Pensieve header: The R-matrix cR for the classical algebra, plus x and y scattering for cR.

Startup

```
In[•]:=   SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\FullDoPeGDO"];
          Once["<< KnotTheory`"];
          << "../Profile/Profile.m";
          << "Engine.m";
          << "Objects.m";
          << "KT.m";
          $k = 0; γ = 1;
          HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, ▮, ▮]];
```

This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.

This version: April 2020. Original version: July 1994.

$In[•]:=$ Timing@HL[$(R_{1,2} \, R_{6,3} \, R_{4,5} \; // \; dm_{1,6\to1} \, dm_{2,4\to2} \, dm_{3,5\to3}) \equiv (R_{2,3} \, R_{1,4} \, R_{5,6} \; // \; dm_{1,5\to1} \, dm_{2,6\to2} \, dm_{3,4\to3})$]

$Out[•]=$

$\{6.6875, \text{True}\}$

$In[•]:=$ Timing@HL[$(R_{1,2} \, R_{6,3} \, R_{4,5} \; // \; cm_{1,6\to1} \, cm_{2,4\to2} \, cm_{3,5\to3}) \equiv (R_{2,3} \, R_{1,4} \, R_{5,6} \; // \; cm_{1,5\to1} \, cm_{2,6\to2} \, cm_{3,4\to3})$]

$Out[•]=$

$\{1.96875, \hbar \, a_2 \, b_1 + \hbar \, a_3 \, (b_1 + b_2) + \hbar \, x_2 \, y_1 + \hbar \, (\hbar \, b_2 + B_2) \, x_3 \, y_1 + \hbar \, B_1 \, x_3 \, y_2 ==$
$\hbar \, a_2 \, b_1 + \hbar \, a_3 \, (b_1 + b_2) + \hbar \, x_2 \, y_1 + \hbar \, x_3 \, y_1 + \hbar \, B_1 \, x_3 \, y_2 \}$

$In[•]:=$ $R_{i,j}$

$Out[•]=$

$\mathbb{E}_{\{\}\to\{i,j\}} [\hbar \, a_j \, b_i + \hbar \, x_j \, y_i]$

$In[•]:=$ $cR_{i\_,j\_} := \mathbb{E}_{\{\}\to\{i,j\}} \left[\hbar \, a_j \, b_i + \frac{B_i - 1}{-b_i} \, x_j \, y_i \right]$

$In[•]:=$ Timing@HL[$(cR_{1,2} \, cR_{6,3} \, cR_{4,5} \; // \; cm_{1,6\to1} \, cm_{2,4\to2} \, cm_{3,5\to3}) \equiv (cR_{2,3} \, cR_{1,4} \, cR_{5,6} \; // \; cm_{1,5\to1} \, cm_{2,6\to2} \, cm_{3,4\to3})$]

$Out[•]=$

$\{2.67188, \text{True}\}$

$\mathbb{E}_{\{\}\to\{1,2\}} [\lambda \, x_1] \, cR_{i,j} \; // \; cm_{1,i\to1} \, cm_{2,j\to2}$

$Out[•]=$

$\mathbb{E}_{\{\}\to\{1,2\}} \left[\hbar \, a_2 \, b_1 + \lambda \, x_1 - \lambda \, (-1 + B_1) \, x_2 - \frac{(-1 + B_1) \, x_2 \, y_1}{b_1} \right]$

x scattering:

$In[\circ]:=$ `lhs = Coefficient[` $\mathbb{E}_{\{\}\to\{i,j\}}[\lambda\, x_i]$ `cR`$_{i1,j1}$ `// cm`$_{i,i1\to i}$ `cm`$_{j,j1\to j}$ `// First,` λ`]`
`rhs = Coefficient[` $\mathbb{E}_{\{\}\to\{i,j\}}[\lambda\,(x_i + (1 - B_i)\, x_j)]$ `cR`$_{i1,j1}$ `// cm`$_{i1,i\to i}$ `cm`$_{j1,j\to j}$ `// First,` λ`]`
`HL[lhs == rhs]`

$Out[\circ]=$
$x_i - (-1 + B_i)\, x_j$

$Out[\circ]=$
$x_i - (-1 + B_i)\, x_j$

$Out[\circ]=$
True

$In[\circ]:=$ `lhs = Coefficient[` $\mathbb{E}_{\{\}\to\{i,j\}}[\lambda\, x_j]$ `cR`$_{i1,j1}$ `// cm`$_{i,i1\to i}$ `cm`$_{j,j1\to j}$ `// First,` λ`]`
`rhs = Coefficient[` $\mathbb{E}_{\{\}\to\{i,j\}}[\lambda\, B_i\, x_j]$ `cR`$_{i1,j1}$ `// cm`$_{i1,i\to i}$ `cm`$_{j1,j\to j}$ `// First,` λ`]`
`HL[lhs == rhs]`

$Out[\circ]=$
$B_i\, x_j$

$Out[\circ]=$
$B_i\, x_j$

$Out[\circ]=$
True

y scattering:

$In[\circ]:=$ `lhs = Coefficient[` $\mathbb{E}_{\{\}\to\{i,j\}}[\lambda\, y_i]$ `cR`$_{i1,j1}$ `// cm`$_{i,i1\to i}$ `cm`$_{j,j1\to j}$ `// First,` λ`]`
`rhs = Coefficient[` $\mathbb{E}_{\{\}\to\{i,j\}}[\lambda\, y_i]$ `cR`$_{i1,j1}$ `// cm`$_{i1,i\to i}$ `cm`$_{j1,j\to j}$ `// First,` λ`]`
`HL[lhs == rhs]`

$Out[\circ]=$
y_i

$Out[\circ]=$
y_i

$Out[\circ]=$
True

In[ ]:= `lhs = Coefficient[`$\mathbb{E}_{\{\}\to\{i,j\}}[\lambda\, y_j]$ `cR`$_{i1,j1}$ `// cm`$_{i,i1\to i}$ `cm`$_{j,j1\to j}$ `// First,` λ`]`

`rhs =`

`Coefficient[`$\mathbb{E}_{\{\}\to\{i,j\}}\left[\lambda\left(\mu\, y_i + B_i^{-1}\, y_j\right)\right]$ `cR`$_{i1,j1}$ `// cm`$_{i1,i\to i}$ `cm`$_{j1,j\to j}$ `// First,` λ`] /.` $\mu \to \dfrac{b_j}{b_i}\left(1 - B_i^{-1}\right)$

`HL[Simplify[lhs == rhs]]`

Out[ ]=

y_j

Out[ ]=

$$\frac{\left(b_j - b_j\, B_i + b_j\left(1 - \frac{1}{B_i}\right) B_i\right) y_i}{b_i\, B_i} + y_j$$

Out[ ]=

<mark>True</mark>

Verifying YB:

In[ ]:= $\mathcal{R}_{i\_,j\_}[\mathcal{E}\_]$ `:= Expand`$\Big[\mathcal{E}$ `/.` $\Big\{$

$x_i \to x_i + (1 - B_i)\, x_j,$

$x_j \to B_i\, x_j,$

$y_i \to y_i,$

$y_j \to \dfrac{b_j}{b_i}\left(1 - B_i^{-1}\right) y_i + B_i^{-1}\, y_j$

$\Big\}\Big]$

In[ ]:= `lhs = {`$x_1, x_2, x_3, y_1, y_2, y_3$`} //` $\mathcal{R}_{1,2}$ `//` $\mathcal{R}_{1,3}$ `//` $\mathcal{R}_{2,3}$

`rhs = {`$x_1, x_2, x_3, y_1, y_2, y_3$`} //` $\mathcal{R}_{2,3}$ `//` $\mathcal{R}_{1,3}$ `//` $\mathcal{R}_{1,2}$

`HL[lhs == rhs]`

Out[ ]=

$$\Big\{x_1 + x_2 - B_1\, x_2 + x_3 - B_1\, x_3,\ B_1\, x_2 + B_1\, x_3 - B_1\, B_2\, x_3,\ B_1\, B_2\, x_3,$$

$$y_1,\ \frac{b_2\, y_1}{b_1} - \frac{b_2\, y_1}{b_1\, B_1} + \frac{y_2}{B_1},\ \frac{b_3\, y_1}{b_1} - \frac{b_3\, y_1}{b_1\, B_1} + \frac{b_3\, y_2}{b_2\, B_1} - \frac{b_3\, y_2}{b_2\, B_1\, B_2} + \frac{y_3}{B_1\, B_2}\Big\}$$

Out[ ]=

$$\Big\{x_1 + x_2 - B_1\, x_2 + x_3 - B_1\, x_3,\ B_1\, x_2 + B_1\, x_3 - B_1\, B_2\, x_3,\ B_1\, B_2\, x_3,$$

$$y_1,\ \frac{b_2\, y_1}{b_1} - \frac{b_2\, y_1}{b_1\, B_1} + \frac{y_2}{B_1},\ \frac{b_3\, y_1}{b_1} - \frac{b_3\, y_1}{b_1\, B_1} + \frac{b_3\, y_2}{b_2\, B_1} - \frac{b_3\, y_2}{b_2\, B_1\, B_2} + \frac{y_3}{B_1\, B_2}\Big\}$$

Out[ ]=

<mark>True</mark>