$$\left\{\begin{pmatrix} \omega & h[a] & h[b] & h[c] \\ t[a] & \alpha & \beta & \theta \\ t[b] & \gamma & \delta & \epsilon \\ t[c] & \phi & \psi & \Xi \end{pmatrix}, \begin{pmatrix} -\omega\,(\theta\, c_a + \epsilon\, c_b) & h[a] & h[b] \\ t[a] & \frac{\alpha\,\theta\, c_a + \alpha\,\epsilon\, c_b + \theta\,\phi\, c_c}{\theta\, c_a + \epsilon\, c_b} & \frac{\beta\,\theta\, c_a + \beta\,\epsilon\, c_b + \theta\,\psi\, c_c}{\theta\, c_a + \epsilon\, c_b} \\ t[b] & \frac{\gamma\,\theta\, c_a + \gamma\,\epsilon\, c_b + \epsilon\,\phi\, c_c}{\theta\, c_a + \epsilon\, c_b} & \frac{\delta\,\theta\, c_a + \delta\,\epsilon\, c_b + \epsilon\,\psi\, c_c}{\theta\, c_a + \epsilon\, c_b} \\ t[c] & 0 & 0 \end{pmatrix}\right\}$$

```
{α0 // dσ[S -> c],
 (α0 // dσ[S -> c] // tr[c])}
```

$$\left\{\begin{pmatrix} \omega & h[a] & h[b] & h[c] \\ t[a] & \alpha & \beta & \theta \\ t[b] & \gamma & \delta & \epsilon \\ t[c] & \phi & \psi & \Xi \end{pmatrix}, \begin{pmatrix} -\omega\,(\theta\, c_a + \epsilon\, c_b) & h[a] & h[b] \\ t[a] & \alpha + \frac{\theta\,\phi\, c_c}{\theta\, c_a + \epsilon\, c_b} & \beta + \frac{\theta\,\psi\, c_c}{\theta\, c_a + \epsilon\, c_b} \\ t[b] & \gamma + \frac{\epsilon\,\phi\, c_c}{\theta\, c_a + \epsilon\, c_b} & \delta + \frac{\epsilon\,\psi\, c_c}{\theta\, c_a + \epsilon\, c_b} \\ t[c] & 0 & 0 \end{pmatrix}\right\}$$

```
{α0 // dm[a, b, c] // tr[c], α0 // dm[b, a, c] // tr[c]}
```

$$\left\{\begin{pmatrix} -\omega\, c_S\,(\phi + \psi + (\beta\,\phi + \gamma\,\psi)\, c_c + \phi\,\psi\, c_S) & h[S] \\ t[c] & 0 \\ t[S] & \frac{(\epsilon+\theta)\, c_c + \Xi\, c_S}{c_S} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} -\omega\, c_S\,(\phi + \psi + (\beta\,\phi + \gamma\,\psi)\, c_c + \phi\,\psi\, c_S) & h[S] \\ t[c] & 0 \\ t[S] & \frac{(\epsilon+\theta)\, c_c + \Xi\, c_S}{c_S} \end{pmatrix}\right\}$$