Pensieve header: Exponentiation in ybax algebras.

Startup

In[ ]:=
```
Date[]
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\FullDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@"../Profile/Profile.m"];
BeginProfile[];
$k = 2;
<< Engine.m
<< Objects.m
<< KT.m
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, ■, ■]];
```

Out[ ]= {2021, 8, 15, 20, 50, 49.0294130}

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.
This version: April 2020. Original version: July 1994.
```

Exponentials

Task. Define $\text{Exp}_m[\text{U\_}(\mathbb{U}_{\{\_\_\_\}\to\{\text{i\_}\}}]$ to compute $e^{\mathbb{O}(U)}$ to order $\epsilon^{\text{Length@}\{U\}-1}$ using the $m_{i,i\to i}$ multiplication, where U is an ϵ-dependent near-docile element, giving the answer in $\mathbb{E}$-form.
Example: $\text{Exp}_{\text{dm},1}[\mathbb{U}_{\{\}\to\{2\}}[b_2\, a_2 + y_2\, x_2, 0]]$ is the exponential of the arrow on strand 2, computed to degree 1.

In[ ]:=
```
Exp_m_[𝕌_{is_→{i_}}[U__]] :=
 Module[{λ, k, n, F, f, j, lhs, rhs, sol, MI (*multi-index*), mis, mi, yax},
  MI /: Coefficient[ℰ_, MI[p_, n_, q_]] :=
   Coefficient[Coefficient[Coefficient[ℰ, y_i, p], a_i, n], x_i, q];
  yax /: yax^MI[p_,n_,q_] := y_i^p a_i^n x_i^q;
  F = 𝔼_{{}→{i}}[];
  Do[
   mis =
    Flatten@Table[MI[p, n, q], {n, 0, k + 1}, {p, 0, 2 k + 2 - 2 n}, {q, 0, 2 k + 2 - 2 n - p}];
   AppendTo[F, Sum[f_mi[λ] yax^mi, {mi, mis}]];
   Echo@F;
   If[k == 0,
    mis = DeleteCases[mis, MI[0, 1, 0]];
    F = F /. f_MI[0,1,0][λ] → λ Coefficient[{U}⟦1⟧, MI[0, 1, 0]]
   ];
   lhs = (∂_μ U2l@Last[F (F /. {λ → μ, i → j}) // m_{i,j→i}]) /. μ → 0 /. f_[0] → 0 /.
     Table[f_mi'[0] → Coefficient[{U}⟦k + 1⟧, mi], {mi, mis}];
   rhs = ∂_λ Last[F];
   F = F /. First@DSolve[Table[Coefficient[lhs - rhs, mi] == 0 ∧ f_mi[0] == 0, {mi, mis}],
       Table[f_mi, {mi, mis}], λ],
   {k, 0, Length[{U}] - 1}
  ];
  CF@l2U[F /. λ → 1]
 ]
```

In[ ]:=
```
Exp_cm[𝕌_{{}→{i}}[ℏ a_i b_i + ℏ x_i y_i, c_1 (x_i + y_i)]]
```

» $\mathbb{E}_{\{\}\to\{i\}}\left[f\$16655_{MI\$16655[0,0,0]}[\lambda\$16655] + x_i\, f\$16655_{MI\$16655[0,0,1]}[\lambda\$16655] + x_i^2\, f\$16655_{MI\$16655[0,0,2]}[\lambda\$16655] + a_i\, f\$16655_{MI\$16655[0,1,0]}[\lambda\$16655] + y_i\, f\$16655_{MI\$16655[1,0,0]}[\lambda\$16655] + x_i\, y_i\, f\$16655_{MI\$16655[1,0,1]}[\lambda\$16655] + y_i^2\, f\$16655_{MI\$16655[2,0,0]}[\lambda\$16655]\right]$

» $\mathbb{E}_{\{\}\to\{i\}}\left[\lambda\$16655\,\hbar\, a_i\, b_i + \frac{e^{-\lambda\$16655\,\hbar\, b_i}\left(-1 + e^{\lambda\$16655\,\hbar\, b_i}\right) x_i\, y_i}{b_i},\right.$
$f\$16655_{MI\$16655[0,0,0]}[\lambda\$16655] + x_i\, f\$16655_{MI\$16655[0,0,1]}[\lambda\$16655] + x_i^2\, f\$16655_{MI\$16655[0,0,2]}[\lambda\$16655] + x_i^3\, f\$16655_{MI\$16655[0,0,3]}[\lambda\$16655] + x_i^4\, f\$16655_{MI\$16655[0,0,4]}[\lambda\$16655] + a_i\, f\$16655_{MI\$16655[0,1,0]}[\lambda\$16655] + a_i\, x_i\, f\$16655_{MI\$16655[0,1,1]}[\lambda\$16655] + a_i\, x_i^2\, f\$16655_{MI\$16655[0,1,2]}[\lambda\$16655] + a_i^2\, f\$16655_{MI\$16655[0,2,0]}[\lambda\$16655] + y_i\, f\$16655_{MI\$16655[1,0,0]}[\lambda\$16655] + x_i\, y_i\, f\$16655_{MI\$16655[1,0,1]}[\lambda\$16655] + x_i^2\, y_i\, f\$16655_{MI\$16655[1,0,2]}[\lambda\$16655] + x_i^3\, y_i\, f\$16655_{MI\$16655[1,0,3]}[\lambda\$16655] + a_i\, y_i\, f\$16655_{MI\$16655[1,1,0]}[\lambda\$16655] + a_i\, x_i\, y_i\, f\$16655_{MI\$16655[1,1,1]}[\lambda\$16655] + y_i^2\, f\$16655_{MI\$16655[2,0,0]}[\lambda\$16655] + x_i\, y_i^2\, f\$16655_{MI\$16655[2,0,1]}[\lambda\$16655] + x_i^2\, y_i^2\, f\$16655_{MI\$16655[2,0,2]}[\lambda\$16655] + a_i\, y_i^2\, f\$16655_{MI\$16655[2,1,0]}[\lambda\$16655] + y_i^3\, f\$16655_{MI\$16655[3,0,0]}[\lambda\$16655] + x_i\, y_i^3\, f\$16655_{MI\$16655[3,0,1]}[\lambda\$16655] + y_i^4\, f\$16655_{MI\$16655[4,0,0]}[\lambda\$16655]\Big]$

Set: Part 4 of Asymptotics`AsymptoticRSolveValueDump`c$18744⟦Asymptotics`AsymptoticRSolveValueDump`i$18744⟧ does not exist.

Set: Part 5 of Asymptotics`AsymptoticRSolveValueDump`c$18744⟦Asymptotics`AsymptoticRSolveValueDump`i$18744⟧ does not exist.

Part: Part 4 of {0, 0, 0} does not exist.

Part: Part 5 of {0, 0, 0} does not exist.

Set: Part 4 of {0, 0, {0, 0, 0}⟦5⟧} does not exist.

General: Further output of Set::partw will be suppressed during this calculation.

Part: Part 4 of {0, 0, {0, 0, 0}⟦5⟧} does not exist.

General: Further output of Part::partw will be suppressed during this calculation.

Inverse: Matrix $\Big\{\{0,1,0,0,0\},\{0,0,1,0,0\},\left\{-2\,e^{-\lambda\$16655\,\hbar\, b_i}\,\hbar,0,0,0,0\right\},\{0,0,\{0,0,0\}[\![4]\!],0,0\},\Big\{0,0,$
$\frac{\{0,0,0\}[\![4]\!] - \{\ll3\gg\}[\![5]\!]^2 + \{\ll3\gg\}[\![5]\!]^2\,\{0,0,\text{Part}[\ll2\gg]\}[\![4]\!]}{\{0,0,0\}[\![5]\!]\,(-1+\{0,0,\text{Part}[\ll2\gg]\}[\![4]\!])}, \frac{\{0,0,\{0,0,0\}[\![5]\!]\}[\![4]\!]}{(-1+\{0,0,\text{Part}[\ll2\gg]\}[\![4]\!])\,\{0,0,\{\ll3\gg\}[\![5]\!]\}[\![5]\!]}, -\frac{}{\{0,0,0\}[\![5]\!]\,(-1+\{(}$
$\Big\}\Big\}$ is singular.

Inverse: Matrix $\Big\{\{0,1,0,0,0\},\{0,0,1,0,0\},\left\{-2\,e^{-\lambda\$16655\,\hbar\, b_i}\,\hbar,0,0,0,0\right\},\{0,0,\{0,0,0\}[\![4]\!],0,0\},\Big\{0,0,$
$\frac{\{0,0,0\}[\![4]\!] - \{\ll3\gg\}[\![5]\!]^2 + \{\ll3\gg\}[\![5]\!]^2\,\{0,0,\text{Part}[\ll2\gg]\}[\![4]\!]}{\{0,0,0\}[\![5]\!]\,(-1+\{0,0,\text{Part}[\ll2\gg]\}[\![4]\!])}, \frac{\{0,0,\{0,0,0\}[\![5]\!]\}[\![4]\!]}{(-1+\{0,0,\text{Part}[\ll2\gg]\}[\![4]\!])\,\{0,0,\{\ll3\gg\}[\![5]\!]\}[\![5]\!]}, -\frac{}{\{0,0,0\}[\![5]\!]\,(-1+\{(}$
$\Big\}\Big\}$ is singular.

Out[ ]= $\mathbb{E}_{\{\}\to\{i\}}\left[\hbar\, a_i\, b_i - \frac{(-1+B_i)\, x_i\, y_i}{b_i},\right.$
$\left. c_1\, x_i + c_1\, y_i - \frac{\hbar\,(-1+B_i)\, x_i\, y_i}{b_i} + \frac{a_i\,(-1+B_i+\hbar\, b_i\, B_i)\, x_i\, y_i}{b_i^2} + \frac{\left(1-4\,B_i+3\,B_i^2+2\,\hbar\, b_i\, B_i^2\right) x_i^2\, y_i^2}{2\, b_i^3}\right]$

In[ ]:= $\text{Exp}_{cm}[\mathbb{U}_{\{\}\to\{i\}}[\hbar\, a_i\, b_i + \hbar\, x_i\, y_i,\ c_1\,(x_i + y_i),\ 0]]$

Out[ ]= $Aborted

Step by step

In[ ]:= $m = cm;\ U = \text{Sequence}[\hbar\, a_i\, b_i + \hbar\, x_i\, y_i,\ c_1\,(x_i + y_i),\ 0]$

Out[ ]= $\text{Sequence}[\hbar\, a_i\, b_i + \hbar\, x_i\, y_i,\ c_1\,(x_i + y_i),\ 0]$

In[ ]:= $\text{MI} \mathbin{/:} \text{Coefficient}[\mathcal{E}\_,\ \text{MI}[p\_,\ n\_,\ q\_]] :=$
$\quad \text{Coefficient}[\text{Coefficient}[\text{Coefficient}[\mathcal{E},\ y_i,\ p],\ a_i,\ n],\ x_i,\ q];$
$\text{yax} \mathbin{/:} \text{yax}^{\text{MI}[p\_,n\_,q\_]} := y_i^p\, a_i^n\, x_i^q;$
$F = \mathbb{E}_{\{\}\to\{i\}}[f[\lambda]\, a_i]$

Out[ ]= $\mathbb{E}_{\{\}\to\{i\}}[f[\lambda]\, a_i]$

In[ ]:= $\text{lhs} = \left(\partial_\mu \text{Last}\left[F\ (F \mathbin{/.} \{\lambda\to\mu,\ i\to j\}) \mathbin{//} m_{i,j\to i}\right]\right) \mathbin{/.}\ \mu\to 0 \mathbin{/.} \{f[0]\to 0,\ f'[0]\to\partial_{a_i}\{U\}[[1]]\}$

Out[ ]= $\hbar\, a_i\, b_i$

In[ ]:= $\text{rhs} = \partial_\lambda \text{Last}[F]$

Out[ ]= $a_i\, f'[\lambda]$

In[ ]:= $F = F \mathbin{/.} \text{First@DSolve}[\text{lhs} == \text{rhs} \land f[0] == 0,\ f,\ \lambda]$

Out[ ]= $\mathbb{E}_{\{\}\to\{i\}}[\lambda\,\hbar\, a_i\, b_i]$

In[ ]:= $\text{mis} = \{\text{MI}[0,0,0],\ \text{MI}[1,0,0],\ \text{MI}[0,0,1],\ \text{MI}[1,0,1]\}$

Out[ ]= $\{\text{MI}[0,0,0],\ \text{MI}[1,0,0],\ \text{MI}[0,0,1],\ \text{MI}[1,0,1]\}$

In[ ]:= $F[[1]] \mathrel{+}= \text{Sum}\left[f_{mi}[\lambda]\ \text{yax}^{mi},\ \{mi,\ mis\}\right]$

Out[ ]= $\lambda\,\hbar\, a_i\, b_i + f_{\text{MI}[0,0,0]}[\lambda] + x_i\, f_{\text{MI}[0,0,1]}[\lambda] + y_i\, f_{\text{MI}[1,0,0]}[\lambda] + x_i\, y_i\, f_{\text{MI}[1,0,1]}[\lambda]$

In[ ]:= $\text{lhs} = \left(\partial_\mu \text{U2l@Last}\left[F\ (F \mathbin{/.} \{\lambda\to\mu,\ i\to j\}) \mathbin{//} m_{i,j\to i}\right]\right) \mathbin{/.}\ \mu\to 0 \mathbin{/.}\ f\_[0]\to 0 \mathbin{/.}$
$\quad \text{Table}[f_{mi}'[0]\to\text{Coefficient}[\{U\}[[1]],\ mi],\ \{mi,\ mis\}]$

Out[ ]= $\hbar\, a_i\, b_i + e^{-\lambda\,\hbar\, b_i}\,\hbar\, x_i\, y_i$

In[ ]:= $\text{rhs} = \partial_\lambda \text{Last}[F]$

Out[ ]= $\hbar\, a_i\, b_i + f_{\text{MI}[0,0,0]}'[\lambda] + x_i\, f_{\text{MI}[0,0,1]}'[\lambda] + y_i\, f_{\text{MI}[1,0,0]}'[\lambda] + x_i\, y_i\, f_{\text{MI}[1,0,1]}'[\lambda]$

In[ ]:= $F = F \mathbin{/.} \text{First@DSolve}[$
$\quad \text{Table}[\text{Coefficient}[\text{lhs} - \text{rhs},\ mi] == 0 \land f_{mi}[0] == 0,\ \{mi,\ mis\}],\ \text{Table}[f_{mi},\ \{mi,\ mis\}],\ \lambda]$

Out[ ]= $\mathbb{E}_{\{\}\to\{i\}}\left[\lambda\,\hbar\, a_i\, b_i + \frac{e^{-\lambda\,\hbar\, b_i}\left(-1 + e^{\lambda\,\hbar\, b_i}\right) x_i\, y_i}{b_i}\right]$

In[ ]:= $\{k,\ \text{Length}[\{U\}] - 1\}$

Out[ ]= $\{k,\ 2\}$

In[∘]:= `k = 1`

Out[∘]= 1

In[∘]:= `mis = Flatten@Table[MI[p, n, q], {n, 0, k + 1}, {p, 0, 2 k + 2 - 2 n}, {q, 0, 2 k + 2 - 2 n - p}]`

Out[∘]= {MI[0, 0, 0], MI[0, 0, 1], MI[0, 0, 2], MI[0, 0, 3], MI[0, 0, 4], MI[1, 0, 0], MI[1, 0, 1], MI[1, 0, 2], MI[1, 0, 3], MI[2, 0, 0], MI[2, 0, 1], MI[2, 0, 2], MI[3, 0, 0], MI[3, 0, 1], MI[4, 0, 0], MI[0, 1, 0], MI[0, 1, 1], MI[0, 1, 2], MI[1, 1, 0], MI[1, 1, 1], MI[2, 1, 0], MI[0, 2, 0]}

In[∘]:= $\texttt{AppendTo}\left[F, \texttt{Sum}\left[f_{mi}[\lambda]\ yax^{mi}, \{mi, mis\}\right]\right]$

Out[∘]=
$$\mathbb{E}_{\{\}\to\{i\}}\Big[\lambda\hbar a_i b_i + \frac{e^{-\lambda\hbar b_i}\left(-1+e^{\lambda\hbar b_i}\right)x_i y_i}{b_i},$$
$$f_{MI[0,0,0]}[\lambda] + x_i f_{MI[0,0,1]}[\lambda] + x_i^2 f_{MI[0,0,2]}[\lambda] + x_i^3 f_{MI[0,0,3]}[\lambda] + x_i^4 f_{MI[0,0,4]}[\lambda] + a_i f_{MI[0,1,0]}[\lambda] + a_i x_i f_{MI[0,1,1]}[\lambda] + a_i x_i^2 f_{MI[0,1,2]}[\lambda] + a_i^2 f_{MI[0,2,0]}[\lambda] + y_i f_{MI[1,0,0]}[\lambda] + x_i y_i f_{MI[1,0,1]}[\lambda] + x_i^2 y_i f_{MI[1,0,2]}[\lambda] + x_i^3 y_i f_{MI[1,0,3]}[\lambda] + a_i y_i f_{MI[1,1,0]}[\lambda] + a_i x_i y_i f_{MI[1,1,1]}[\lambda] + y_i^2 f_{MI[2,0,0]}[\lambda] + x_i y_i^2 f_{MI[2,0,1]}[\lambda] + x_i^2 y_i^2 f_{MI[2,0,2]}[\lambda] + a_i y_i^2 f_{MI[2,1,0]}[\lambda] + y_i^3 f_{MI[3,0,0]}[\lambda] + x_i y_i^3 f_{MI[3,0,1]}[\lambda] + y_i^4 f_{MI[4,0,0]}[\lambda]\Big]$$

In[∘]:= $\texttt{lhs} = \left(\partial_\mu \texttt{U2l@Last}\left[F\ (F\ /.\ \{\lambda\to\mu, i\to j\})\ //\ m_{i,j\to i}\right]\right)$ /. $\mu\to 0$ /. f_[0] → 0 /. Table[f_{mi}'[0] → Coefficient[{U}⟦k + 1⟧, mi], {mi, mis}]

Out[∘]=
$$c_1 y_i - \hbar b_i x_i f_{MI[0,0,1]}[\lambda] + x_i\left(c_1 + \hbar b_i f_{MI[0,0,1]}[\lambda]\right) - \hbar b_i x_i y_i f_{MI[1,0,1]}[\lambda] + \frac{e^{-\lambda\hbar b_i} x_i y_i\left(-\hbar + e^{\lambda\hbar b_i}\hbar + \lambda\hbar^2 b_i - \hbar b_i f_{MI[0,1,0]}[\lambda] + \hbar b_i f_{MI[0,2,0]}[\lambda] + e^{\lambda\hbar b_i}\hbar b_i^2 f_{MI[1,0,1]}[\lambda]\right)}{b_i} -$$
$$2\hbar b_i x_i^2 y_i f_{MI[1,0,2]}[\lambda] + \frac{e^{-\lambda\hbar b_i} x_i^2 y_i\left(-\hbar b_i f_{MI[0,1,1]}[\lambda] + 2 e^{\lambda\hbar b_i}\hbar b_i^2 f_{MI[1,0,2]}[\lambda]\right)}{b_i} -$$
$$3\hbar b_i x_i^3 y_i f_{MI[1,0,3]}[\lambda] + \frac{e^{-\lambda\hbar b_i} x_i^3 y_i\left(-\hbar b_i f_{MI[0,1,2]}[\lambda] + 3 e^{\lambda\hbar b_i}\hbar b_i^2 f_{MI[1,0,3]}[\lambda]\right)}{b_i} -$$
$$2\hbar a_i b_i x_i y_i f_{MI[1,1,1]}[\lambda] + \frac{e^{-2\lambda\hbar b_i} a_i x_i y_i\left(-e^{\lambda\hbar b_i}\lambda\hbar^2 b_i^2 - 2 e^{\lambda\hbar b_i}\hbar b_i^2 f_{MI[0,2,0]}[\lambda] + 2 e^{2\lambda\hbar b_i}\hbar b_i^3 f_{MI[1,1,1]}[\lambda]\right)}{b_i^2} -$$
$$\hbar b_i x_i y_i^2 f_{MI[2,0,1]}[\lambda] + \frac{e^{-2\lambda\hbar b_i} x_i y_i^2\left(-e^{\lambda\hbar b_i}\hbar b_i f_{MI[1,1,0]}[\lambda] + e^{2\lambda\hbar b_i}\hbar b_i^2 f_{MI[2,0,1]}[\lambda]\right)}{b_i} -$$
$$7\hbar b_i x_i^2 y_i^2 f_{MI[2,0,2]}[\lambda] + \frac{1}{2 b_i^3} e^{-7\lambda\hbar b_i} x_i^2 y_i^2\Big(-2 e^{5\lambda\hbar b_i}\hbar b_i + 2 e^{6\lambda\hbar b_i}\hbar b_i - 2 e^{5\lambda\hbar b_i}\lambda\hbar^2 b_i^2 - 2 e^{6\lambda\hbar b_i}\hbar b_i^3 f_{MI[1,1,1]}[\lambda] + 14 e^{7\lambda\hbar b_i}\hbar b_i^4 f_{MI[2,0,2]}[\lambda]\Big) -$$
$$\hbar b_i x_i y_i^3 f_{MI[3,0,1]}[\lambda] + \frac{e^{-3\lambda\hbar b_i} x_i y_i^3\left(-e^{2\lambda\hbar b_i}\hbar b_i f_{MI[2,1,0]}[\lambda] + e^{3\lambda\hbar b_i}\hbar b_i^2 f_{MI[3,0,1]}[\lambda]\right)}{b_i}$$

In[∘]:= **CF[lhs]**

Out[∘]=
$$c_1 x_i + c_1 y_i - e^{-\lambda \hbar b_i} \hbar x_i^2 y_i f_{\mathrm{MI}[0,1,1]}[\lambda] - e^{-\lambda \hbar b_i} \hbar x_i^3 y_i f_{\mathrm{MI}[0,1,2]}[\lambda] - e^{-\lambda \hbar b_i} \hbar a_i x_i y_i (\lambda \hbar + 2 f_{\mathrm{MI}[0,2,0]}[\lambda]) + \frac{e^{-\lambda \hbar b_i} \hbar x_i y_i \left(-1 + e^{\lambda \hbar b_i} + \lambda \hbar b_i - b_i f_{\mathrm{MI}[0,1,0]}[\lambda] + b_i f_{\mathrm{MI}[0,2,0]}[\lambda]\right)}{b_i} - e^{-\lambda \hbar b_i} \hbar x_i y_i^2 f_{\mathrm{MI}[1,1,0]}[\lambda] - \frac{e^{-2 \lambda \hbar b_i} \hbar x_i^2 y_i^2 \left(1 - e^{\lambda \hbar b_i} + \lambda \hbar b_i + e^{\lambda \hbar b_i} b_i^2 f_{\mathrm{MI}[1,1,1]}[\lambda]\right)}{b_i^2} - e^{-\lambda \hbar b_i} \hbar x_i y_i^3 f_{\mathrm{MI}[2,1,0]}[\lambda]$$

In[∘]:= **rhs = ∂_λLast[F]**

Out[∘]=
$$f_{\mathrm{MI}[0,0,0]}'[\lambda] + x_i f_{\mathrm{MI}[0,0,1]}'[\lambda] + x_i^2 f_{\mathrm{MI}[0,0,2]}'[\lambda] + x_i^3 f_{\mathrm{MI}[0,0,3]}'[\lambda] + x_i^4 f_{\mathrm{MI}[0,0,4]}'[\lambda] + a_i f_{\mathrm{MI}[0,1,0]}'[\lambda] + a_i x_i f_{\mathrm{MI}[0,1,1]}'[\lambda] + a_i x_i^2 f_{\mathrm{MI}[0,1,2]}'[\lambda] + a_i^2 f_{\mathrm{MI}[0,2,0]}'[\lambda] + y_i f_{\mathrm{MI}[1,0,0]}'[\lambda] + x_i y_i f_{\mathrm{MI}[1,0,1]}'[\lambda] + x_i^2 y_i f_{\mathrm{MI}[1,0,2]}'[\lambda] + x_i^3 y_i f_{\mathrm{MI}[1,0,3]}'[\lambda] + a_i y_i f_{\mathrm{MI}[1,1,0]}'[\lambda] + a_i x_i y_i f_{\mathrm{MI}[1,1,1]}'[\lambda] + y_i^2 f_{\mathrm{MI}[2,0,0]}'[\lambda] + x_i y_i^2 f_{\mathrm{MI}[2,0,1]}'[\lambda] + x_i^2 y_i^2 f_{\mathrm{MI}[2,0,2]}'[\lambda] + a_i y_i^2 f_{\mathrm{MI}[2,1,0]}'[\lambda] + y_i^3 f_{\mathrm{MI}[3,0,0]}'[\lambda] + x_i y_i^3 f_{\mathrm{MI}[3,0,1]}'[\lambda] + y_i^4 f_{\mathrm{MI}[4,0,0]}'[\lambda]$$

In[∘]:= **Table[mi → Simplify@Coefficient[lhs − rhs, mi], {mi, mis}] // ColumnForm**

Out[∘]=

MI[0, 0, 0] → $-f_{\mathrm{MI}[0,0,0]}'[\lambda]$

MI[0, 0, 1] → $c_1 - f_{\mathrm{MI}[0,0,1]}'[\lambda]$

MI[0, 0, 2] → $-f_{\mathrm{MI}[0,0,2]}'[\lambda]$

MI[0, 0, 3] → $-f_{\mathrm{MI}[0,0,3]}'[\lambda]$

MI[0, 0, 4] → $-f_{\mathrm{MI}[0,0,4]}'[\lambda]$

MI[1, 0, 0] → $c_1 - f_{\mathrm{MI}[1,0,0]}'[\lambda]$

MI[1, 0, 1] → $\frac{e^{-\lambda \hbar b_i} \left(\left(-1+e^{\lambda \hbar b_i}\right) \hbar + b_i \left(\lambda \hbar^2 - \hbar f_{\mathrm{MI}[0,1,0]}[\lambda] + \hbar f_{\mathrm{MI}[0,2,0]}[\lambda] - e^{\lambda \hbar b_i} f_{\mathrm{MI}[1,0,1]}'[\lambda]\right)\right)}{b_i}$

MI[1, 0, 2] → $-e^{-\lambda \hbar b_i} \hbar f_{\mathrm{MI}[0,1,1]}[\lambda] - f_{\mathrm{MI}[1,0,2]}'[\lambda]$

MI[1, 0, 3] → $-e^{-\lambda \hbar b_i} \hbar f_{\mathrm{MI}[0,1,2]}[\lambda] - f_{\mathrm{MI}[1,0,3]}'[\lambda]$

MI[2, 0, 0] → $-f_{\mathrm{MI}[2,0,0]}'[\lambda]$

MI[2, 0, 1] → $-e^{-\lambda \hbar b_i} \hbar f_{\mathrm{MI}[1,1,0]}[\lambda] - f_{\mathrm{MI}[2,0,1]}'[\lambda]$

MI[2, 0, 2] → $\frac{e^{-2 \lambda \hbar b_i} \left(\left(-1+e^{\lambda \hbar b_i}\right) \hbar - \lambda \hbar^2 b_i - e^{\lambda \hbar b_i} b_i^2 \left(\hbar f_{\mathrm{MI}[1,1,1]}[\lambda] + e^{\lambda \hbar b_i} f_{\mathrm{MI}[2,0,2]}'[\lambda]\right)\right)}{b_i^2}$

MI[3, 0, 0] → $-f_{\mathrm{MI}[3,0,0]}'[\lambda]$

MI[3, 0, 1] → $-e^{-\lambda \hbar b_i} \hbar f_{\mathrm{MI}[2,1,0]}[\lambda] - f_{\mathrm{MI}[3,0,1]}'[\lambda]$

MI[4, 0, 0] → $-f_{\mathrm{MI}[4,0,0]}'[\lambda]$

MI[0, 1, 0] → $-f_{\mathrm{MI}[0,1,0]}'[\lambda]$

MI[0, 1, 1] → $-f_{\mathrm{MI}[0,1,1]}'[\lambda]$

MI[0, 1, 2] → $-f_{\mathrm{MI}[0,1,2]}'[\lambda]$

MI[1, 1, 0] → $-f_{\mathrm{MI}[1,1,0]}'[\lambda]$

MI[1, 1, 1] → $-e^{-\lambda \hbar b_i} \left(\lambda \hbar^2 + 2 \hbar f_{\mathrm{MI}[0,2,0]}[\lambda] + e^{\lambda \hbar b_i} f_{\mathrm{MI}[1,1,1]}'[\lambda]\right)$

MI[2, 1, 0] → $-f_{\mathrm{MI}[2,1,0]}'[\lambda]$

MI[0, 2, 0] → $-f_{\mathrm{MI}[0,2,0]}'[\lambda]$

```
In[•]:= F = F /. First@DSolve[
      Table[Coefficient[lhs - rhs, mi] == 0 ∧ f_mi[0] == 0, {mi, mis}], Table[f_mi, {mi, mis}], λ]
```

Set: Part 4 of Asymptotics`AsymptoticRSolveValueDump`c$22614⟦Asymptotics`AsymptoticRSolveValueDump`i$22614⟧ does not exist.

Set: Part 5 of Asymptotics`AsymptoticRSolveValueDump`c$22614⟦Asymptotics`AsymptoticRSolveValueDump`i$22614⟧ does not exist.

Part: Part 4 of {0, 0, 0} does not exist.

Part: Part 5 of {0, 0, 0} does not exist.

Set: Part 4 of {0, 0, {0, 0, 0}⟦5⟧} does not exist.

General: Further output of Set::partw will be suppressed during this calculation.

Part: Part 4 of {0, 0, {0, 0, 0}⟦5⟧} does not exist.

General: Further output of Part::partw will be suppressed during this calculation.

Inverse: Matrix $\{\{0, 1, 0, 0, 0\}, \{0, 0, 1, 0, 0\}, \{-2 e^{-\lambda \hbar b_i} \hbar, 0, 0, 0, 0\}, \{0, 0, \{0, 0, 0\}⟦4⟧, 0, 0\}, \{0, 0, \frac{\{0, 0, 0\}⟦4⟧ - \{\ll 3 \gg\}⟦5⟧^2 + \{\ll 3 \gg\}⟦5⟧^2 \{0, 0, \text{Part}[\ll 2 \gg]\}⟦4⟧}{\{0, 0, 0\}⟦5⟧ (-1 + \{0, 0, \text{Part}[\ll 2 \gg]\}⟦4⟧)}, \frac{\{0, 0, \{0, 0, 0\}⟦5⟧\}⟦4⟧}{(-1 + \{0, 0, \text{Part}[\ll 2 \gg]\}⟦4⟧) \{0, 0, \{\ll 3 \gg\}⟦5⟧\}⟦5⟧}, -\frac{}{\{0, 0, 0\}⟦5⟧ (-1 + \{(}\}\}$ is singular.

Inverse: Matrix $\{\{0, 1, 0, 0, 0\}, \{0, 0, 1, 0, 0\}, \{-2 e^{-\lambda \hbar b_i} \hbar, 0, 0, 0, 0\}, \{0, 0, \{0, 0, 0\}⟦4⟧, 0, 0\}, \{0, 0, \frac{\{0, 0, 0\}⟦4⟧ - \{\ll 3 \gg\}⟦5⟧^2 + \{\ll 3 \gg\}⟦5⟧^2 \{0, 0, \text{Part}[\ll 2 \gg]\}⟦4⟧}{\{0, 0, 0\}⟦5⟧ (-1 + \{0, 0, \text{Part}[\ll 2 \gg]\}⟦4⟧)}, \frac{\{0, 0, \{0, 0, 0\}⟦5⟧\}⟦4⟧}{(-1 + \{0, 0, \text{Part}[\ll 2 \gg]\}⟦4⟧) \{0, 0, \{\ll 3 \gg\}⟦5⟧\}⟦5⟧}, -\frac{}{\{0, 0, 0\}⟦5⟧ (-1 + \{(}\}\}$ is singular.

Out[•]= $$\mathbb{E}_{\{\}\to\{i\}}\left[\lambda \hbar a_i b_i + \frac{e^{-\lambda \hbar b_i}\left(-1 + e^{\lambda \hbar b_i}\right) x_i y_i}{b_i},\ \lambda c_1 x_i + \lambda c_1 y_i + \frac{e^{-\lambda \hbar b_i}\left(-1 + e^{\lambda \hbar b_i}\right) \lambda \hbar x_i y_i}{b_i} - \frac{e^{-\lambda \hbar b_i} a_i \left(-1 + e^{\lambda \hbar b_i} - \lambda \hbar b_i\right) x_i y_i}{b_i^2} + \frac{e^{-2 \lambda \hbar b_i}\left(3 - 4 e^{\lambda \hbar b_i} + e^{2 \lambda \hbar b_i} + 2 \lambda \hbar b_i\right) x_i^2 y_i^2}{2 b_i^3}\right]$$

```
In[•]:= k = 2
Out[•]= 2
```

```
In[•]:= mis = Flatten@Table[MI[p, n, q], {n, 0, 2 k + 2, 2}, {p, 0, 2 k + 2 - 2 n}, {q, 0, 2 k + 2 - 2 n - p}]
Out[•]= {MI[0, 0, 0], MI[0, 0, 1], MI[0, 0, 2], MI[0, 0, 3], MI[0, 0, 4], MI[0, 0, 5], MI[0, 0, 6],
  MI[1, 0, 0], MI[1, 0, 1], MI[1, 0, 2], MI[1, 0, 3], MI[1, 0, 4], MI[1, 0, 5], MI[2, 0, 0],
  MI[2, 0, 1], MI[2, 0, 2], MI[2, 0, 3], MI[2, 0, 4], MI[3, 0, 0], MI[3, 0, 1], MI[3, 0, 2],
  MI[3, 0, 3], MI[4, 0, 0], MI[4, 0, 1], MI[4, 0, 2], MI[5, 0, 0], MI[5, 0, 1], MI[6, 0, 0],
  MI[0, 2, 0], MI[0, 2, 1], MI[0, 2, 2], MI[1, 2, 0], MI[1, 2, 1], MI[2, 2, 0]}
```

In[∘]:= **AppendTo[F, Sum[f_{mi}[λ] yax^{mi}, {mi, mis}]]**

Out[∘]= $\mathbb{E}_{\{\}\to\{i\}}\Big[\lambda\hbar a_i b_i + \frac{e^{-\lambda\hbar b_i}\left(-1+e^{\lambda\hbar b_i}\right)x_i y_i}{b_i},$

$\lambda c_1 x_i + \lambda c_1 y_i + \frac{e^{-\lambda\hbar b_i}\left(-1+e^{\lambda\hbar b_i}\right)\lambda\hbar x_i y_i}{b_i} + \frac{e^{-2\lambda\hbar b_i}\left(3-4e^{\lambda\hbar b_i}+e^{2\lambda\hbar b_i}+2\lambda\hbar b_i\right)x_i^2 y_i^2}{4 b_i^3},$

$f_{MI[0,0,0]}[\lambda] + x_i f_{MI[0,0,1]}[\lambda] + x_i^2 f_{MI[0,0,2]}[\lambda] + x_i^3 f_{MI[0,0,3]}[\lambda] + x_i^4 f_{MI[0,0,4]}[\lambda] +$
$x_i^5 f_{MI[0,0,5]}[\lambda] + x_i^6 f_{MI[0,0,6]}[\lambda] + a_i^2 f_{MI[0,2,0]}[\lambda] + a_i^2 x_i f_{MI[0,2,1]}[\lambda] + a_i^2 x_i^2 f_{MI[0,2,2]}[\lambda] +$
$y_i f_{MI[1,0,0]}[\lambda] + x_i y_i f_{MI[1,0,1]}[\lambda] + x_i^2 y_i f_{MI[1,0,2]}[\lambda] + x_i^3 y_i f_{MI[1,0,3]}[\lambda] + x_i^4 y_i f_{MI[1,0,4]}[\lambda] +$
$x_i^5 y_i f_{MI[1,0,5]}[\lambda] + a_i^2 y_i f_{MI[1,2,0]}[\lambda] + a_i^2 x_i y_i f_{MI[1,2,1]}[\lambda] + y_i^2 f_{MI[2,0,0]}[\lambda] + x_i y_i^2 f_{MI[2,0,1]}[\lambda] +$
$x_i^2 y_i^2 f_{MI[2,0,2]}[\lambda] + x_i^3 y_i^2 f_{MI[2,0,3]}[\lambda] + x_i^4 y_i^2 f_{MI[2,0,4]}[\lambda] + a_i^2 y_i^2 f_{MI[2,2,0]}[\lambda] +$
$y_i^3 f_{MI[3,0,0]}[\lambda] + x_i y_i^3 f_{MI[3,0,1]}[\lambda] + x_i^2 y_i^3 f_{MI[3,0,2]}[\lambda] + x_i^3 y_i^3 f_{MI[3,0,3]}[\lambda] + y_i^4 f_{MI[4,0,0]}[\lambda] +$
$x_i y_i^4 f_{MI[4,0,1]}[\lambda] + x_i^2 y_i^4 f_{MI[4,0,2]}[\lambda] + y_i^5 f_{MI[5,0,0]}[\lambda] + x_i y_i^5 f_{MI[5,0,1]}[\lambda] + y_i^6 f_{MI[6,0,0]}[\lambda]\Big]$

In[∘]:=
```
lhs = (∂_μ U2l@Last[F (F /. {λ → μ, i → j}) // m_{i,j→i}]) /. μ → 0 /. f_[0] → 0 /.
   Table[f_mi'[0] → Coefficient[{U}[[k + 1]], mi], {mi, mis}]
```

Out[∘]=

$$\lambda b_i c_1^2 + \lambda \hbar c_1 y_i - \frac{e^{-\lambda \hbar b_i} a_i \left(1 - e^{\lambda \hbar b_i} + \lambda \hbar b_i\right) c_1 y_i}{b_i} - \hbar b_i x_i f_{MI[0,0,1]}[\lambda] +$$
$$x_i \left(\lambda \hbar c_1 + \hbar b_i f_{MI[0,0,1]}[\lambda]\right) - \frac{e^{-2\lambda \hbar b_i} a_i x_i y_i \left(e^{\lambda \hbar b_i} \lambda^2 \hbar^3 b_i^2 + 2 e^{\lambda \hbar b_i} \hbar b_i^2 f_{MI[0,2,0]}[\lambda]\right)}{b_i^2} -$$
$$2 e^{-\lambda \hbar b_i} \hbar a_i x_i^2 y_i f_{MI[0,2,1]}[\lambda] - 2 e^{-\lambda \hbar b_i} \hbar a_i x_i^3 y_i f_{MI[0,2,2]}[\lambda] -$$
$$2 \hbar b_i x_i y_i f_{MI[1,0,1]}[\lambda] + \frac{1}{2 b_i} e^{-2\lambda \hbar b_i} x_i y_i \left(-2 e^{\lambda \hbar b_i} \lambda \hbar^2 + 2 e^{2\lambda \hbar b_i} \lambda \hbar^2 + e^{\lambda \hbar b_i} \lambda^2 \hbar^3 b_i + 2 e^{\lambda \hbar b_i} \hbar b_i f_{MI[0,2,0]}[\lambda] + 4 e^{2\lambda \hbar b_i} \hbar b_i^2 f_{MI[1,0,1]}[\lambda]\right) - 5 \hbar b_i x_i^2 y_i f_{MI[1,0,2]}[\lambda] +$$
$$\frac{e^{-3\lambda \hbar b_i} x_i^2 y_i \left(2 e^{2\lambda \hbar b_i} \hbar b_i^2 f_{MI[0,2,1]}[\lambda] + 10 e^{3\lambda \hbar b_i} \hbar b_i^3 f_{MI[1,0,2]}[\lambda]\right)}{2 b_i^2} - 3 \hbar b_i x_i^3 y_i f_{MI[1,0,3]}[\lambda] +$$
$$\frac{e^{-\lambda \hbar b_i} x_i^3 y_i \left(\hbar b_i f_{MI[0,2,2]}[\lambda] + 3 e^{\lambda \hbar b_i} \hbar b_i^2 f_{MI[1,0,3]}[\lambda]\right)}{b_i} - 2 e^{-\lambda \hbar b_i} \hbar a_i x_i y_i^2 f_{MI[1,2,0]}[\lambda] -$$
$$\hbar a_i^2 b_i x_i y_i f_{MI[1,2,1]}[\lambda] + \frac{e^{-\lambda \hbar b_i} a_i^2 x_i y_i \left(\lambda^2 \hbar^3 b_i + 2 e^{\lambda \hbar b_i} \hbar b_i^2 f_{MI[1,2,1]}[\lambda]\right)}{2 b_i} -$$
$$\frac{e^{-7\lambda \hbar b_i} a_i x_i^2 y_i^2 \left(-2 e^{5\lambda \hbar b_i} \lambda \hbar^2 b_i^2 + 2 e^{6\lambda \hbar b_i} \lambda \hbar^2 b_i^2 - 2 e^{5\lambda \hbar b_i} \lambda^2 \hbar^3 b_i^3 + 4 e^{6\lambda \hbar b_i} \hbar b_i^4 f_{MI[1,2,1]}[\lambda]\right)}{2 b_i^4} -$$
$$3 \hbar b_i x_i y_i^2 f_{MI[2,0,1]}[\lambda] + \frac{1}{2 b_i^2} e^{-5\lambda \hbar b_i} x_i y_i^2 \left(-e^{3\lambda \hbar b_i} c_1 + 2 e^{4\lambda \hbar b_i} c_1 - e^{5\lambda \hbar b_i} c_1 + 2 e^{4\lambda \hbar b_i} \hbar b_i^2 f_{MI[1,2,0]}[\lambda] + 6 e^{5\lambda \hbar b_i} \hbar b_i^3 f_{MI[2,0,1]}[\lambda]\right) - 7 \hbar b_i$$
$$x_i^2 y_i^2 f_{MI[2,0,2]}[\lambda] + \frac{1}{8 b_i^4} e^{-7\lambda \hbar b_i} x_i^2 y_i^2 \left(4 e^{5\lambda \hbar b_i} \hbar b_i - 8 e^{6\lambda \hbar b_i} \hbar b_i + 4 e^{7\lambda \hbar b_i} \hbar b_i - 16 e^{5\lambda \hbar b_i} \lambda \hbar^2 b_i^2 + 16 e^{6\lambda \hbar b_i} \lambda \hbar^2 b_i^2 - 20 e^{5\lambda \hbar b_i} \lambda^2 \hbar^3 b_i^3 + 8 e^{6\lambda \hbar b_i} \hbar b_i^4 f_{MI[1,2,1]}[\lambda] + 56 e^{7\lambda \hbar b_i} \hbar b_i^5 f_{MI[2,0,2]}[\lambda]\right) -$$
$$2 e^{-\lambda \hbar b_i} \hbar a_i x_i y_i^3 f_{MI[2,2,0]}[\lambda] - \hbar b_i x_i y_i^3 f_{MI[3,0,1]}[\lambda] +$$
$$\frac{e^{-3\lambda \hbar b_i} x_i y_i^3 \left(e^{2\lambda \hbar b_i} \hbar b_i f_{MI[2,2,0]}[\lambda] + e^{3\lambda \hbar b_i} \hbar b_i^2 f_{MI[3,0,1]}[\lambda]\right)}{b_i} - 18 \hbar b_i x_i^3 y_i^3 f_{MI[3,0,3]}[\lambda] +$$
$$\frac{1}{12 b_i^5} e^{-18\lambda \hbar b_i} x_i^3 y_i^3 \left(33 e^{15\lambda \hbar b_i} \hbar b_i - 48 e^{16\lambda \hbar b_i} \hbar b_i + 15 e^{17\lambda \hbar b_i} \hbar b_i + 42 e^{15\lambda \hbar b_i} \lambda \hbar^2 b_i^2 - 24 e^{16\lambda \hbar b_i} \lambda \hbar^2 b_i^2 + 18 e^{15\lambda \hbar b_i} \lambda^2 \hbar^3 b_i^3 + 216 e^{18\lambda \hbar b_i} \hbar b_i^6 f_{MI[3,0,3]}[\lambda]\right)$$

In[∘]:= `rhs = ∂_λ Last[F]`

Out[∘]= $f_{MI[0,0,0]}'[\lambda] + x_i f_{MI[0,0,1]}'[\lambda] + x_i^2 f_{MI[0,0,2]}'[\lambda] + x_i^3 f_{MI[0,0,3]}'[\lambda] + x_i^4 f_{MI[0,0,4]}'[\lambda] + x_i^5 f_{MI[0,0,5]}'[\lambda] + x_i^6 f_{MI[0,0,6]}'[\lambda] + a_i^2 f_{MI[0,2,0]}'[\lambda] + a_i^2 x_i f_{MI[0,2,1]}'[\lambda] + a_i^2 x_i^2 f_{MI[0,2,2]}'[\lambda] + y_i f_{MI[1,0,0]}'[\lambda] + x_i y_i f_{MI[1,0,1]}'[\lambda] + x_i^2 y_i f_{MI[1,0,2]}'[\lambda] + x_i^3 y_i f_{MI[1,0,3]}'[\lambda] + x_i^4 y_i f_{MI[1,0,4]}'[\lambda] + x_i^5 y_i f_{MI[1,0,5]}'[\lambda] + a_i^2 y_i f_{MI[1,2,0]}'[\lambda] + a_i^2 x_i y_i f_{MI[1,2,1]}'[\lambda] + y_i^2 f_{MI[2,0,0]}'[\lambda] + x_i y_i^2 f_{MI[2,0,1]}'[\lambda] + x_i^2 y_i^2 f_{MI[2,0,2]}'[\lambda] + x_i^3 y_i^2 f_{MI[2,0,3]}'[\lambda] + x_i^4 y_i^2 f_{MI[2,0,4]}'[\lambda] + a_i^2 y_i^2 f_{MI[2,2,0]}'[\lambda] + y_i^3 f_{MI[3,0,0]}'[\lambda] + x_i y_i^3 f_{MI[3,0,1]}'[\lambda] + x_i^2 y_i^3 f_{MI[3,0,2]}'[\lambda] + x_i^3 y_i^3 f_{MI[3,0,3]}'[\lambda] + y_i^4 f_{MI[4,0,0]}'[\lambda] + x_i y_i^4 f_{MI[4,0,1]}'[\lambda] + x_i^2 y_i^4 f_{MI[4,0,2]}'[\lambda] + y_i^5 f_{MI[5,0,0]}'[\lambda] + x_i y_i^5 f_{MI[5,0,1]}'[\lambda] + y_i^6 f_{MI[6,0,0]}'[\lambda]$

In[∘]:= `F = F /. First@DSolve[Table[Coefficient[lhs - rhs, mi] == 0 ∧ f_mi[0] == 0, {mi, mis}], Table[f_mi, {mi, mis}], λ]`

Out[∘]=
$$\mathbb{E}_{\{\}\to\{i\}}\Big[\lambda\hbar a_i b_i + \frac{e^{-\lambda\hbar b_i}\left(-1+e^{\lambda\hbar b_i}\right)x_i y_i}{b_i},\ \lambda c_1 x_i + \lambda c_1 y_i + \frac{e^{-\lambda\hbar b_i}\left(-1+e^{\lambda\hbar b_i}\right)\lambda\hbar x_i y_i}{b_i} + \frac{e^{-2\lambda\hbar b_i}\left(3-4e^{\lambda\hbar b_i}+e^{2\lambda\hbar b_i}+2\lambda\hbar b_i\right)x_i^2 y_i^2}{4b_i^3},$$
$$\frac{1}{2}\lambda^2 b_i c_1^2 + \frac{1}{2}\lambda^2\hbar c_1 x_i + \frac{1}{2}\lambda^2\hbar c_1 y_i + \frac{e^{-\lambda\hbar b_i}\left(-1+e^{\lambda\hbar b_i}\right)\lambda^2\hbar^2 x_i y_i}{2b_i} + \frac{e^{-\lambda\hbar b_i}a_i^2\left(-2+2e^{\lambda\hbar b_i}-2\lambda\hbar b_i-\lambda^2\hbar^2 b_i^2\right)x_i y_i}{2b_i^3} - \frac{e^{-2\lambda\hbar b_i}\left(-1+4e^{\lambda\hbar b_i}-3e^{2\lambda\hbar b_i}+2e^{2\lambda\hbar b_i}\lambda\hbar b_i\right)c_1 x_i y_i^2}{4\hbar b_i^3} +$$
$$\frac{e^{-2\lambda\hbar b_i}\left(7-8e^{\lambda\hbar b_i}+e^{2\lambda\hbar b_i}+12\lambda\hbar b_i-8e^{\lambda\hbar b_i}\lambda\hbar b_i+2e^{2\lambda\hbar b_i}\lambda\hbar b_i+6\lambda^2\hbar^2 b_i^2\right)x_i^2 y_i^2}{4b_i^4} + \frac{e^{-3\lambda\hbar b_i}\left(-17+30e^{\lambda\hbar b_i}-15e^{2\lambda\hbar b_i}+2e^{3\lambda\hbar b_i}-18\lambda\hbar b_i+12e^{\lambda\hbar b_i}\lambda\hbar b_i-6\lambda^2\hbar^2 b_i^2\right)x_i^3 y_i^3}{12b_i^5}\Big]$$

In[∘]:= `CF@l2U[F /. λ → 1]`

Out[∘]=
$$\mathbb{E}_{\{\}\to\{i\}}\Big[\hbar a_i b_i - \frac{(-1+B_i)x_i y_i}{b_i},\ c_1 x_i + c_1 y_i - \frac{\hbar(-1+B_i)x_i y_i}{b_i} + \frac{\left(1-4B_i+3B_i^2+2\hbar b_i B_i^2\right)x_i^2 y_i^2}{4b_i^3},$$
$$\frac{1}{2}b_i c_1^2 + \frac{1}{2}\hbar c_1 x_i + \frac{1}{2}\hbar c_1 y_i - \frac{\hbar^2(-1+B_i)x_i y_i}{2b_i} - \frac{a_i^2\left(-2+2B_i+2\hbar b_i B_i+\hbar^2 b_i^2 B_i\right)x_i y_i}{2b_i^3} - \frac{\left(-3+2\hbar b_i+4B_i-B_i^2\right)c_1 x_i y_i^2}{4\hbar b_i^3} + \frac{\left(1+2\hbar b_i-8B_i-8\hbar b_i B_i+7B_i^2+12\hbar b_i B_i^2+6\hbar^2 b_i^2 B_i^2\right)x_i^2 y_i^2}{4b_i^4} - \frac{\left(-2+15B_i-30B_i^2-12\hbar b_i B_i^2+17B_i^3+18\hbar b_i B_i^3+6\hbar^2 b_i^2 B_i^3\right)x_i^3 y_i^3}{12b_i^5}\Big]$$