Balloons and Hoops and their Universal Finite Type Invariant, BF Theory, and an Ultimate Alexander Invariant

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http://www.math.toronto.edu/~drorbn/papers/KBH /
{ bch, chic1, chic2, ham, mo, ox, tor, viet }

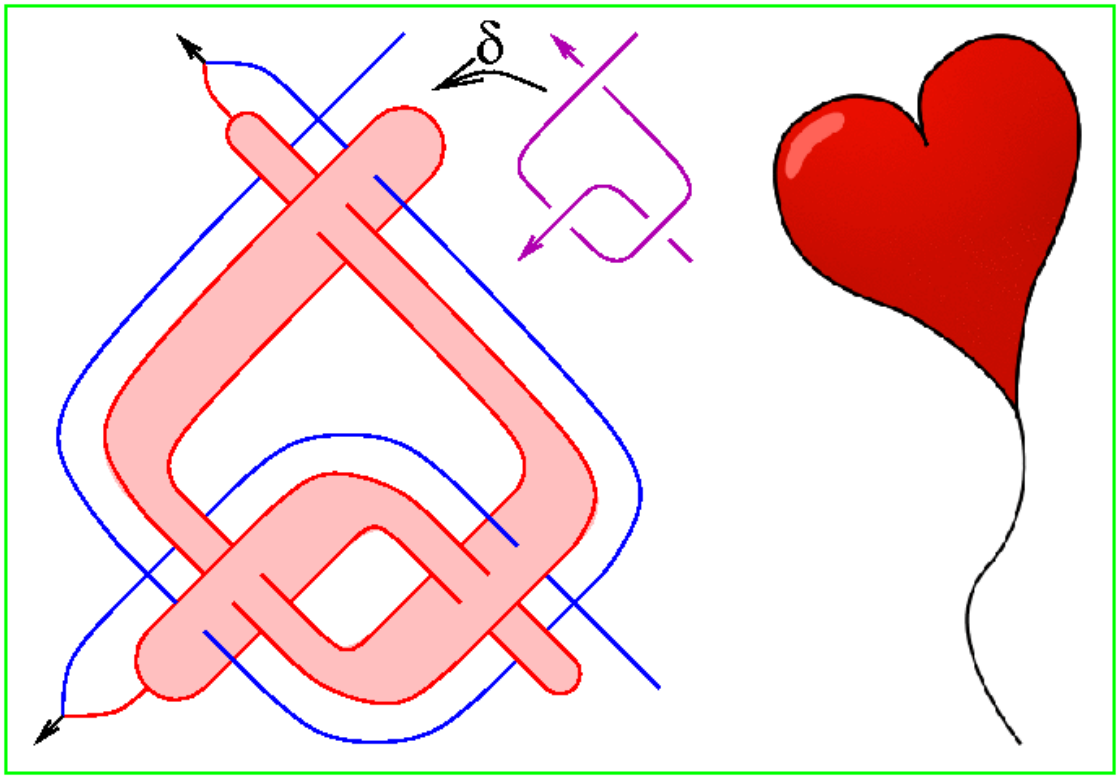
Abstract. Balloons are two-dimensional spheres. Hoops are one dimensional loops. Knotted Balloons and Hoops (KBH) in 4-space behave much like the first and second fundamental groups of a topological space - hoops can be composed as in π_1, balloons as in π_2, and hoops "act" on balloons as π_1 acts on π_2. We observe that ordinary knots and tangles in 3-space map into KBH in 4-space and become amalgams of both balloons and hoops.

We give an ansatz for a tree and wheel (that is, free-Lie and cyclic word) -valued invariant ζ of (ribbon) KBHs in terms of the said compositions and action and we explain its relationship with finite type invariants. We speculate that ζ is a complete evaluation of the BF topological quantum field theory in 4D, though we are not sure what that means. We show that a certain "reduction and repackaging" of ζ is an "ultimate Alexander invariant" that contains the Alexander polynomial (multivariable, if you wish), has extremely good composition properties, is evaluated in a topologically meaningful way, and is least-wasteful in a computational sense. If you believe in categorification, that should be a wonderful playground.

The paper. KBH.pdf, KBH.zip.

Related Mathematica Notebooks. "The free-Lie meta-monoid-action structure" (Source, PDF). "A free-Lie calculator" (Source, PDF).

Related Scratch Work is under Pensieve: KBH.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