Dror Bar-Natan — Most Significant Contributions and Publications

Construct, implement, and document poly-time computable polynomial invariants of knots and tangles

Most Significant Contributions.

(in relation to the proposed project)

Contribution A. My paper *On the Vassiliev Knot Invariants* [4]. This is an old paper, yet it is worth noting as in it I have first laid the foundations for the substitution "diagram-valued invariants of knots" instead of "an invariant for each Lie algebra and representation thereof". This substitution is one of the keys to the realization of the goals of my project: the Lie-theory approach is intrinsically exponential time, yet as I argue in the "Detailed Project Description", the diagrammatic approach (properly modernized) affords quotients that are of polynomial complexity.

Contribution B. My paper with S. Garoufalidis *On the Melvin-Morton-Rozansky Conjecture* [7]. This is the first place where the Alexander polynomial (poly-time) was proven to be dominated by the coloured Jones polynomial (highly exponential). One aspect of my proposed project is the realization that there ought to be further poly-time sections of the coloured Jones polynomial.

Contribution C. My paper *Fast Khovanov Homology Computations* [26] which is a direct continuation of [25]. This paper describes my mathematically-sophisticated methodology for the computation of Khovanov homology. While not poly-time, it is many orders of magnitude more efficient than the naive approach, and it made Khovanov homology computable even for rather large knots (knots with up to 50-70 crossings). Regrettably, Khovanov homology fails criteria C5 of my "Detailed Project Description", and hence the need to do even better.

Contribution D. My paper with Z. Dancso *Finite Type Invariants of W-Knotted Objects I* [36] and its sequels in the same series *II* [37], *III* (in preparation), and *IV* (arXiv:1511.05624). These papers fully analyze the quotient $\mathcal{A}^w$ that is mentioned in the "Detailed Project Description", relating it to 4-dimensional topology, and to Lie algebras and the Kashiwara-Vergne problem. Most importantly from the perspective of the current proposal is that in this series we set up an intricate theoretical framework for computations in $\mathcal{A}^w$. The purpose of the current project is to lift that framework one level higher, to the quotient denoted $\mathcal{A}^{2,2}$ in the detailed description.

Contribution E. I believe in complete transparency and open access, and therefore nearly every talk that I have given in the last 15 years, many with direct relations to this project, was accompanied by an openly-available informational handout and many of the talks are available on video. See ωεβ/Talks. Likewise almost every internal note or computer program that I have written in relation to this project or otherwise in the last 8 years is at ωεβ/AP. Altogether my personal web site ωεβ/me contains several thousand documents and serves as a resource and repository for the knot theory community.

Publications.

(In mathematics authors are normally listed alphabetically)

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[16] D. Bar-Natan, S. Garoufalidis, L. Rozansky, and D. P. Thurston, *The Århus Integral of Rational Homology 3-Spheres II: Invariance and Universality,* Selecta Mathematica, New Series **8** (2002) 341–371.

[17] D. Bar-Natan, B. McKay, M. Bar-Hillel and G. Kalai, *Solving the Bible Code Puzzle,* Statistical Science **14-2** (1999) 150–173.

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