Pensieve header: Exponentiation in ybax algebras.

Startup

In[•]:=
```
Date[]
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\FullDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@"../Profile/Profile.m"];
BeginProfile[];
$k = 2;
<< Engine.m
<< Objects.m
<< KT.m
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, , ]];
```

Out[•]= {2021, 8, 13, 15, 3, 21.4316886}

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.

This version: April 2020. Original version: July 1994.
```

Exponentials

Task. Define $\mathrm{Exp}_m[\mathrm{U}_-(\mathbb{U}_{\{\_\_\}\to\{i\_\}}]$ to compute $e^{\mathbb{D}(U)}$ to order $\epsilon^{\mathrm{Length@}\{U\}-1}$ using the $m_{i,i\to i}$ multiplication, where U is an ϵ-dependent near-docile element, giving the answer in $\mathbb{E}$-form.
Example: $\mathrm{Exp}_{\mathrm{dm},1}[\mathbb{U}_{\{\}\to\{2\}}[b_2\,a_2 + y_2\,x_2,\,0]]$ is the exponential of the arrow on strand 2, computed to degree 1.

```
In[•]:= Exp_m_[𝕎_is_→{i_}[U__]] :=
 Module[{λ, k, n, F, f, j, lhs, rhs, sol, MI(*multi-index*), mis, mi, yax},
  MI /: Coefficient[ℰ_, MI[p_, n_, q_]] :=
   Coefficient[Coefficient[Coefficient[ℰ, y_i, p], a_i, n], x_i, q];
  yax /: yax^MI[p_,n_,q_] := y_i^p a_i^n x_i^q;
  F = 𝔼_{}→{i}[f[λ] a_i];
  lhs =
   (∂_μ Last[F (F /. {λ → μ, i → j}) // m_{i,j→i}]) /. μ → 0 /. {f[0] → 0, f'[0] → ∂_{a_i} {U}[[1]]};
  rhs = ∂_λ Last[F];
  F = F /. First@DSolve[lhs == rhs ∧ f[0] == 0, f, λ];
  mis = {MI[0, 0, 0], MI[1, 0, 0], MI[0, 0, 1], MI[1, 0, 1]};
  F[[1]] += Sum[f_mi[λ] yax^mi, {mi, mis}];
  lhs = (∂_μ U2l@Last[F (F /. {λ → μ, i → j}) // m_{i,j→i}]) /. μ → 0 /. f_[0] → 0 /.
    Table[f_mi'[0] → Coefficient[{U}[[1]], mi], {mi, mis}];
  rhs = ∂_λ Last[F];
  F = F /. First@DSolve[Table[Coefficient[lhs - rhs, mi] == 0 ∧ f_mi[0] == 0, {mi, mis}],
      Table[f_mi, {mi, mis}], λ];
  Do[
   mis =
    Flatten@Table[MI[p, n, q], {n, 0, 2 k + 2, 2}, {p, 0, 2 k + 2 - 2 n}, {q, 0, 2 k + 2 - 2 n - p}];
   AppendTo[F, Sum[f_mi[λ] yax^mi, {mi, mis}]];
   lhs = (∂_μ U2l@Last[F (F /. {λ → μ, i → j}) // m_{i,j→i}]) /. μ → 0 /. f_[0] → 0 /.
     Table[f_mi'[0] → Coefficient[{U}[[k + 1]], mi], {mi, mis}];
   rhs = ∂_λ Last[F];
   F = F /. First@DSolve[Table[Coefficient[lhs - rhs, mi] == 0 ∧ f_mi[0] == 0, {mi, mis}],
       Table[f_mi, {mi, mis}], λ],
   {k, Length[{U}] - 1}
  ];
  CF@l2U[F /. λ → 1]
 ]
```

```
In[•]:= Exp_cm[𝕎_{}→{i}[ℏ a_i b_i + ℏ x_i y_i, c_1 (x_i + y_i)]]
```

$$\text{Out[•]= } \mathbb{E}_{\{\}\to\{i\}}\left[\hbar\, a_i\, b_i - \frac{(-1+B_i)\, x_i\, y_i}{b_i},\ c_1\, x_i + c_1\, y_i - \frac{\hbar\,(-1+B_i)\, x_i\, y_i}{b_i} + \frac{\left(1-4\,B_i+3\,B_i^2+2\,\hbar\, b_i\, B_i^2\right) x_i^2\, y_i^2}{4\, b_i^3}\right]$$

Step by step

```
In[•]:= m = cm; U = Sequence[ℏ a_i b_i + ℏ x_i y_i, c_1 (x_i + y_i), 0]
```

```
Out[•]= Sequence[ℏ a_i b_i + ℏ x_i y_i, c_1 (x_i + y_i), 0]
```

```
In[•]:= MI /: Coefficient[ℰ_, MI[p_, n_, q_]] :=
          Coefficient[Coefficient[Coefficient[ℰ, y_i, p], a_i, n], x_i, q];
        yax /: yax^MI[p_,n_,q_] := y_i^p a_i^n x_i^q;
        F = 𝔼_{}→{i}[f[λ] a_i]
Out[•]= 𝔼_{}→{i}[f[λ] a_i]

In[•]:= lhs = (∂_μ Last[F (F /. {λ → μ, i → j}) // m_{i,j→i}]) /. μ → 0 /. {f[0] → 0, f'[0] → ∂_{a_i} {U}⟦1⟧}
Out[•]= ħ a_i b_i

In[•]:= rhs = ∂_λ Last[F]
Out[•]= a_i f'[λ]

In[•]:= F = F /. First@DSolve[lhs == rhs ∧ f[0] == 0, f, λ]
Out[•]= 𝔼_{}→{i}[λ ħ a_i b_i]

In[•]:= mis = {MI[0, 0, 0], MI[1, 0, 0], MI[0, 0, 1], MI[1, 0, 1]}
Out[•]= {MI[0, 0, 0], MI[1, 0, 0], MI[0, 0, 1], MI[1, 0, 1]}

In[•]:= F⟦1⟧ += Sum[f_mi[λ] yax^mi, {mi, mis}]
Out[•]= λ ħ a_i b_i + f_MI[0,0,0][λ] + x_i f_MI[0,0,1][λ] + y_i f_MI[1,0,0][λ] + x_i y_i f_MI[1,0,1][λ]

In[•]:= lhs = (∂_μ U2l@Last[F (F /. {λ → μ, i → j}) // m_{i,j→i}]) /. μ → 0 /. f_[0] → 0 /.
            Table[f_mi'[0] → Coefficient[{U}⟦1⟧, mi], {mi, mis}]
Out[•]= ħ a_i b_i + ℯ^(-λ ħ b_i) ħ x_i y_i

In[•]:= rhs = ∂_λ Last[F]
Out[•]= ħ a_i b_i + f_MI[0,0,0]'[λ] + x_i f_MI[0,0,1]'[λ] + y_i f_MI[1,0,0]'[λ] + x_i y_i f_MI[1,0,1]'[λ]

In[•]:= F = F /. First@DSolve[
              Table[Coefficient[lhs - rhs, mi] == 0 ∧ f_mi[0] == 0, {mi, mis}], Table[f_mi, {mi, mis}], λ]
Out[•]= 𝔼_{}→{i}[λ ħ a_i b_i + (ℯ^(-λ ħ b_i) (-1 + ℯ^(λ ħ b_i)) x_i y_i)/b_i]

In[•]:= {k, Length[{U}] - 1}
Out[•]= {1, 2}

In[•]:= k = 1
Out[•]= 1

In[•]:= mis = Flatten@Table[MI[p, n, q], {n, 0, 2 k + 2, 2}, {p, 0, 2 k + 2 - 2 n}, {q, 0, 2 k + 2 - 2 n - p}]
Out[•]= {MI[0, 0, 0], MI[0, 0, 1], MI[0, 0, 2], MI[0, 0, 3], MI[0, 0, 4],
         MI[1, 0, 0], MI[1, 0, 1], MI[1, 0, 2], MI[1, 0, 3], MI[2, 0, 0],
         MI[2, 0, 1], MI[2, 0, 2], MI[3, 0, 0], MI[3, 0, 1], MI[4, 0, 0], MI[0, 2, 0]}
```

In[•]:=
```
AppendTo[F, Sum[f_mi[λ] yax^mi, {mi, mis}]]
```

Out[•]=
$$\mathbb{E}_{\{\}\to\{i\}}\Big[\lambda\hbar\,a_i\,b_i+\frac{e^{-\lambda\hbar\,b_i}\left(-1+e^{\lambda\hbar\,b_i}\right)x_i\,y_i}{b_i},$$
$$f_{MI[0,0,0]}[\lambda]+x_i\,f_{MI[0,0,1]}[\lambda]+x_i^2\,f_{MI[0,0,2]}[\lambda]+x_i^3\,f_{MI[0,0,3]}[\lambda]+x_i^4\,f_{MI[0,0,4]}[\lambda]+a_i^2\,f_{MI[0,2,0]}[\lambda]+$$
$$y_i\,f_{MI[1,0,0]}[\lambda]+x_i\,y_i\,f_{MI[1,0,1]}[\lambda]+x_i^2\,y_i\,f_{MI[1,0,2]}[\lambda]+x_i^3\,y_i\,f_{MI[1,0,3]}[\lambda]+y_i^2\,f_{MI[2,0,0]}[\lambda]+$$
$$x_i\,y_i^2\,f_{MI[2,0,1]}[\lambda]+x_i^2\,y_i^2\,f_{MI[2,0,2]}[\lambda]+y_i^3\,f_{MI[3,0,0]}[\lambda]+x_i\,y_i^3\,f_{MI[3,0,1]}[\lambda]+y_i^4\,f_{MI[4,0,0]}[\lambda]\Big]$$

In[•]:=
```
lhs = (∂_μ U2l@Last[F (F /. {λ → μ, i → j}) // m_{i,j→i}]) /. μ → 0 /. f_[0] → 0 /.
  Table[f_mi'[0] → Coefficient[{U}[[k + 1]], mi], {mi, mis}]
```

Out[•]=
$$c_1\,y_i-\hbar\,b_i\,x_i\,f_{MI[0,0,1]}[\lambda]+x_i\,(c_1+\hbar\,b_i\,f_{MI[0,0,1]}[\lambda])-$$
$$\frac{e^{-2\lambda\hbar\,b_i}\,a_i\,x_i\,y_i\left(e^{\lambda\hbar\,b_i}\,\lambda\hbar^2\,b_i^2+2\,e^{\lambda\hbar\,b_i}\,\hbar\,b_i^2\,f_{MI[0,2,0]}[\lambda]\right)}{b_i^2}-\hbar\,b_i\,x_i\,y_i\,f_{MI[1,0,1]}[\lambda]+$$
$$\frac{e^{-\lambda\hbar\,b_i}\,x_i\,y_i\left(-\hbar+e^{\lambda\hbar\,b_i}\,\hbar+\lambda\hbar^2\,b_i+\hbar\,b_i\,f_{MI[0,2,0]}[\lambda]+e^{\lambda\hbar\,b_i}\,\hbar\,b_i^2\,f_{MI[1,0,1]}[\lambda]\right)}{b_i}-$$
$$7\,\hbar\,b_i\,x_i^2\,y_i^2\,f_{MI[2,0,2]}[\lambda]+$$
$$\frac{e^{-7\lambda\hbar\,b_i}\,x_i^2\,y_i^2\left(-2\,e^{5\lambda\hbar\,b_i}\,\hbar\,b_i+2\,e^{6\lambda\hbar\,b_i}\,\hbar\,b_i-2\,e^{5\lambda\hbar\,b_i}\,\lambda\hbar^2\,b_i^2+14\,e^{7\lambda\hbar\,b_i}\,\hbar\,b_i^4\,f_{MI[2,0,2]}[\lambda]\right)}{2\,b_i^3}$$

In[•]:=
```
CF[lhs]
```

Out[•]=
$$c_1\,x_i+c_1\,y_i+\frac{e^{-2\lambda\hbar\,b_i}\,\hbar\left(-1+e^{\lambda\hbar\,b_i}-\lambda\hbar\,b_i\right)x_i^2\,y_i^2}{b_i^2}-$$
$$e^{-\lambda\hbar\,b_i}\,\hbar\,a_i\,x_i\,y_i\,(\lambda\hbar+2\,f_{MI[0,2,0]}[\lambda])+\frac{e^{-\lambda\hbar\,b_i}\,\hbar\,x_i\,y_i\left(-1+e^{\lambda\hbar\,b_i}+\lambda\hbar\,b_i+b_i\,f_{MI[0,2,0]}[\lambda]\right)}{b_i}$$

In[•]:=
```
rhs = ∂_λ Last[F]
```

Out[•]=
$$f_{MI[0,0,0]}'[\lambda]+x_i\,f_{MI[0,0,1]}'[\lambda]+x_i^2\,f_{MI[0,0,2]}'[\lambda]+x_i^3\,f_{MI[0,0,3]}'[\lambda]+$$
$$x_i^4\,f_{MI[0,0,4]}'[\lambda]+a_i^2\,f_{MI[0,2,0]}'[\lambda]+y_i\,f_{MI[1,0,0]}'[\lambda]+x_i\,y_i\,f_{MI[1,0,1]}'[\lambda]+$$
$$x_i^2\,y_i\,f_{MI[1,0,2]}'[\lambda]+x_i^3\,y_i\,f_{MI[1,0,3]}'[\lambda]+y_i^2\,f_{MI[2,0,0]}'[\lambda]+x_i\,y_i^2\,f_{MI[2,0,1]}'[\lambda]+$$
$$x_i^2\,y_i^2\,f_{MI[2,0,2]}'[\lambda]+y_i^3\,f_{MI[3,0,0]}'[\lambda]+x_i\,y_i^3\,f_{MI[3,0,1]}'[\lambda]+y_i^4\,f_{MI[4,0,0]}'[\lambda]$$

In[•]:=
```
F = F /. First@DSolve[
    Table[Coefficient[lhs - rhs, mi] == 0 ∧ f_mi[0] == 0, {mi, mis}], Table[f_mi, {mi, mis}], λ]
```

Out[•]=
$$\mathbb{E}_{\{\}\to\{i\}}\Big[\lambda\hbar\,a_i\,b_i+\frac{e^{-\lambda\hbar\,b_i}\left(-1+e^{\lambda\hbar\,b_i}\right)x_i\,y_i}{b_i},$$
$$\lambda\,c_1\,x_i+\lambda\,c_1\,y_i+\frac{e^{-\lambda\hbar\,b_i}\left(-1+e^{\lambda\hbar\,b_i}\right)\lambda\hbar\,x_i\,y_i}{b_i}+\frac{e^{-2\lambda\hbar\,b_i}\left(3-4\,e^{\lambda\hbar\,b_i}+e^{2\lambda\hbar\,b_i}+2\,\lambda\hbar\,b_i\right)x_i^2\,y_i^2}{4\,b_i^3}\Big]$$

In[•]:=
```
k = 2
```

Out[•]=
2

In[•]:=
```
mis = Flatten@Table[MI[p, n, q], {n, 0, 2 k + 2, 2}, {p, 0, 2 k + 2 - 2 n}, {q, 0, 2 k + 2 - 2 n - p}]
```

Out[•]=
```
{MI[0, 0, 0], MI[0, 0, 1], MI[0, 0, 2], MI[0, 0, 3], MI[0, 0, 4], MI[0, 0, 5], MI[0, 0, 6],
 MI[1, 0, 0], MI[1, 0, 1], MI[1, 0, 2], MI[1, 0, 3], MI[1, 0, 4], MI[1, 0, 5], MI[2, 0, 0],
 MI[2, 0, 1], MI[2, 0, 2], MI[2, 0, 3], MI[2, 0, 4], MI[3, 0, 0], MI[3, 0, 1], MI[3, 0, 2],
 MI[3, 0, 3], MI[4, 0, 0], MI[4, 0, 1], MI[4, 0, 2], MI[5, 0, 0], MI[5, 0, 1], MI[6, 0, 0],
 MI[0, 2, 0], MI[0, 2, 1], MI[0, 2, 2], MI[1, 2, 0], MI[1, 2, 1], MI[2, 2, 0]}
```

In[•]:=
```
AppendTo[F, Sum[f_mi[λ] yax^mi, {mi, mis}]]
```

Out[•]=
$$\mathbb{E}_{\{\}\to\{i\}}\left[\lambda\hbar\, a_i\, b_i + \frac{e^{-\lambda\hbar\, b_i}\left(-1+e^{\lambda\hbar\, b_i}\right) x_i\, y_i}{b_i},\right.$$
$$\lambda\, c_1\, x_i + \lambda\, c_1\, y_i + \frac{e^{-\lambda\hbar\, b_i}\left(-1+e^{\lambda\hbar\, b_i}\right)\lambda\hbar\, x_i\, y_i}{b_i} + \frac{e^{-2\lambda\hbar\, b_i}\left(3-4\,e^{\lambda\hbar\, b_i}+e^{2\lambda\hbar\, b_i}+2\lambda\hbar\, b_i\right) x_i^2\, y_i^2}{4\, b_i^3},$$
$$f_{MI[0,0,0]}[\lambda] + x_i\, f_{MI[0,0,1]}[\lambda] + x_i^2\, f_{MI[0,0,2]}[\lambda] + x_i^3\, f_{MI[0,0,3]}[\lambda] + x_i^4\, f_{MI[0,0,4]}[\lambda] +$$
$$x_i^5\, f_{MI[0,0,5]}[\lambda] + x_i^6\, f_{MI[0,0,6]}[\lambda] + a_i^2\, f_{MI[0,2,0]}[\lambda] + a_i^2\, x_i\, f_{MI[0,2,1]}[\lambda] + a_i^2\, x_i^2\, f_{MI[0,2,2]}[\lambda] +$$
$$y_i\, f_{MI[1,0,0]}[\lambda] + x_i\, y_i\, f_{MI[1,0,1]}[\lambda] + x_i^2\, y_i\, f_{MI[1,0,2]}[\lambda] + x_i^3\, y_i\, f_{MI[1,0,3]}[\lambda] + x_i^4\, y_i\, f_{MI[1,0,4]}[\lambda] +$$
$$x_i^5\, y_i\, f_{MI[1,0,5]}[\lambda] + a_i^2\, y_i\, f_{MI[1,2,0]}[\lambda] + a_i^2\, x_i\, y_i\, f_{MI[1,2,1]}[\lambda] + y_i^2\, f_{MI[2,0,0]}[\lambda] + x_i\, y_i^2\, f_{MI[2,0,1]}[\lambda] +$$
$$x_i^2\, y_i^2\, f_{MI[2,0,2]}[\lambda] + x_i^3\, y_i^2\, f_{MI[2,0,3]}[\lambda] + x_i^4\, y_i^2\, f_{MI[2,0,4]}[\lambda] + a_i^2\, y_i^2\, f_{MI[2,2,0]}[\lambda] +$$
$$y_i^3\, f_{MI[3,0,0]}[\lambda] + x_i\, y_i^3\, f_{MI[3,0,1]}[\lambda] + x_i^2\, y_i^3\, f_{MI[3,0,2]}[\lambda] + x_i^3\, y_i^3\, f_{MI[3,0,3]}[\lambda] + y_i^4\, f_{MI[4,0,0]}[\lambda] +$$
$$\left.x_i\, y_i^4\, f_{MI[4,0,1]}[\lambda] + x_i^2\, y_i^4\, f_{MI[4,0,2]}[\lambda] + y_i^5\, f_{MI[5,0,0]}[\lambda] + x_i\, y_i^5\, f_{MI[5,0,1]}[\lambda] + y_i^6\, f_{MI[6,0,0]}[\lambda]\right]$$

```
In[•]:= lhs = (∂_μ U2l@Last[F (F /. {λ → μ, i → j}) // m_{i,j→i}]) /. μ → 0 /. f_[0] → 0 /.
            Table[f_mi'[0] → Coefficient[{U}[[k + 1]], mi], {mi, mis}]
```

Out[•]=

$$\lambda\, b_i\, c_1^2 + \lambda\, \hbar\, c_1\, y_i - \frac{e^{-\lambda \hbar b_i}\, a_i \left(1 - e^{\lambda \hbar b_i} + \lambda\, \hbar\, b_i\right) c_1\, y_i}{b_i} - \hbar\, b_i\, x_i\, f_{MI[0,0,1]}[\lambda] +$$

$$x_i \left(\lambda\, \hbar\, c_1 + \hbar\, b_i\, f_{MI[0,0,1]}[\lambda]\right) - \frac{e^{-2\lambda \hbar b_i}\, a_i\, x_i\, y_i \left(e^{\lambda \hbar b_i}\, \lambda^2\, \hbar^3\, b_i^2 + 2\, e^{\lambda \hbar b_i}\, \hbar\, b_i^2\, f_{MI[0,2,0]}[\lambda]\right)}{b_i^2} -$$

$$2\, e^{-\lambda \hbar b_i}\, \hbar\, a_i\, x_i^2\, y_i\, f_{MI[0,2,1]}[\lambda] - 2\, e^{-\lambda \hbar b_i}\, \hbar\, a_i\, x_i^3\, y_i\, f_{MI[0,2,2]}[\lambda] -$$

$$2\, \hbar\, b_i\, x_i\, y_i\, f_{MI[1,0,1]}[\lambda] + \frac{1}{2\, b_i}\, e^{-2\lambda \hbar b_i}\, x_i\, y_i \left(-2\, e^{\lambda \hbar b_i}\, \lambda\, \hbar^2 + 2\, e^{2\lambda \hbar b_i}\, \lambda\, \hbar^2 + e^{\lambda \hbar b_i}\, \lambda^2\, \hbar^3\, b_i + \right.$$

$$\left. 2\, e^{\lambda \hbar b_i}\, \hbar\, b_i\, f_{MI[0,2,0]}[\lambda] + 4\, e^{2\lambda \hbar b_i}\, \hbar\, b_i^2\, f_{MI[1,0,1]}[\lambda]\right) - 5\, \hbar\, b_i\, x_i^2\, y_i\, f_{MI[1,0,2]}[\lambda] +$$

$$\frac{e^{-3\lambda \hbar b_i}\, x_i^2\, y_i \left(2\, e^{2\lambda \hbar b_i}\, \hbar\, b_i^2\, f_{MI[0,2,1]}[\lambda] + 10\, e^{3\lambda \hbar b_i}\, \hbar\, b_i^3\, f_{MI[1,0,2]}[\lambda]\right)}{2\, b_i^2} - 3\, \hbar\, b_i\, x_i^3\, y_i\, f_{MI[1,0,3]}[\lambda] +$$

$$\frac{e^{-\lambda \hbar b_i}\, x_i^3\, y_i \left(\hbar\, b_i\, f_{MI[0,2,2]}[\lambda] + 3\, e^{\lambda \hbar b_i}\, \hbar\, b_i^2\, f_{MI[1,0,3]}[\lambda]\right)}{b_i} - 2\, e^{-\lambda \hbar b_i}\, \hbar\, a_i\, x_i\, y_i^2\, f_{MI[1,2,0]}[\lambda] -$$

$$\hbar\, a_i^2\, b_i\, x_i\, y_i\, f_{MI[1,2,1]}[\lambda] + \frac{e^{-\lambda \hbar b_i}\, a_i^2\, x_i\, y_i \left(\lambda^2\, \hbar^3\, b_i + 2\, e^{\lambda \hbar b_i}\, \hbar\, b_i^2\, f_{MI[1,2,1]}[\lambda]\right)}{2\, b_i} -$$

$$\frac{e^{-7\lambda \hbar b_i}\, a_i\, x_i^2\, y_i^2 \left(-2\, e^{5\lambda \hbar b_i}\, \lambda\, \hbar^2\, b_i^2 + 2\, e^{6\lambda \hbar b_i}\, \lambda\, \hbar^2\, b_i^2 - 2\, e^{5\lambda \hbar b_i}\, \lambda^2\, \hbar^3\, b_i^3 + 4\, e^{6\lambda \hbar b_i}\, \hbar\, b_i^4\, f_{MI[1,2,1]}[\lambda]\right)}{2\, b_i^4} -$$

$$3\, \hbar\, b_i\, x_i\, y_i^2\, f_{MI[2,0,1]}[\lambda] + \frac{1}{2\, b_i^2}\, e^{-5\lambda \hbar b_i}\, x_i\, y_i^2$$

$$\left(-e^{3\lambda \hbar b_i}\, c_1 + 2\, e^{4\lambda \hbar b_i}\, c_1 - e^{5\lambda \hbar b_i}\, c_1 + 2\, e^{4\lambda \hbar b_i}\, \hbar\, b_i^2\, f_{MI[1,2,0]}[\lambda] + 6\, e^{5\lambda \hbar b_i}\, \hbar\, b_i^3\, f_{MI[2,0,1]}[\lambda]\right) - 7\, \hbar\, b_i$$

$$x_i^2\, y_i^2\, f_{MI[2,0,2]}[\lambda] + \frac{1}{8\, b_i^4}\, e^{-7\lambda \hbar b_i}\, x_i^2\, y_i^2 \left(4\, e^{5\lambda \hbar b_i}\, \hbar\, b_i - 8\, e^{6\lambda \hbar b_i}\, \hbar\, b_i + 4\, e^{7\lambda \hbar b_i}\, \hbar\, b_i - 16\, e^{5\lambda \hbar b_i}\, \lambda\, \hbar^2\, b_i^2 + \right.$$

$$\left. 16\, e^{6\lambda \hbar b_i}\, \lambda\, \hbar^2\, b_i^2 - 20\, e^{5\lambda \hbar b_i}\, \lambda^2\, \hbar^3\, b_i^3 + 8\, e^{6\lambda \hbar b_i}\, \hbar\, b_i^4\, f_{MI[1,2,1]}[\lambda] + 56\, e^{7\lambda \hbar b_i}\, \hbar\, b_i^5\, f_{MI[2,0,2]}[\lambda]\right) -$$

$$2\, e^{-\lambda \hbar b_i}\, \hbar\, a_i\, x_i\, y_i^3\, f_{MI[2,2,0]}[\lambda] - \hbar\, b_i\, x_i\, y_i^3\, f_{MI[3,0,1]}[\lambda] +$$

$$\frac{e^{-3\lambda \hbar b_i}\, x_i\, y_i^3 \left(e^{2\lambda \hbar b_i}\, \hbar\, b_i\, f_{MI[2,2,0]}[\lambda] + e^{3\lambda \hbar b_i}\, \hbar\, b_i^2\, f_{MI[3,0,1]}[\lambda]\right)}{b_i} - 18\, \hbar\, b_i\, x_i^3\, y_i^3\, f_{MI[3,0,3]}[\lambda] +$$

$$\frac{1}{12\, b_i^5}\, e^{-18\lambda \hbar b_i}\, x_i^3\, y_i^3 \left(33\, e^{15\lambda \hbar b_i}\, \hbar\, b_i - 48\, e^{16\lambda \hbar b_i}\, \hbar\, b_i + 15\, e^{17\lambda \hbar b_i}\, \hbar\, b_i + \right.$$

$$\left. 42\, e^{15\lambda \hbar b_i}\, \lambda\, \hbar^2\, b_i^2 - 24\, e^{16\lambda \hbar b_i}\, \lambda\, \hbar^2\, b_i^2 + 18\, e^{15\lambda \hbar b_i}\, \lambda^2\, \hbar^3\, b_i^3 + 216\, e^{18\lambda \hbar b_i}\, \hbar\, b_i^6\, f_{MI[3,0,3]}[\lambda]\right)$$

In[•]:=
```
rhs = ∂λ Last[F]
```

Out[•]=
$$f_{MI[0,0,0]}'[\lambda] + x_i\, f_{MI[0,0,1]}'[\lambda] + x_i^2\, f_{MI[0,0,2]}'[\lambda] + x_i^3\, f_{MI[0,0,3]}'[\lambda] + x_i^4\, f_{MI[0,0,4]}'[\lambda] +$$
$$x_i^5\, f_{MI[0,0,5]}'[\lambda] + x_i^6\, f_{MI[0,0,6]}'[\lambda] + a_i^2\, f_{MI[0,2,0]}'[\lambda] + a_i^2\, x_i\, f_{MI[0,2,1]}'[\lambda] + a_i^2\, x_i^2\, f_{MI[0,2,2]}'[\lambda] +$$
$$y_i\, f_{MI[1,0,0]}'[\lambda] + x_i\, y_i\, f_{MI[1,0,1]}'[\lambda] + x_i^2\, y_i\, f_{MI[1,0,2]}'[\lambda] + x_i^3\, y_i\, f_{MI[1,0,3]}'[\lambda] + x_i^4\, y_i\, f_{MI[1,0,4]}'[\lambda] +$$
$$x_i^5\, y_i\, f_{MI[1,0,5]}'[\lambda] + a_i^2\, y_i\, f_{MI[1,2,0]}'[\lambda] + a_i^2\, x_i\, y_i\, f_{MI[1,2,1]}'[\lambda] + y_i^2\, f_{MI[2,0,0]}'[\lambda] + x_i\, y_i^2\, f_{MI[2,0,1]}'[\lambda] +$$
$$x_i^2\, y_i^2\, f_{MI[2,0,2]}'[\lambda] + x_i^3\, y_i^2\, f_{MI[2,0,3]}'[\lambda] + x_i^4\, y_i^2\, f_{MI[2,0,4]}'[\lambda] + a_i^2\, y_i^2\, f_{MI[2,2,0]}'[\lambda] +$$
$$y_i^3\, f_{MI[3,0,0]}'[\lambda] + x_i\, y_i^3\, f_{MI[3,0,1]}'[\lambda] + x_i^2\, y_i^3\, f_{MI[3,0,2]}'[\lambda] + x_i^3\, y_i^3\, f_{MI[3,0,3]}'[\lambda] + y_i^4\, f_{MI[4,0,0]}'[\lambda] +$$
$$x_i\, y_i^4\, f_{MI[4,0,1]}'[\lambda] + x_i^2\, y_i^4\, f_{MI[4,0,2]}'[\lambda] + y_i^5\, f_{MI[5,0,0]}'[\lambda] + x_i\, y_i^5\, f_{MI[5,0,1]}'[\lambda] + y_i^6\, f_{MI[6,0,0]}'[\lambda]$$

In[•]:=
```
F = F /. First@DSolve[
    Table[Coefficient[lhs - rhs, mi] == 0 ∧ f_mi[0] == 0, {mi, mis}], Table[f_mi, {mi, mis}], λ]
```

Out[•]=
$$\mathbb{E}_{\{\}\to\{i\}}\Bigg[\lambda\hbar\, a_i\, b_i + \frac{e^{-\lambda\hbar b_i}\left(-1+e^{\lambda\hbar b_i}\right) x_i\, y_i}{b_i},$$
$$\lambda\, c_1\, x_i + \lambda\, c_1\, y_i + \frac{e^{-\lambda\hbar b_i}\left(-1+e^{\lambda\hbar b_i}\right)\lambda\hbar\, x_i\, y_i}{b_i} + \frac{e^{-2\lambda\hbar b_i}\left(3-4\,e^{\lambda\hbar b_i}+e^{2\lambda\hbar b_i}+2\lambda\hbar\, b_i\right) x_i^2\, y_i^2}{4\, b_i^3},$$
$$\frac{1}{2}\lambda^2\, b_i\, c_1^2 + \frac{1}{2}\lambda^2\hbar\, c_1\, x_i + \frac{1}{2}\lambda^2\hbar\, c_1\, y_i + \frac{e^{-\lambda\hbar b_i}\left(-1+e^{\lambda\hbar b_i}\right)\lambda^2\hbar^2\, x_i\, y_i}{2\, b_i} +$$
$$\frac{e^{-\lambda\hbar b_i}\, a_i^2\left(-2+2\,e^{\lambda\hbar b_i}-2\lambda\hbar\, b_i-\lambda^2\hbar^2\, b_i^2\right) x_i\, y_i}{2\, b_i^3} -$$
$$\frac{e^{-2\lambda\hbar b_i}\left(-1+4\,e^{\lambda\hbar b_i}-3\,e^{2\lambda\hbar b_i}+2\,e^{2\lambda\hbar b_i}\lambda\hbar\, b_i\right) c_1\, x_i\, y_i^2}{4\hbar\, b_i^3} +$$
$$\frac{e^{-2\lambda\hbar b_i}\left(7-8\,e^{\lambda\hbar b_i}+e^{2\lambda\hbar b_i}+12\lambda\hbar\, b_i-8\,e^{\lambda\hbar b_i}\lambda\hbar\, b_i+2\,e^{2\lambda\hbar b_i}\lambda\hbar\, b_i+6\lambda^2\hbar^2\, b_i^2\right) x_i^2\, y_i^2}{4\, b_i^4} +$$
$$\frac{e^{-3\lambda\hbar b_i}\left(-17+30\,e^{\lambda\hbar b_i}-15\,e^{2\lambda\hbar b_i}+2\,e^{3\lambda\hbar b_i}-18\lambda\hbar\, b_i+12\,e^{\lambda\hbar b_i}\lambda\hbar\, b_i-6\lambda^2\hbar^2\, b_i^2\right) x_i^3\, y_i^3}{12\, b_i^5}\Bigg]$$

In[•]:=
```
CF@l2U[F /. λ → 1]
```

Out[•]=
$$\mathbb{E}_{\{\}\to\{i\}}\Bigg[\hbar\, a_i\, b_i - \frac{(-1+B_i)\, x_i\, y_i}{b_i},\ c_1\, x_i + c_1\, y_i - \frac{\hbar\,(-1+B_i)\, x_i\, y_i}{b_i} + \frac{\left(1-4\,B_i+3\,B_i^2+2\hbar\, b_i\, B_i^2\right) x_i^2\, y_i^2}{4\, b_i^3},$$
$$\frac{1}{2}\, b_i\, c_1^2 + \frac{1}{2}\hbar\, c_1\, x_i + \frac{1}{2}\hbar\, c_1\, y_i - \frac{\hbar^2\,(-1+B_i)\, x_i\, y_i}{2\, b_i} - \frac{a_i^2\left(-2+2\,B_i+2\hbar\, b_i\, B_i+\hbar^2\, b_i^2\, B_i\right) x_i\, y_i}{2\, b_i^3} -$$
$$\frac{\left(-3+2\hbar\, b_i+4\,B_i-B_i^2\right) c_1\, x_i\, y_i^2}{4\hbar\, b_i^3} + \frac{\left(1+2\hbar\, b_i-8\,B_i-8\hbar\, b_i\, B_i+7\,B_i^2+12\hbar\, b_i\, B_i^2+6\hbar^2\, b_i^2\, B_i^2\right) x_i^2\, y_i^2}{4\, b_i^4} -$$
$$\frac{\left(-2+15\,B_i-30\,B_i^2-12\hbar\, b_i\, B_i^2+17\,B_i^3+18\hbar\, b_i\, B_i^3+6\hbar^2\, b_i^2\, B_i^3\right) x_i^3\, y_i^3}{12\, b_i^5}\Bigg]$$