AlgebraRelations package

A subpackage for QuantumGroups v2.
Version 2.0, June 11, 2005, Scott Morrison

Introduction

This package produces the relations for a quantum group, and checks that they hold in a given representation.

Implementation

Start of package

Specify package dependencies:

```
BeginPackage["QuantumGroups`AlgebraRelations`",
  {"QuantumGroups`", "QuantumGroups`RootSystems`", "QuantumGroups`Utilities`Debugging`",
   "QuantumGroups`Utilities`MatrixWrapper`", "QuantumGroups`MatrixPresentations`",
   "QuantumGroups`Algebra`", "QuantumGroups`Representations`"}];
```

Usage messages

```
Relations::usage = "";
```

```
CheckRelations::usage = "";
```

Internals

```
Begin["`Private`"];
```

```
q = Global`q
```

$$\texttt{qBinomial}[n\_, k\_][q\_] := \frac{\texttt{qFactorial}[n][q]}{\texttt{qFactorial}[n-k][q]\ \texttt{qFactorial}[k][q]}$$

```
BasicRelations[Γ_] := With[{cm = CartanMatrix[Γ], cf = CartanFactors[Γ], n = Rank[Γ]},
  DeleteCases[Flatten[Join[
      Table[K_i ** K_j == K_j ** K_i, {i, 1, n}, {j, i + 1, n}],
      Table[K_i ** K_i^-1 == 1, {i, 1, n}],
      Table[K_i^-1 ** K_i == 1, {i, 1, n}],
      Table[K_i ** SuperPlus[X_j] ** K_i^-1 == q^(cf⟦i⟧ cm⟦i,j⟧) SuperPlus[X_j], {i, 1, n}, {j, 1, n}],
      Table[
       K_i ** SuperMinus[X_j] ** K_i^-1 == q^(-cf⟦i⟧ cm⟦i,j⟧) SuperMinus[X_j], {i, 1, n}, {j, 1, n}],
      Table[SuperPlus[X_i] ** SuperMinus[X_j] - SuperMinus[X_j] ** SuperPlus[X_i] ==
        DiscreteDelta[i - j] (K_i - K_i^-1)/(q^cf⟦i⟧ - q^-cf⟦i⟧), {i, 1, n}, {j, 1, n}]
      ]] /. {0 → O}, True]
 ]

SerreRelations[Γ_] := With[{cm = CartanMatrix[Γ], cf = CartanFactors[Γ], n = Rank[Γ]},
  DeleteCases[Flatten[Join[
      Table[Sum[(-1)^r qBinomial[1 - cm⟦i, j⟧, r][q^cf⟦i⟧] NonCommutativePower[SuperPlus[X_i],
          1 - cm⟦i, j⟧ - r] ** SuperPlus[X_j] ** NonCommutativePower[SuperPlus[X_i], r],
        {r, 0, 1 - cm⟦i, j⟧}] == O, {i, 1, n}, {j, 1, n}],
      Table[Sum[(-1)^r qBinomial[1 - cm⟦i, j⟧, r][q^cf⟦i⟧] NonCommutativePower[SuperMinus[X_i],
          1 - cm⟦i, j⟧ - r] ** SuperMinus[X_j] ** NonCommutativePower[SuperMinus[X_i], r],
        {r, 0, 1 - cm⟦i, j⟧}] == O, {i, 1, n}, {j, 1, n}]
     ]] /. {0 → O}, True]
 ]
```

```
(*SerreRelations[Γ_]:=With[{cm=CartanMatrix[Γ],cf=CartanFactors[Γ],n=Rank[Γ]},
  DeleteCases[Flatten[Join[
      Table[∑_{r=0}^{1-cm⟦i,j⟧} (-1)^r qBinomial[1-cm⟦i,j⟧,r][q^cf⟦i⟧]
          NonCommutativePower[SuperPlus[X_i],1-cm⟦i,j⟧-r]**SuperPlus[X_j]**
           NonCommutativePower[SuperPlus[X_i],r]==O,{i,1,n},{j,1,n}],
      Table[∑_{r=0}^{1-cm⟦i,j⟧} (-1)^r qBinomial[1-cm⟦i,j⟧,r][q^cf⟦i⟧]
          NonCommutativePower[SuperMinus[X_i],1-cm⟦i,j⟧-r]**SuperMinus[X_j]**
           NonCommutativePower[SuperMinus[X_i],r]==O,{i,1,n},{j,1,n}]
     ]]/.{0→O},True]
 ]*)
```

```
Relations[Γ_] := BasicRelations[Γ] ~Join~ SerreRelations[Γ]
```

```
CheckRelationsInternal[Γ_][V_, β_, λ_] := (Simplify[
    Relations[Γ] /. {a_ == O :> ZeroMatrixQ[Simplify[MatrixPresentation[Γ][a][V, β, λ]]],
      a_ == b_ :> ZeroMatrixQ[Simplify[MatrixPresentation[Γ][a - b][V, β, λ]]]}])
```

```
CheckBasicRelations[Γ_][V_, β_, λ_] := (Simplify[BasicRelations[Γ] /.
    {a_ == 0 :> ZeroMatrixQ[Simplify[MatrixPresentation[Γ][a][V, β, λ]]],
     a_ == b_ :> ZeroMatrixQ[Simplify[MatrixPresentation[Γ][a - b][V, β, λ]]]}])
```

```
CheckRelations[Γ_][V_, β_, λ_] := And @@ (Simplify[
    Relations[Γ] /. {a_ == 0 :> ZeroMatrixQ[Simplify[MatrixPresentation[Γ][a][V, β, λ]]],
      a_ == b_ :> ZeroMatrixQ[Simplify[MatrixPresentation[Γ][a - b][V, β, λ]]]}])
```

```
CheckRelations[Γ_][V_, β_] := And @@ (CheckRelations[Γ][V, β, #] & /@ Weights[Γ, V])
```

```
End[];
```

End of package