Pensieve header: Improving the Z Program.

Startup

In[•]:=
```
Date[]
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\FullDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@"../Profile/Profile.m"];
BeginProfile[];
$k = 1;
<< Engine.m
<< Objects.m
<< KT.m
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, , ]];
LogCF[ℰ_] :=
 ℰ //. {c_ Log[a_] :↦ Log[a^c], Log[a_] + Log[b_] :↦ Log[a b], Log[a_] :↦ Log[CF[a]]}
```

Out[•]= {2021, 7, 13, 21, 27, 46.7615360}

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.

This version: April 2020. Original version: July 1994.
```

In[•]:=
```
tab5Z = Timing[Table[K → LogCF@Z[K], {K, AllKnots[{3, 5}]}]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[•]=
$$\left\{35.4375,\ \left\{\text{Knot}[3,1] \to \mathbb{E}_{\{\}\to\{0\}}\left[-2\,t\,\hbar + \text{Log}\left[\frac{T^3}{1-T+T^2}\right],\right.\right.\right.$$
$$\left.\frac{2\,a\,(-1+T)\times(1+T)\,\hbar}{1-T+T^2} + \frac{(-1+T)\times\left(2-T+T^2\right)\hbar}{\left(1-T+T^2\right)^2} - \frac{2\times(1+T)\,x\,y\,\hbar^2}{1-T+T^2}\right],\ \text{Knot}[4,1] \to \mathbb{E}_{\{\}\to\{0\}}\left[\right.$$
$$-t\,\hbar + \text{Log}\left[-\frac{T^2}{1-3\,T+T^2}\right],\ \frac{(-1+T)\times(1+T)\,\hbar}{1-3\,T+T^2} + \frac{2\,a\,(-1+T)\times(1+T)\,\hbar}{1-3\,T+T^2} - \frac{2\times(1+T)\,x\,y\,\hbar^2}{1-3\,T+T^2}\Big],$$
$$\text{Knot}[5,1] \to \mathbb{E}_{\{\}\to\{0\}}\left[-3\,t\,\hbar + \text{Log}\left[\frac{T^5}{1-T+T^2-T^3+T^4}\right],\ \frac{2\,a\,(-1+T)\times(1+T)\times\left(2-T+2\,T^2\right)\hbar}{1-T+T^2-T^3+T^4} + \right.$$
$$\left.\frac{(-1+T)\times\left(4-3\,T+5\,T^2-3\,T^3+3\,T^4-T^5+T^6\right)\hbar}{\left(1-T+T^2-T^3+T^4\right)^2} - \frac{2\times(1+T)\times\left(2-T+2\,T^2\right)x\,y\,\hbar^2}{1-T+T^2-T^3+T^4}\right],$$
$$\text{Knot}[5,2] \to \mathbb{E}_{\{\}\to\{0\}}\left[-3\,t\,\hbar + \text{Log}\left[\frac{T^4}{2-3\,T+2\,T^2}\right],\right.$$
$$\left.\left.\left.\frac{4\,a\,(-1+T)\times(1+T)\,\hbar}{2-3\,T+2\,T^2} - \frac{(-1+T)\times\left(-9+11\,T-7\,T^2+T^3\right)\hbar}{\left(2-3\,T+2\,T^2\right)^2} - \frac{4\times(1+T)\,x\,y\,\hbar^2}{2-3\,T+2\,T^2}\right]\right\}\right\}$$

```
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := (*Z[rvk]=*)
 Monitor[PP"Z"@Module[{todo, n, rots, ℊ, done, st, cx, ℊ1, i, j, k, k1, k2, k3},
    {todo, rots} = List @@ rvk;
    AppendTo[rots, 0];
    n = Length[todo];
    ℊ = dη0;
    done = {0};
    st = Range[0, 2 n + 1];
    While[{} =!= ($M = todo),
     {cx} = MaximalBy[todo, Length[done ⋂ {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
     {i, j} = List @@ cx;
     ℊ1 = Switch[Head[cx],
        Xp, (kR_{i,j} \[OverBar]kKink_k) // km_{j,k→j},
        Xm, (\[OverBar]kR_{i,j} kKink_k) // km_{j,k→j}
       ];
     ℊ1 = (rot[k, rots[[i]]] ℊ1) // km_{k,i→i}; rots[[i]] = 0;
     ℊ1 = (ℊ1 rot[k, rots[[i + 1]]]) // km_{i,k→i}; rots[[i + 1]] = 0;
     ℊ1 = (rot[k, rots[[j]]] ℊ1) // km_{k,j→j}; rots[[j]] = 0;
     ℊ1 = (ℊ1 rot[k, rots[[j + 1]]]) // km_{j,k→j}; rots[[j + 1]] = 0;
     ℊ *= ℊ1;
     If[MemberQ[done, i], ℊ = ℊ // km_{i,i+1→i}; st = st /. st[[i + 2]] → st[[i + 1]]];
     If[MemberQ[done, i - 1], ℊ = ℊ // km_{st[[i]],i→st[[i]]}; st = st /. st[[i + 1]] → st[[i]]];
     If[MemberQ[done, j], ℊ = ℊ // km_{j,j+1→j}; st = st /. st[[j + 2]] → st[[j + 1]]];
     If[MemberQ[done, j - 1], ℊ = ℊ // km_{st[[j]],j→st[[j]]}; st = st /. st[[j + 1]] → st[[j]]];
     done = done ⋃ {i - 1, i, j - 1, j};
     todo = DeleteCases[todo, cx]
    ];
    CF /@ (ℊ /. {x0 → x, y0 → y, a0 → a})
   ], $M]
```

```
Z1[K_] := Z1[RVK@K];
Z1[rvk_RVK] := Monitor[ PP"z"@Module[{todo, rots, ℊ, done, st, cx, ℊ1, i, j, k},
    {todo, rots} = List @@ rvk;
    ℊ = dη₀;
    done = {0};
    st = Range[0, 2 Length[todo]];
    While[{} =!= ($M = todo),
     cx =
      RandomChoice@MaximalBy[todo, Length[done ⋂ {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &];
     {i, j} = List @@ cx;
     ℊ1 = (cx /. {_Xp :> kR_{i,j} kKink₀, _Xm :> kR̄_{i,j} kKink₀}) // km_{j,0→j};
     Do[ℊ1 = (rot[0, rots[[k]]] ℊ1) // km_{0,k→k}, {k, {i, j}}];
     ℊ *= ℊ1;
     Do[
      If[MemberQ[done, k], ℊ = ℊ // km_{st[[k+1]],k+1→st[[k+1]]}; st = st /. st[[k + 2]] → st[[k + 1]]],
      {k, {i, i - 1, j, j - 1}}];
     done = done ⋃ {i - 1, i, j - 1, j};
     todo = DeleteCases[todo, cx]
    ];
    CF /@ (ℊ /. {x₀ → x, y₀ → y, a₀ → a})
   ], $M]
```

In[•]:= `tab5Z1 = Timing[Table[K → LogCF@Z1[K], {K, AllKnots[{3, 5}]}]]`

Out[•]=

$$\Big\{23.0469,\ \Big\{\text{Knot}[3,1] \to \mathbb{E}_{\{\}\to\{0\}}\Big[-2\,t\,\hbar + \text{Log}\Big[\frac{T^3}{1-T+T^2}\Big],\ \frac{2\,a\,(-1+T)\times(1+T)\,\hbar}{1-T+T^2} + \frac{(-1+T)\times\left(2-T+T^2\right)\hbar}{\left(1-T+T^2\right)^2} - \frac{2\times(1+T)\,x\,y\,\hbar^2}{1-T+T^2}\Big],\ \text{Knot}[4,1] \to \mathbb{E}_{\{\}\to\{0\}}\Big[$$

$$-t\,\hbar + \text{Log}\Big[-\frac{T^2}{1-3\,T+T^2}\Big],\ \frac{(-1+T)\times(1+T)\,\hbar}{1-3\,T+T^2} + \frac{2\,a\,(-1+T)\times(1+T)\,\hbar}{1-3\,T+T^2} - \frac{2\times(1+T)\,x\,y\,\hbar^2}{1-3\,T+T^2}\Big],$$

$$\text{Knot}[5,1] \to \mathbb{E}_{\{\}\to\{0\}}\Big[-3\,t\,\hbar + \text{Log}\Big[\frac{T^5}{1-T+T^2-T^3+T^4}\Big],\ \frac{2\,a\,(-1+T)\times(1+T)\times\left(2-T+2\,T^2\right)\hbar}{1-T+T^2-T^3+T^4} +$$

$$\frac{(-1+T)\times\left(4-3\,T+5\,T^2-3\,T^3+3\,T^4-T^5+T^6\right)\hbar}{\left(1-T+T^2-T^3+T^4\right)^2} - \frac{2\times(1+T)\times\left(2-T+2\,T^2\right)x\,y\,\hbar^2}{1-T+T^2-T^3+T^4}\Big],$$

$$\text{Knot}[5,2] \to \mathbb{E}_{\{\}\to\{0\}}\Big[-3\,t\,\hbar + \text{Log}\Big[\frac{T^4}{2-3\,T+2\,T^2}\Big],$$

$$\frac{4\,a\,(-1+T)\times(1+T)\,\hbar}{2-3\,T+2\,T^2} - \frac{(-1+T)\times\left(-9+11\,T-7\,T^2+T^3\right)\hbar}{\left(2-3\,T+2\,T^2\right)^2} - \frac{4\times(1+T)\,x\,y\,\hbar^2}{2-3\,T+2\,T^2}\Big]\Big\}\Big\}$$

In[•]:= `HL[tab5Z[[2]] == tab5Z1[[2]]]`

Out[•]= True

```
In[•]:= tab7Z = Timing[Table[K → LogCF@Z[K], {K, AllKnots[{3, 7}]}]];

In[•]:= tab7Z1 = Timing[Table[K → LogCF@Z1[K], {K, AllKnots[{3, 7}]}]];
        HL[tab7Z[[2]] == tab7Z1[[2]]]

Out[•]= True

In[•]:= Timing[tab10Z = Table[K → LogCF@Z[K], {K, AllKnots[{3, 10}]}];]

Out[•]= {7519.83, Null}

In[•]:= Timing[tab10Z1 = Table[K → LogCF@Z1[K], {K, AllKnots[{3, 10}]}];]
        HL[tab10Z[[2]] == tab10Z1[[2]]]

Out[•]= {6947.72, Null}

Out[•]= True
```