

Pensieve header: Testing the KnotSignature function. Continued pensieve://Projects/Signatures/.

```

In[ ]:= << KnotTheory`

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

In[ ]:= KS[PD[Loop[_]]] = 0;
KS[pd_PD] := KS[pd] = Module[{spd, a, s = 0, c, cs, A, es},
  spd = (Times @@ pd) /. X[i_, j_, k_, l_] => If[j - l == 1 || l - j > 1, Xp, Xm][i, j, k, l];
  cs = spd /. {Xp[i_, j_, k_, l_] => a[j, ++s, i] a[k, ++s, -j] a[-l, ++s, -k] a[-i, ++s, l],
    Xm[i_, j_, k_, l_] => a[-j, ++s, i] a[k, ++s, j] a[l, ++s, -k] a[-i, ++s, -l]} /.
    a[i_, x_, j_] a[j_, y_, k_] => a[i, x, y, k] /. a[i_, x_, j_] => a[x];
  A = Table[0, {Length[cs]}, {Length[cs]}];
  Do[indices = Position[cs, #][[1, 1]] & /@ (4 i - 4 + {1, 2, 3, 4});
    A[[indices, indices]] +=
      If[Head[spd[[i]]] === Xp, {{0, 0, 0, 0}, {1, -1, 0, 0}, {0, -1, 0, 1}, {-1, 2, 0, -1}},
        {{1, -1, 0, 0}, {0, 0, 0, 0}, {-2, 1, 1, 0}, {1, 0, -1, 0}}, {i, Length[spd]}];
  es = Re[Eigenvalues[N[A + Transpose[A], 40]]] /. x_Real /; Abs[x] < 10^-20 -> 0;
  -Plus @@ Sign /@ es]
KS[K_] := KS[PD[K]]

```

```

In[ ]:= Total[(KS[#] == KnotSignature[#]) & /@ AllKnots[]]

```

KnotTheory: Loading precomputed data in PD4Knots`.

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[ ]:= 802 True



```

In[ ]:= Clear[KS1];
KS1[PD[Loop[_]]] = 0;
KS1[pd_PD] := KS1[pd] = Module[{spd, a, s = 0, c, cs, A},
  spd = (Times @@ pd) /. X[i_, j_, k_, L_] => If[j - L == 1 || L - j > 1, Xp, Xm][i, j, k, L];
  cs = spd /. {
    Xp[i_, j_, k_, L_] => a[j, ++s, i] a[k, ++s, -j] a[-L, ++s, -k] a[-i, ++s, L],
    Xm[i_, j_, k_, L_] => a[-j, ++s, i] a[k, ++s, j] a[L, ++s, -k] a[-i, ++s, -L]
  } /. a[i_, x_, j_] a[j_, y_, k_] => a[i, x, y, k] /. a[i_, x_, j_] => a[x];
  A = Table[0, {Length[cs]}, {Length[cs]}];
  Do[indices = Position[cs, #][[1, 1]] & /@ (4 i - 4 + {1, 2, 3, 4});

  A[[indices, indices]] += If[Head[spd][[i]] === Xp,

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 2 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix}],
    {i, Length[spd]}
  ];
  Total[If[# > 0, -1, 1] & /@ DeleteCases[Eigenvalues[A + Transpose[A]], 0]]
]
KS1[K_] := KS1[PD[K]]$$

```

```

In[ ]:= Total[(KS1[#] == KnotSignature[#]) & /@ AllKnots[]]

```

KnotTheory: Loading precomputed data in PD4Knots`.

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[ ]:= 802 True

```

In[ ]:= t = f = 0; fails1 = {};
Monitor[
  Do[If[KS1[K] == KnotSignature[K], ++t, ++f; AppendTo[fails1, K]],
    {K, AllKnots[{3, 16}]}],
  {K, t, f}
];
{t, f}

```

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

KnotTheory: Loading precomputed data in KnotTheory/13A.dts.

General: Further output of KnotTheory:loading will be suppressed during this calculation.

In[ ]:= {t, f, fails1}

Out[ ]:= {1701935, 0, {}}

## Sample Runs

```
In[ ]:= pd = PD@Knot[3, 1]
```

```
Out[ ]:= PD[X[1, 4, 2, 5], X[3, 6, 4, 1], X[5, 2, 6, 3]]
```

```
In[ ]:= spd = (Times @@ pd) /. X[i_, j_, k_, L_] => If[j - L == 1 || L - j > 1, Xp, Xm][i, j, k, L]
```

```
Out[ ]:= Xm[1, 4, 2, 5] Xm[3, 6, 4, 1] Xm[5, 2, 6, 3]
```

```
In[ ]:= s = 0;
```

```
cs = spd /. {
  Xp[i_, j_, k_, L_] => a[j, ++s, i] a[k, ++s, -j] a[-L, ++s, -k] a[-i, ++s, L],
  Xm[i_, j_, k_, L_] => a[-j, ++s, i] a[k, ++s, j] a[L, ++s, -k] a[-i, ++s, -L]
} /. a[i_, x_, j_] a[j_, y_, k_] => a[i, x, y, k] /. a[i_, x_, j_] => a[x]
```

```
Out[ ]:= a[1, 7] a[5, 11] a[9, 3] a[4, 12, 8] a[10, 2, 6]
```

```
In[ ]:= MatrixForm[A = Table[0, {Length[cs]}, {Length[cs]}]]
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= Do[
```

```
indices = Echo[Position[cs, #][[1, 1]] & /@ {4, 3, 2, 1, 0}];
```

```
A[[indices, indices]] += If[Head[spd[[i]]] == Xp,  $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 2 & 0 & -1 \end{pmatrix}$ ,  $\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix}$ ];
```

```
Echo@MatrixForm[A],
```

```
{i, Length[spd]}
```

```
]
```

» {1, 5, 3, 4}

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ -2 & 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

» {2, 5, 1, 4}

$$\begin{pmatrix} 2 & -2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ -2 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

» {3, 5, 2, 4}

$$\begin{pmatrix} 2 & -2 & 0 & 0 & 0 \\ 0 & 2 & -2 & 0 & 0 \\ -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[ ]:= Eigenvalues[A + Transpose[A]]

Out[ ]:= {6, 6, 0, 0, 0}

In[ ]:= Total[If[# > 0, -1, 1] & /@ DeleteCases[Eigenvalues[A + Transpose[A]], 0]]

Out[ ]:= -2

In[ ]:= pd = PD@Knot[8, 17]

Out[ ]:= PD[X[6, 2, 7, 1], X[14, 8, 15, 7], X[8, 3, 9, 4], X[2, 13, 3, 14],  
X[12, 5, 13, 6], X[4, 9, 5, 10], X[16, 12, 1, 11], X[10, 16, 11, 15]]

In[ ]:= spd = (Times@@pd) /. X[i\_, j\_, k\_, l\_] => If[j - l == 1 || l - j > 1, Xp, Xm][i, j, k, l]

Out[ ]:= Xm[2, 13, 3, 14] Xm[4, 9, 5, 10] Xm[8, 3, 9, 4] Xm[12, 5, 13, 6]  
Xp[6, 2, 7, 1] Xp[10, 16, 11, 15] Xp[14, 8, 15, 7] Xp[16, 12, 1, 11]

In[ ]:= s = 0;

cs = spd /. {

Xp[i\_, j\_, k\_, l\_] => a[j, ++s, i] a[k, ++s, -j] a[-l, ++s, -k] a[-i, ++s, l],  
Xm[i\_, j\_, k\_, l\_] => a[-j, ++s, i] a[k, ++s, j] a[l, ++s, -k] a[-i, ++s, -l]  
} /. a[i\_, x\_, j\_] a[j\_, y\_, k\_] => a[i, x, y, k] /. a[i\_, x\_, j\_] => a[x]

Out[ ]:= a[5, 11] a[32, 22] a[3, 9, 25] a[15, 1, 17] a[18, 4, 28] a[30, 16, 20]  
a[7, 13, 29, 21] a[8, 24, 26, 12] a[10, 2, 14, 6] a[23, 31, 19, 27]

```
In[ ]:= MatrixForm[A = Table[0, {Length[cs]}, {Length[cs]}]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= Do[
  Echo@i;
  indices = Echo[Position[cs, #][[1, 1]] & /@ (4 i - {3, 2, 1, 0})];
  A[[indices, indices]] += If[Head[spd[[i]]] === Xp,
    {{0, 0, 0, 0}, {1, -1, 0, 0}, {0, -1, 0, 1}, {-1, 2, 0, -1}},
    {{1, -1, 0, 0}, {0, 0, 0, 0}, {-2, 1, 1, 0}, {1, 0, -1, 0}}
  ];
  Echo@MatrixForm[A],
  {i, Length[spd]}
]
```

» 1

» {4, 9, 3, 5}

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

» 2

» {1, 9, 7, 8}

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

» 3

» {3, 9, 1, 8}

$$\gg \begin{pmatrix} 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

» 4

» {7, 9, 4, 6}

$$\gg \begin{pmatrix} 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

» 5

» {4, 5, 10, 6}

$$\gg \begin{pmatrix} 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 2 & -1 & 1 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

» 6

» {7, 2, 10, 8}

$$\gg \begin{pmatrix} 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 2 & -1 & 1 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & -2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

» 7

» {3, 8, 10, 5}

$$\gg \begin{pmatrix} 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -2 & 2 & -2 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -2 & 2 & -1 & 1 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 & -2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\gg$  8

$\gg$  {7, 6, 10, 2}

$$\gg \begin{pmatrix} 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -2 & 2 & -2 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -2 & 2 & -2 & 2 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 & -2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[ ]:= **Eigenvalues** [A + Transpose[A]] // N

Out[ ]:= {-8.59564, 8.59564, -5.23441, 5.23441, -4.32618, 4.32618, 0., 0., 0., 0.}

In[ ]:= **Total** [If[# > 0, -1, 1] & /@ **DeleteCases** [Eigenvalues [A + Transpose[A]], 0]]

Out[ ]:= 0