

Pensieve header: The GST-48 knot. Continues pensieve://2016-09/.

(Alt) In[]:=

```
Date []
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\FullDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@"..\\Profile\\Profile.m"];
BeginProfile[];
$k = 1;
<< Engine.m
<< Objects.m
<< KT.m
```

(Alt) Out[]:= {2021, 8, 18, 14, 20, 49.8429650}

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

» CCFLogFile is CCFLog-2021-08-18T14-20-50.m

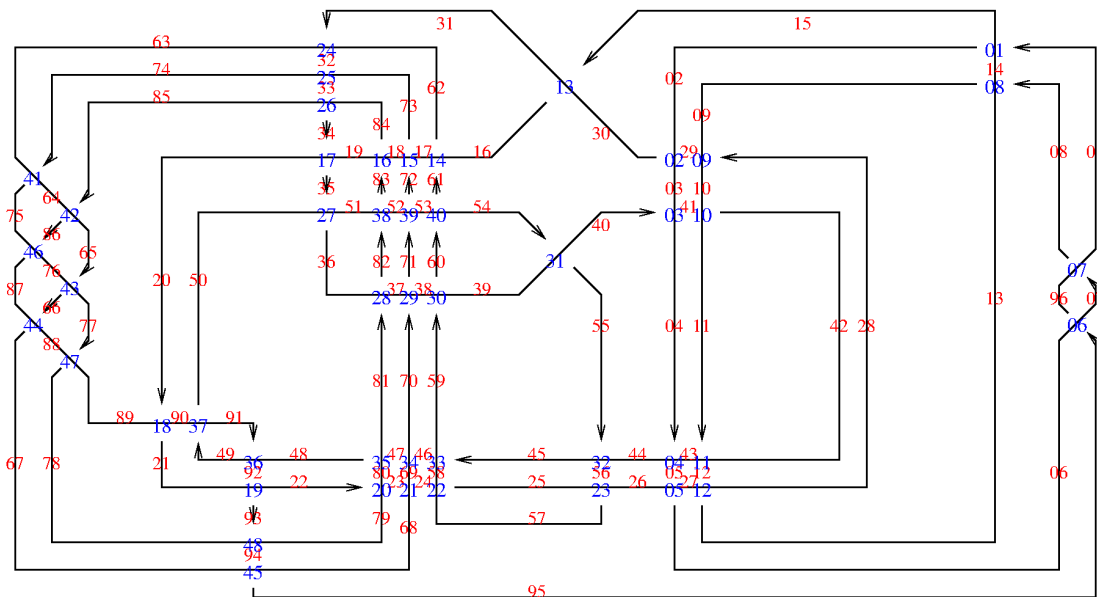
(Alt) In[]:=

```
HL[ε_] := Style[ε, Background → If[TrueQ@ε, █, █]]];
```

(Alt) In[]:=

```
Import["../2016-09/GST48-Marked.png"]
```

(Alt) Out[]:=



```
(Alt) In[ ]:= PD[GST48] = PD[
  X[01, 15, 02, 14], X[29, 02, 30, 03],
  X[40, 04, 41, 03], X[04, 44, 05, 43], X[05, 26, 06, 27],
  X[95, 07, 96, 06], X[07, 01, 08, 96], X[08, 14, 09, 13],
  X[28, 09, 29, 10], X[41, 11, 42, 10],
  X[11, 43, 12, 42], X[12, 27, 13, 28], X[15, 31, 16, 30],
  X[61, 16, 62, 17], X[72, 17, 73, 18],
  X[83, 18, 84, 19], X[34, 20, 35, 19], X[20, 89, 21, 90],
  X[92, 21, 93, 22], X[22, 79, 23, 80],
  X[23, 68, 24, 69], X[24, 57, 25, 58], X[56, 25, 57, 26],
  X[31, 63, 32, 62], X[32, 74, 33, 73],
  X[33, 85, 34, 84], X[35, 50, 36, 51], X[81, 37, 82, 36],
  X[70, 38, 71, 37], X[59, 39, 60, 38],
  X[54, 39, 55, 40], X[55, 45, 56, 44], X[45, 59, 46, 58],
  X[46, 70, 47, 69], X[47, 81, 48, 80],
  X[91, 49, 92, 48], X[49, 91, 50, 90], X[82, 52, 83, 51],
  X[71, 53, 72, 52], X[60, 54, 61, 53],
  X[74, 63, 75, 64], X[85, 64, 86, 65], X[65, 76, 66, 77],
  X[66, 87, 67, 88], X[94, 67, 95, 68],
  X[86, 75, 87, 76], X[77, 88, 78, 89], X[93, 78, 94, 79] ];
```

```
(Alt) In[ ]:= Alexander[GST48][t] // Factor
```

$$(Alt) Out[]:= - \frac{(-1 + 2t - t^2 - t^3 + 2t^4 - t^5 + t^8) \times (-1 + t^3 - 2t^4 + t^5 + t^6 - 2t^7 + t^8)}{t^8}$$

```
(Alt) In[ ]:= KnotSignature[GST48]
```

```
(Alt) Out[ ]:= 0
```

```
(Alt) In[ ]:= With[{f = -1 + 2t - t^2 - t^3 + 2t^4 - t^5 + t^8},
  HL@Simplify[Alexander[GST48][t] == f (f /. t -> 1/t)]]
```

```
(Alt) Out[ ]:= True
```

```
(Alt) In[ ]:= Timing@Block[{$k = 1}, Z[Knot[3, 1]]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

- » 4
- » 0.015
- » 0.016
- » 0.016
- » 0.016
- » 0.031
- » 0.078
- » 0.203

(Alt) Out[]:= {17.6406,

$$\mathbb{E}_{\{\} \rightarrow \{1\}} \left[-2 t \hbar + \text{Log} \left[\frac{T^3}{1 - T + T^2} \right], \frac{2 a (-\hbar + T^2 \hbar)}{1 - T + T^2} + \frac{-2 \hbar + 3 T \hbar - 2 T^2 \hbar + T^3 \hbar}{(1 - T + T^2)^2} - \frac{2 x y (\hbar^2 + T \hbar^2)}{1 - T + T^2} \right]$$

(Alt) In[]:= Timing@Block[{\$k = 2}, Z[Knot[3, 1]]]

- » 4
- » 0.25
- » 0.313
- » 1.657
- » 3.328
- » 19.343

(Alt) Out[]:= {128.922,

$$\mathbb{E}_{\{\} \rightarrow \{1\}} \left[-2 t \hbar + \text{Log} \left[\frac{T^3}{1 - T + T^2} \right], \frac{2 a (-\hbar + T^2 \hbar)}{1 - T + T^2} + \frac{-2 \hbar + 3 T \hbar - 2 T^2 \hbar + T^3 \hbar}{(1 - T + T^2)^2} - \frac{2 x y (\hbar^2 + T \hbar^2)}{1 - T + T^2}, \right. \\ \frac{2 a^2 (T \hbar^2 - 4 T^2 \hbar^2 + T^3 \hbar^2)}{(1 - T + T^2)^2} + \frac{2 a (T \hbar^2 - 7 T^2 \hbar^2 + 6 T^3 \hbar^2 - 3 T^4 \hbar^2 + T^5 \hbar^2)}{(1 - T + T^2)^3} + \\ \frac{T \hbar^2 - 11 T^2 \hbar^2 + 16 T^3 \hbar^2 - 12 T^4 \hbar^2 + 8 T^5 \hbar^2 - 3 T^6 \hbar^2 + T^7 \hbar^2}{2 (1 - T + T^2)^4} - \frac{4 a x y (-2 T \hbar^3 + 2 T^2 \hbar^3 + T^3 \hbar^3)}{(1 - T + T^2)^2} + \\ \left. \frac{2 x y (-\hbar^3 - T^2 \hbar^3 - 3 T^3 \hbar^3 + T^5 \hbar^3)}{(1 - T + T^2)^3} + \frac{x^2 y^2 (\hbar^4 + 5 T \hbar^4 + T^2 \hbar^4)}{(1 - T + T^2)^2} \right]$$

(Alt) In[]:= **RVK[GST48]**

(Alt) Out[]:= RVK[{Xp[14, 1], Xm[2, 29], Xp[3, 40], Xp[43, 4], Xm[26, 5], Xp[6, 95], Xp[96, 7],
 Xp[13, 8], Xm[9, 28], Xp[10, 41], Xp[42, 11], Xm[27, 12], Xp[30, 15], Xm[16, 61],
 Xm[17, 72], Xm[18, 83], Xp[19, 34], Xm[89, 20], Xm[21, 92], Xm[79, 22], Xm[68, 23],
 Xm[57, 24], Xm[25, 56], Xp[62, 31], Xp[73, 32], Xp[84, 33], Xm[50, 35], Xp[36, 81],
 Xp[37, 70], Xp[38, 59], Xm[39, 54], Xp[44, 55], Xp[58, 45], Xp[69, 46], Xp[80, 47],
 Xp[48, 91], Xp[90, 49], Xp[51, 82], Xp[52, 71], Xp[53, 60], Xm[63, 74], Xm[64, 85],
 Xm[76, 65], Xm[87, 66], Xm[67, 94], Xm[75, 86], Xm[88, 77], Xm[78, 93]},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 -1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 1, 0, 0, 0, 0, 0, 0, -1,
 0, 0, 0, 0, 0, 0, -1, 1, -1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, -1, 0, 1, 0,
 0, 0, 0, 0, 0, 0, 1, -1, 0, 1, 0, 0, 0, 0, 0, 0, 1, -1, 0, 0, 0, 0, 0}]

(Alt) In[]:= **Timing@Block[{\$k = 1}, Z[GST48]]**

» 12

(Alt) Out[]:= {4485.94,

$$\mathbb{E}_{\{t\} \rightarrow \{1\}} \left[3 t \hbar + \text{Log} \left[\left(\sqrt{\frac{T^{12}}{(1 - 2 T + T^2 + 6 T^3 - 23 T^4 + 34 T^5 - 17 T^6 - 25 T^7 + 48 T^8 - 26 T^9 - 13 T^{10} + 31 T^{11} - 25 T^{12} + 11 T^{13} - T^{14} - 2 T^{15} + T^{16})^2}} \right) (1 - 2 T + T^2 + 6 T^3 - 23 T^4 + 34 T^5 - 17 T^6 - 25 T^7 + 48 T^8 - 26 T^9 - 13 T^{10} + 31 T^{11} - 25 T^{12} + 11 T^{13} - T^{14} - 2 T^{15} + T^{16}) \right] \right. \\
\left. (T (-1 + 2 T - T^2 - T^3 + 2 T^4 - T^5 + T^8)) \times (-1 + T^3 - 2 T^4 + T^5 + T^6 - 2 T^7 + T^8) \right], \\
(2 a (-8 \hbar + 14 T \hbar - 6 T^2 \hbar - 8 T^4 \hbar + 15 T^5 \hbar - 4 T^6 \hbar - 7 T^7 \hbar + 7 T^9 \hbar + 4 T^{10} \hbar - 15 T^{11} \hbar + 8 T^{12} \hbar + 6 T^{14} \hbar - 14 T^{15} \hbar + 8 T^{16} \hbar)) / \\
(1 - 2 T + T^2 + 2 T^4 - 5 T^5 + 2 T^6 + 7 T^7 - 13 T^8 + 7 T^9 + 2 T^{10} - 5 T^{11} + 2 T^{12} + T^{14} - 2 T^{15} + T^{16}) + \\
(-13 \hbar + 58 T \hbar - 116 T^2 \hbar + 142 T^3 \hbar - 129 T^4 \hbar + 93 T^5 \hbar + 4 T^6 \hbar - 140 T^7 \hbar + 64 T^8 \hbar + 372 T^9 \hbar - 707 T^{10} \hbar + 316 T^{11} \hbar + 562 T^{12} \hbar - 984 T^{13} \hbar + 592 T^{14} \hbar - 33 T^{15} \hbar - 50 T^{16} \hbar - 235 T^{17} \hbar + 474 T^{18} \hbar - 480 T^{19} \hbar + 202 T^{20} \hbar + 336 T^{21} \hbar - 809 T^{22} \hbar + 792 T^{23} \hbar - 304 T^{24} \hbar - 176 T^{25} \hbar + 284 T^{26} \hbar - 105 T^{27} \hbar - 69 T^{28} \hbar + 90 T^{29} \hbar - 32 T^{30} \hbar - 2 T^{31} \hbar + 3 T^{32} \hbar) / \\
(1 - 2 T + T^2 + 2 T^4 - 5 T^5 + 2 T^6 + 7 T^7 - 13 T^8 + 7 T^9 + 2 T^{10} - 5 T^{11} + 2 T^{12} + T^{14} - 2 T^{15} + T^{16})^2 - \\
(2 x y (8 \hbar^2 - 6 T \hbar^2 + 8 T^4 \hbar^2 - 7 T^5 \hbar^2 - 3 T^6 \hbar^2 + 4 T^7 \hbar^2 + 4 T^8 \hbar^2 - 3 T^9 \hbar^2 - 7 T^{10} \hbar^2 + 8 T^{11} \hbar^2 - 6 T^{14} \hbar^2 + 8 T^{15} \hbar^2)) / \\
(1 - 2 T + T^2 + 2 T^4 - 5 T^5 + 2 T^6 + 7 T^7 - 13 T^8 + 7 T^9 + 2 T^{10} - 5 T^{11} + 2 T^{12} + T^{14} - 2 T^{15} + T^{16}) \Big]$$

(Alt) In[]:= **PrintProfile[]**

(Alt) Out[]:= ProfileRoot is root. Profiled time: 4630.75
 (3) 11.616/ 4630.735 above Z
 (4) 0.015/ 0.015 above RVK
 CF: called 109852 times, time in 2198.39/4316.86
 (1006) 269.524/ 544.953 under Z
 (204) 0.467/ 0.672 under Boot
 (1001) 1415.714/ 2737.065 under EZip3
 (642) 5.713/ 16.193 under Zip1

```

( 25733) 210.299/ 415.153 under Zip2
( 81266) 296.676/ 602.825 under Zip3
( 279805) 2118.468/ 2118.468 above CCF
CCF: called 279805 times, time in 2118.47/2118.47
( 279805) 2118.468/ 2118.468 under CF
EZip3: called 321 times, time in 195.56/3033.17
( 270) 195.342/ 3025.364 under Z
( 51) 0.218/ 7.808 under Boot
( 1001) 1415.714/ 2737.065 above CF
( 321) 22.691/ 100.547 above Zip3
Zip3: called 642 times, time in 79.925/682.75
( 270) 54.184/ 574.058 under Z
( 51) 3.050/ 8.145 under Boot
( 321) 22.691/ 100.547 under EZip3
( 81266) 296.676/ 602.825 above CF
Zip2: called 321 times, time in 18.176/433.329
( 270) 16.630/ 428.701 under Z
( 51) 1.546/ 4.628 under Boot
( 25733) 210.299/ 415.153 above CF
Z: called 3 times, time in 11.616/4630.74
( 3) 11.616/ 4630.735 under ProfileRoot
( 13) 0.032/ 24.455 above Boot
( 1006) 269.524/ 544.953 above CF
( 270) 195.342/ 3025.364 above EZip3
( 270) 5.884/ 21.588 above Zip1
( 270) 16.630/ 428.701 above Zip2
( 270) 54.184/ 574.058 above Zip3
Zip1: called 321 times, time in 8.191/24.384
( 270) 5.884/ 21.588 under Z
( 51) 2.307/ 2.796 under Boot
( 642) 5.713/ 16.193 above CF
Boot: called 48 times, time in 0.406/64.363
( 13) 0.032/ 24.455 under Z
( 35) 0.374/ 39.908 under Boot
( 35) 0.374/ 39.908 above Boot
( 204) 0.467/ 0.672 above CF
( 51) 0.218/ 7.808 above EZip3
( 51) 2.307/ 2.796 above Zip1
( 51) 1.546/ 4.628 above Zip2
( 51) 3.050/ 8.145 above Zip3
RVK: called 4 times, time in 0.015/0.015
( 4) 0.015/ 0.015 under ProfileRoot

```

```
(Alt) In[ ]:= Timing@Block[{$k = 2}, Z[GST48]]
```

```
» 12
```

```
(Alt) In[ ]:= PrintProfile[]
```

```

(Alt) Out[ ] = ProfileRoot is root. Profiled time: 776083.
  ( 4) 3.722/ 1845.001 above Z
  ( 5) 0.031/ 0.031 above RVK
CCF: called 3015476 times, time in 563667./563667.
  ( 3015476) 563666.678/ 563666.678 under CF
CF: called 194237 times, time in 207500./771110.
  ( 2269) 5095.857/ 11937.173 under Z
  ( 225) 0.281/ 0.344 under Boot
  ( 1953) 190079.201/ 727773.191 under EZip3
  ( 1118) 7.048/ 19.953 under Zip1
  ( 43713) 3846.584/ 8931.960 under Zip2
  ( 144959) 8470.808/ 22447.569 under Zip3
  ( 3015476) 563666.678/ 563666.678 above CCF
EZip3: called 559 times, time in 3782.68/732075.
  ( 503) 3782.523/ 732070.863 under Z
  ( 56) 0.158/ 4.615 under Boot
  ( 1953) 190079.201/ 727773.191 above CF
  ( 559) 54.346/ 537.559 above Zip3
Zip3: called 1117 times, time in 980.536/23428.1
  ( 502) 924.330/ 22885.379 under Z
  ( 56) 1.860/ 5.167 under Boot
  ( 559) 54.346/ 537.559 under EZip3
  ( 144959) 8470.808/ 22447.569 above CF
Zip2: called 559 times, time in 137.594/9069.55
  ( 503) 136.871/ 9066.618 under Z
  ( 56) 0.723/ 2.936 under Boot
  ( 43713) 3846.584/ 8931.960 above CF
Zip1: called 559 times, time in 11.211/31.164
  ( 503) 9.726/ 29.400 under Z
  ( 56) 1.485/ 1.764 under Boot
  ( 1118) 7.048/ 19.953 above CF
Z: called 4 times, time in 3.722/1845.
  ( 4) 3.722/ 1845.001 under ProfileRoot
  ( 16) 0.048/ 15.203 above Boot
  ( 2269) 5095.857/ 11937.173 above CF
  ( 503) 3782.523/ 732070.863 above EZip3
  ( 503) 9.726/ 29.400 above Zip1
  ( 503) 136.871/ 9066.618 above Zip2
  ( 502) 924.330/ 22885.379 above Zip3
Boot: called 53 times, time in 0.377/38.903
  ( 16) 0.048/ 15.203 under Z
  ( 37) 0.329/ 23.700 under Boot
  ( 37) 0.329/ 23.700 above Boot
  ( 225) 0.281/ 0.344 above CF
  ( 56) 0.158/ 4.615 above EZip3
  ( 56) 1.485/ 1.764 above Zip1
  ( 56) 0.723/ 2.936 above Zip2

```

```
(      56)      1.860/      5.167 above Zip3  
RVK: called 5 times, time in 0.031/0.031  
(      5)      0.031/      0.031 under ProfileRoot
```

```
(Alt) In[ ]:= Block[{$k = 3}, Z[GST48]]
```

```
PrintProfile[]
```

```
(Alt) In[ ]:= Exit[]
```