

Pensieve header: Profile with encapsulation of inner Zip2 and Zip3. Time to K31@\$k=3: 7112.92.

Startup

```
(Alt) In[ ]:=
Date[]
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\FullDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@"../Profile/Profile.m"];
$k = 1;
<< Objects.m
<< KT.m
```

```
(Alt) Out[ ]:= {2021, 1, 3, 9, 53, 30.2035381}
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

Engine

Canonical Forms:

```
(Alt) In[ ]:=
CCF[ $\mathcal{E}$ _] := PPCCF@ExpandDenominator@ExpandNumerator@Together[ $\mathcal{E}$ ];
(*Coefficient Canonical Form *)
CF[ $\mathcal{E}$ _] := PPCF@Module[
  {vs = Cases[ $\mathcal{E}$ , (y | a | x |  $\eta$  |  $\beta$  |  $\tau$  |  $\xi$ )_ ,  $\infty$ ]  $\cup$  {y, a, x,  $\eta$ ,  $\beta$ ,  $\tau$ ,  $\xi$ }},
  Total[(CCF[#[[2]]]  $\times$  (Times@@vs#[[1]])) & /@ CoefficientRules[ $\mathcal{E}$ , vs]]
];
CF[ $\mathcal{E}$ _E] := CF /@  $\mathcal{E}$ ;
CF[ $\mathcal{E}$ _List] := CF /@  $\mathcal{E}$ ;
CF[ $\mathbb{E}$ sp___[ $\mathcal{ES}$ ___]] := CF /@  $\mathbb{E}$ sp[ $\mathcal{ES}$ ];
```

Variables and their duals:

```
(Alt) In[ ]:=
{t*, b*, y*, a*, x*, z*,  $\tau$ *,  $\beta$ *,  $\eta$ *,  $\alpha$ *,  $\xi$ *,  $\zeta$ *} = { $\tau$ ,  $\beta$ ,  $\eta$ ,  $\alpha$ ,  $\xi$ ,  $\zeta$ , t, b, y, a, x, z};
(vs_List)* := (v  $\mapsto$  v*) /@ vs;
(u_i)* := (u*)i;
```

Weights:

```
(Alt) In[ ]:=
Clear[Wt];
Evaluate[Wt /@ {y, b, t, a, x,  $\eta$ ,  $\beta$ ,  $\tau$ ,  $\alpha$ ,  $\xi$ }] = {1, 0, 0, 2, 1, 1, 2, 2, 0, 1};
Wt[u_i] := Wt[u];
```

The maximal weight \$n, i.e. the n of $gl(n)$. Initially and for a long while this will not be tested beyond \$n == 2.

```
(Alt) In[ ]:=
$n = 2;
```

Upper to lower and lower to Upper:

```
(Alt) In[ ]:=
U21[ε_] := ε /. {B_i^p_ := e^{-p h b_i}, B^p_ := e^{-p h b}, T_i^p_ := e^{p h t_i}, T^p_ := e^{p h t}, A_i^p_ := e^{p α_i}, A^p_ := e^{p α}};
L2U[ε_] := ε //. {e^{c_· b_i + d_} := B_i^{-c/h} e^d, e^{c_· b + d_} := B^{-c/h} e^d, e^{c_· t_i + d_} := T_i^{c/h} e^d, e^{c_· t + d_} := T^{c/h} e^d,
e^{c_· α_i + d_} := A_i^c e^d, e^{c_· α + d_} := A^c e^d, e^χ := e^{Expand@χ}};
L2U[r_Rule] := Module[{U = r[[1]] /. {b → B, t → T, α → A}}, U → L2U[U21[U] /. r]];
AlsoUpper[rs_List] := rs ∪ (L2U/@rs);
```

Derivatives in the presence of exponentiated variables:

```
(Alt) In[ ]:=
D_b[f_] := ∂_b f - h B ∂_B f; D_b_i[f_] := ∂_{b_i} f - h B_i ∂_{B_i} f;
D_t[f_] := ∂_t f + h T ∂_T f; D_t_i[f_] := ∂_{t_i} f + h T_i ∂_{T_i} f;
D_α[f_] := ∂_α f + A ∂_A f; D_α_i[f_] := ∂_{α_i} f + A_i ∂_{A_i} f;
D_v[f_] := ∂_v f;
```

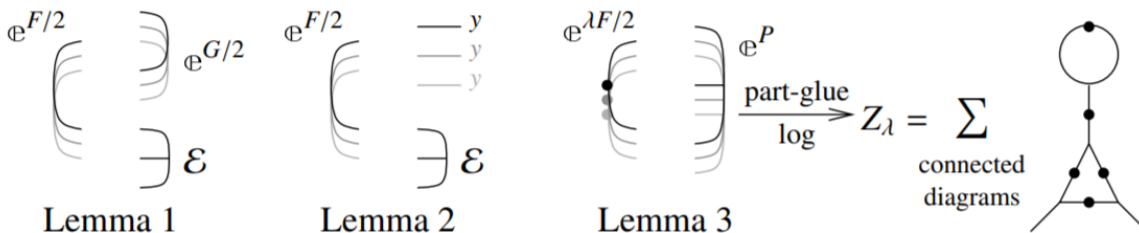
E operations:

```
(Alt) In[ ]:=
ε_E[$] := Length[ε] - 1; E_[εS___] [$] := E[εS] [$];
ε_E[k_Integer] := ε[[k+1]]; E_[εS___] [k_Integer] := {εS}[[k+1]];
E /: ε1_E ≡ ε2_E := Inner[CF@#1 == CF@#2 &, ε1, ε2, And];
E_{d1→r1}[ε1S___] ≡ E_{d2→r2}[ε2S___] ^:= (d1 == d2) ∧ (r1 == r2) ∧ (E[ε1S] ≡ E[ε2S]);
E /: ε1_E * ε2_E := E @@ Table[CF[ε1[kk] + ε2[kk]], {kk, 0, Min[ε1[$], ε2[$]]};
E_{d1→r1}[ε1S___] E_{d2→r2}[ε2S___] ^:= E_{(d1∪d2)→(r1∪r2)} @@ (E[ε1S] × E[ε2S]);
```

```
(Alt) In[ ]:=
E_{d1→r1}[ε1S___] // E_{d2→r2}[ε2S___] := Module[{is = r1 ∩ d2, lvs},
lvs = Flatten@Table[{y_{ei}, b_{ei}, t_{ei}, a_{ei}, x_{ei}}, {i, is}];
E_{(d1∪Complement[d2,is])→(r2∪Complement[r1,is])} @@ (Zip_{lvs∪lvs} [{lvs*.lvs, Times[
E[ε1S] /. Table[(v : b | B | t | T | a | x | y)_i → v_{ei}, {i, is}],
E[ε2S] /. Table[(v : β | τ | α | A | ε | η)_i → v_{ei}, {i, is}]
]}])
]
```

```
(Alt) In[ ]:=
Λ2E_{d→r}[A_] := Module[{k}, E_{d→r} @@ L2U@Table[SeriesCoefficient[A, {ε, 0, k}], {k, 0, $k}];
```

Ziping! Lemmas 2 and 3 are combined, yet they must be applied first to the middle weight variables and then to the heavy and light variables.



```
(Alt) In[ ]:=
Zip_{vs_} [{F_, ε_}] :=
{F, ε} // Zip1_{vs} // EZip23_{Select[vs, (0 < Wt[#] < $n) &]} // Zip2_{Select[vs, (Wt[#] == 0 ∨ Wt[#] == $n) &]} //
Zip3_{Select[vs, (Wt[#] == 0 ∨ Wt[#] == $n) &]} // Last;
```

Getting rid of the quadratic.

Lemma 1. With convergences left to the reader,

$$\left\langle F: \mathcal{E}_{\mathbb{E}^{\frac{1}{2} \sum_{i,j \in B} G_{ij} z_i z_j}} \right\rangle_B = \det(1 - GF)^{-1/2} \left\langle F(1 - GF)^{-1}: \mathcal{E} \right\rangle_B$$

(Alt) In[]:=

```

Zip1{} = Identity;
Zip1vs_@{F_, E[Q_, P___]} := PPZip1@Module[{I, F, G, u, v},
  I = IdentityMatrix@Length@vs;
  F = Table[If[Wt[u] + Wt[v] == $n,  $\partial_{u^*, v^*} \mathcal{F}$ ,  $\theta$ ], {u, vs}, {v, vs}] ;
  G = Table[If[Wt[u] + Wt[v] == $n,  $\partial_{u, v} Q$ ,  $\theta$ ], {u, vs}, {v, vs}] ;
  {CF[vs*.(F.Inverse[I - G.F]).vs* / 2], E[CF[Q - Log[Det[I - G.F]] / 2 - vs.G.vs / 2], P]}
]

```

Getting rid of linear terms.

Lemma 2. $\left\langle F: \mathcal{E}_{\mathbb{E}^{\sum_{i \in B} y_i z_i}} \right\rangle_B = \mathbb{E}^{\frac{1}{2} \sum_{i,j \in B} F_{ij} y_i y_j} \left\langle F: \mathcal{E}_{|z_B \rightarrow z_B + F y_B} \right\rangle_B$.

(Alt) In[]:=

```

Zip2{} = Identity;
Zip2vs_@{F_, E[Q_, P___]} := PPZip2@Module[{F, Y, u, v},
  F = Table[If[Wt[u] + Wt[v] == $n,  $\partial_{u^*, v^*} \mathcal{F}$ ,  $\theta$ ], {u, vs}, {v, vs}] ;
  Y = Table[ $\partial_v Q$ , {v, vs}] /. AlsoUpper@Table[v →  $\theta$ , {v, vs}] ;
  CF /@ ({F_, E[Q - Y.vs + Y.F.Y / 2, P]} /. AlsoUpper@Thread[vs → vs + F.Y])
]

```

Dealing with Feynman diagrams.

Lemma 3. With an extra variable λ , $Z_\lambda := \log[\lambda F: \mathbb{E}^P]_B$ satisfies and is determined by the following PDE / IVP:

$$Z_0 = P \quad \text{and} \quad \partial_\lambda Z_\lambda = \frac{1}{2} \sum_{i,j \in B} F_{ij} \left(\partial_{z_i} \partial_{z_j} Z_\lambda + (\partial_{z_i} Z_\lambda)(\partial_{z_j} Z_\lambda) \right).$$

Note that the power m of λ is at most $k - 1 + \frac{2k+2}{2} = 2k$. We write $Z_\lambda = \sum Z[m] \lambda^m$.

```

(Alt) In[ ]:= Zip3vs@{ $\mathcal{F}$ _,  $\mathcal{E}$ _E} := PPZip3@Module[
  {F, u, v, Z, $k, kk, jj, $m = 0, m, n},
  $k = Length[ $\mathcal{E}$ ] - 1;
  Do[Z[0, kk] =  $\mathcal{E}$ [[kk + 1]], {kk, 0, $k}];
  F[u_, v_] := F[u, v] = CF@If[Wt[u] + Wt[v] == $n,  $\partial_{u^*, v^*} \mathcal{F}$ , 0];
  Z[m_, kk_, u_] := Z[m, kk, u] = Du[Z[m, kk]];
  Z[m_, kk_, u_, v_] := Z[m, kk, u, v] = Dv[Z[m, kk, u]];
  For[m = 0, m ≤ 2 $m, ++m, For[kk = 0, kk ≤ $k, ++kk,
    Z[m + 1, kk] = CF@Sum[
      If[F[u, v] == 0, 0,  $\frac{F[u, v]}{2(m+1)}$ 
        (Z[m, kk, u, v] + Sum[Z[n, jj, u] * Z[m - n, kk - jj, v], {n, 0, m}, {jj, 0, kk}])],
      {u, vs}, {v, vs}];
    If[Z[m + 1, kk] != 0, $m = m + 1
  ]];
  CF/@({
     $\mathcal{F}$  - Sum[F[u, v] u* v* / 2, {u, vs}, {v, vs}],
    E@@Table[Sum[Z[m, kk], {m, 0, $m}], {kk, 0, $k}]
  ) /. AlsoUpper@Table[v → 0, {v, vs}]
]

```

Encapsulation.

```

(Alt) In[ ]:= EZip3vs@{ $\mathcal{F}$ _,  $\mathcal{E}$ _E} := PPEZip3@Module[
  {n $\mathcal{E}$ , n $\mathcal{F}$ , rc, ps, rr = {(*release rules*)}},
  rc = 0; n $\mathcal{E}$  = Total[
    CoefficientRules[#, vs] /. (ps_ → c_) ⇒ (AppendTo[rr, c $\mathcal{E}$ [++rc] → c]; c $\mathcal{E}$ [rc] × (Times@@vsps))
  ] & /@  $\mathcal{E}$ ;
  rc = 0; n $\mathcal{F}$  = Total[CoefficientRules[ $\mathcal{F}$ , vs*] /.
    (ps_ → c_) ⇒ (AppendTo[rr, c $\mathcal{F}$ [++rc] → c]; c $\mathcal{F}$ [rc] × (Times@@(vs*)ps))];
  CF[Expand[{n $\mathcal{F}$ , n $\mathcal{E}$ } // Zip3vs] /. rr]
]

```

```

(Alt) In[ ]:= EZip23vs@{ $\mathcal{F}$ _,  $\mathcal{E}$ _E} := PPEZip3@Module[
  {n $\mathcal{E}$ , n $\mathcal{F}$ , rc, ps, rr = {(*release rules*)}},
  rc = 0; n $\mathcal{E}$  = Total[
    CoefficientRules[#, vs] /. (ps_ → c_) ⇒ (AppendTo[rr, c $\mathcal{E}$ [++rc] → c]; c $\mathcal{E}$ [rc] × (Times@@vsps))
  ] & /@  $\mathcal{E}$ ;
  rc = 0; n $\mathcal{F}$  = Total[CoefficientRules[ $\mathcal{F}$ , vs*] /.
    (ps_ → c_) ⇒ (AppendTo[rr, c $\mathcal{F}$ [++rc] → c]; c $\mathcal{F}$ [rc] × (Times@@(vs*)ps))];
  CF[Expand[{n $\mathcal{F}$ , n $\mathcal{E}$ } // Zip2vs // Zip3vs] /. rr]
]

```

Profiling

(Alt) In[]:= **BeginProfile**[];

(Alt) In[]:= **cm**_{1,2→1} // **cm**_{1,3→1}

(Alt) Out[]:= $\mathbb{E}_{\{1,2,3\} \rightarrow \{1\}}$ [

$$\begin{aligned} & \mathbf{a}_1 (\alpha_1 + \alpha_2 + \alpha_3) + \mathbf{b}_1 \beta_1 + \mathbf{b}_1 \beta_2 + \mathbf{b}_1 \beta_3 + \mathbf{y}_1 \eta_1 + \frac{\mathbf{y}_1 \eta_2}{\mathcal{A}_1} + \frac{\mathbf{y}_1 \eta_3}{\mathcal{A}_1 \mathcal{A}_2} + \frac{\mathbf{x}_1 \xi_1}{\mathcal{A}_2 \mathcal{A}_3} + \mathbf{b}_1 \eta_2 \xi_1 + \frac{\mathbf{b}_1 \eta_3 \xi_1}{\mathcal{A}_2} + \frac{\mathbf{x}_1 \xi_2}{\mathcal{A}_3} + \mathbf{b}_1 \eta_3 \xi_2 + \mathbf{x}_1 \xi_3, \\ & - \frac{\mathbf{y}_1 \beta_1 \eta_2}{\mathcal{A}_1} - \frac{\mathbf{y}_1 \beta_1 \eta_3}{\mathcal{A}_1 \mathcal{A}_2} - \frac{\mathbf{y}_1 \beta_2 \eta_3}{\mathcal{A}_1 \mathcal{A}_2} - \frac{\mathbf{x}_1 \beta_2 \xi_1}{\mathcal{A}_2 \mathcal{A}_3} - \frac{\mathbf{x}_1 \beta_3 \xi_1}{\mathcal{A}_2 \mathcal{A}_3} + \mathbf{a}_1 \eta_2 \xi_1 - \frac{\mathbf{y}_1 \eta_2^2 \xi_1}{\mathcal{A}_1} + \frac{\mathbf{a}_1 \eta_3 \xi_1}{\mathcal{A}_2} - \\ & \frac{\mathbf{b}_1 \beta_2 \eta_3 \xi_1}{\mathcal{A}_2} - \frac{2 \mathbf{y}_1 \eta_2 \eta_3 \xi_1}{\mathcal{A}_1 \mathcal{A}_2} - \frac{\mathbf{y}_1 \eta_3^2 \xi_1}{\mathcal{A}_1 \mathcal{A}_2^2} - \frac{\mathbf{x}_1 \eta_2 \xi_1^2}{\mathcal{A}_2 \mathcal{A}_3} - \frac{1}{2} \mathbf{b}_1 \eta_2^2 \xi_1^2 - \frac{\mathbf{x}_1 \eta_3 \xi_1^2}{\mathcal{A}_2^2 \mathcal{A}_3} - \frac{\mathbf{b}_1 \eta_2 \eta_3 \xi_1^2}{\mathcal{A}_2} - \\ & \left. \frac{\mathbf{b}_1 \eta_3^2 \xi_1^2}{2 \mathcal{A}_2^2} - \frac{\mathbf{x}_1 \beta_3 \xi_2}{\mathcal{A}_3} + \mathbf{a}_1 \eta_3 \xi_2 - \frac{\mathbf{y}_1 \eta_3^2 \xi_2}{\mathcal{A}_1 \mathcal{A}_2} - \frac{2 \mathbf{x}_1 \eta_3 \xi_1 \xi_2}{\mathcal{A}_2 \mathcal{A}_3} - \frac{\mathbf{b}_1 \eta_3^2 \xi_1 \xi_2}{\mathcal{A}_2} - \frac{\mathbf{x}_1 \eta_3 \xi_2^2}{\mathcal{A}_3} - \frac{1}{2} \mathbf{b}_1 \eta_3^2 \xi_2^2 \right] \end{aligned}$$

(Alt) In[]:= **Timing@Block**[{**\$k** = 1}, **Z**[**Knot**[3, 1]]]

KnotTheory: Loading precomputed data in PD4Knots`.

$$\begin{aligned} \text{(Alt) Out[]:= } & \left\{ 16.0313, \mathbb{E}_{\{1\} \rightarrow \{0\}} \left[\frac{1}{2} \times \left(-4 t \hbar - \text{Log} \left[\left(\frac{1}{T^3} - \frac{2}{T^2} + \frac{2}{T} \right)^2 \right] - \text{Log} \left[\left(1 + \frac{T}{1 - 2T + 2T^2} - \frac{T^2}{1 - 2T + 2T^2} \right)^2 \right] \right) \right], \right. \\ & \left. \frac{\mathbf{a} (-2 \hbar + 2 T^2 \hbar)}{1 - T + T^2} + \frac{-2 \hbar + 3 T \hbar - 2 T^2 \hbar + T^3 \hbar}{1 - 2 T + 3 T^2 - 2 T^3 + T^4} + \frac{\mathbf{x} \mathbf{y} (-2 \hbar^2 - 2 T \hbar^2)}{1 - T + T^2} \right\} \end{aligned}$$

(Alt) In[]:= **PrintProfile**[]

(Alt) Out[]:= ProfileRoot is root. Profiled time: 16.36

```
( 1) 0.079/ 15.990 above Z
( 1) 0.219/ 0.219 above Boot
( 2) 0.015/ 0.015 above CF
( 1) 0/ 0.047 above EZip3
( 1) 0/ 0 above RVK
( 1) 0/ 0.016 above Zip1
( 1) 0/ 0.031 above Zip2
( 1) 0.016/ 0.047 above Zip3
```

CF: called 13144 times, time in 5.962/10.51

```
( 84) 0.265/ 0.469 under Z
( 76) 0.094/ 0.202 under Boot
( 138) 0.610/ 2.153 under EZip3
( 2) 0.015/ 0.015 under ProfileRoot
( 92) 0.172/ 0.393 under Zip1
( 276) 0.577/ 1.809 under Zip2
( 12476) 4.229/ 5.469 under Zip3
( 7621) 4.548/ 4.548 above CCF
```

CCF: called 7621 times, time in 4.548/4.548

```
( 7621) 4.548/ 4.548 under CF
```

Zip3: called 92 times, time in 2.894/8.363

```
( 22) 0.879/ 2.999 under Z
( 23) 1.078/ 2.674 under Boot
( 46) 0.921/ 2.643 under EZip3
( 1) 0.016/ 0.047 under ProfileRoot
```

```

( 12476) 4.229/ 5.469 above CF
Zip1: called 46 times, time in 1.248/1.641
( 22) 0.499/ 0.686 under Z
( 23) 0.749/ 0.939 under Boot
( 1) 0/ 0.016 under ProfileRoot
( 92) 0.172/ 0.393 above CF
Zip2: called 92 times, time in 1.015/2.824
( 22) 0.300/ 1.050 under Z
( 23) 0.357/ 0.574 under Boot
( 46) 0.358/ 1.169 under EZip3
( 1) 0/ 0.031 under ProfileRoot
( 276) 0.577/ 1.809 above CF
Boot: called 24 times, time in 0.314/17.655
( 5) 0.016/ 6.594 under Z
( 18) 0.079/ 10.840 under Boot
( 1) 0.219/ 0.219 under ProfileRoot
( 18) 0.079/ 10.840 above Boot
( 76) 0.094/ 0.202 above CF
( 23) 0.109/ 2.110 above EZip3
( 23) 0.749/ 0.939 above Zip1
( 23) 0.357/ 0.574 above Zip2
( 23) 1.078/ 2.674 above Zip3
EZip3: called 46 times, time in 0.3/6.265
( 22) 0.191/ 4.108 under Z
( 23) 0.109/ 2.110 under Boot
( 1) 0/ 0.047 under ProfileRoot
( 138) 0.610/ 2.153 above CF
( 46) 0.358/ 1.169 above Zip2
( 46) 0.921/ 2.643 above Zip3
Z: called 1 times, time in 0.079/15.985
( 1) 0.079/ 15.990 under ProfileRoot
( 5) 0.016/ 6.594 above Boot
( 84) 0.265/ 0.469 above CF
( 22) 0.191/ 4.108 above EZip3
( 22) 0.499/ 0.686 above Zip1
( 22) 0.300/ 1.050 above Zip2
( 22) 0.879/ 2.999 above Zip3
RVK: called 1 times, time in 0./0.
( 1) 0/ 0 under ProfileRoot

```

(Alt) In[]:= **Timing@Block**[{**\$k = 1**}, **Z[Knot[8, 17]]**]

$$\begin{aligned}
 (\text{Alt}) \text{Out}[*]= & \left\{ 52.9531, \mathbb{E}_{\{\} \rightarrow \{\emptyset\}} \left[\frac{1}{2} \times \left[-2 t \hbar - \text{Log} \left[\left(-1 - \frac{1}{T^4} + \frac{4}{T^3} - \frac{6}{T^2} + \frac{5}{T} \right)^2 \right] - \right. \right. \\
 & \text{Log} \left[\left(1 + \frac{T}{1 - 4 T + 6 T^2 - 5 T^3 + T^4} - \frac{2 T^2}{1 - 4 T + 6 T^2 - 5 T^3 + T^4} + \frac{T^3}{1 - 4 T + 6 T^2 - 5 T^3 + T^4} \right)^2 \right] - \\
 & \text{Log} \left[\left(1 - \frac{T}{1 - 3 T + 4 T^2 - 4 T^3 + T^4} + \frac{4 T^2}{1 - 3 T + 4 T^2 - 4 T^3 + T^4} - \frac{7 T^3}{1 - 3 T + 4 T^2 - 4 T^3 + T^4} + \right. \right. \\
 & \left. \left. \frac{7 T^4}{1 - 3 T + 4 T^2 - 4 T^3 + T^4} - \frac{4 T^5}{1 - 3 T + 4 T^2 - 4 T^3 + T^4} + \frac{T^6}{1 - 3 T + 4 T^2 - 4 T^3 + T^4} \right)^2 \right] \right], \\
 & \frac{-3 \hbar + 8 T \hbar - 8 T^2 \hbar + 8 T^4 \hbar - 8 T^5 \hbar + 3 T^6 \hbar}{1 - 4 T + 8 T^2 - 11 T^3 + 8 T^4 - 4 T^5 + T^6} + \frac{a (-6 \hbar + 16 T \hbar - 16 T^2 \hbar + 16 T^4 \hbar - 16 T^5 \hbar + 6 T^6 \hbar)}{1 - 4 T + 8 T^2 - 11 T^3 + 8 T^4 - 4 T^5 + T^6} \\
 & \left. \left. \times y \left(\frac{-6 \hbar^2 + 10 T \hbar^2 - 6 T^2 \hbar^2 - 6 T^3 \hbar^2 + 10 T^4 \hbar^2 - 6 T^5 \hbar^2}{1 - 4 T + 8 T^2 - 11 T^3 + 8 T^4 - 4 T^5 + T^6} \right) \right] \right\}
 \end{aligned}$$

(Alt) In[*]:= **PrintProfile[]**

(Alt) Out[*]= ProfileRoot is root. Profiled time: 69.313

- (2) 0.358/ 68.938 above Z
- (1) 0.219/ 0.219 above Boot
- (2) 0.015/ 0.015 above CF
- (1) 0/ 0.047 above EZip3
- (2) 0/ 0 above RVK
- (1) 0/ 0.016 above Zip1
- (1) 0/ 0.031 above Zip2
- (1) 0.016/ 0.047 above Zip3

CCF: called 21248 times, time in 28.875/28.875

- (21248) 28.875/ 28.875 under CF

CF: called 27375 times, time in 26.763/55.638

- (298) 1.436/ 3.190 under Z
- (88) 0.094/ 0.249 under Boot
- (321) 7.910/ 20.043 under EZip3
- (2) 0.015/ 0.015 under ProfileRoot
- (214) 0.422/ 1.191 under Zip1
- (642) 3.898/ 10.281 under Zip2
- (25810) 12.988/ 20.669 under Zip3
- (21248) 28.875/ 28.875 above CCF

Zip3: called 214 times, time in 6.973/27.642

- (79) 3.535/ 18.683 under Z
- (27) 1.204/ 2.956 under Boot
- (107) 2.218/ 5.956 under EZip3
- (1) 0.016/ 0.047 under ProfileRoot
- (25810) 12.988/ 20.669 above CF

Zip1: called 107 times, time in 2.465/3.656

- (79) 1.654/ 2.592 under Z
- (27) 0.811/ 1.048 under Boot
- (1) 0/ 0.016 under ProfileRoot
- (214) 0.422/ 1.191 above CF

Zip2: called 214 times, time in 2.342/12.623

- (79) 1.127/ 8.661 under Z
- (27) 0.403/ 0.683 under Boot

```
( 107) 0.812/ 3.248 under EZip3
( 1) 0/ 0.031 under ProfileRoot
( 642) 3.898/ 10.281 above CF
```

EZip3: called 107 times, time in 1.193/30.44

```
( 79) 1.084/ 28.001 under Z
( 27) 0.109/ 2.392 under Boot
( 1) 0/ 0.047 under ProfileRoot
( 321) 7.910/ 20.043 above CF
( 107) 0.812/ 3.248 above Zip2
( 107) 2.218/ 5.956 above Zip3
```

Z: called 2 times, time in 0.358/68.938

```
( 2) 0.358/ 68.938 under ProfileRoot
( 7) 0.031/ 7.453 above Boot
( 298) 1.436/ 3.190 above CF
( 79) 1.084/ 28.001 above EZip3
( 79) 1.654/ 2.592 above Zip1
( 79) 1.127/ 8.661 above Zip2
( 79) 3.535/ 18.683 above Zip3
```

Boot: called 28 times, time in 0.344/19.014

```
( 7) 0.031/ 7.453 under Z
( 20) 0.094/ 11.342 under Boot
( 1) 0.219/ 0.219 under ProfileRoot
( 20) 0.094/ 11.342 above Boot
( 88) 0.094/ 0.249 above CF
( 27) 0.109/ 2.392 above EZip3
( 27) 0.811/ 1.048 above Zip1
( 27) 0.403/ 0.683 above Zip2
( 27) 1.204/ 2.956 above Zip3
```

RVK: called 2 times, time in 0./0.

```
( 2) 0/ 0 under ProfileRoot
```

(Alt) In[]:= **Timing@Block**[{**\$k = 2**}, **Z[Knot**[3, 1]]]

$$\begin{aligned}
 & \left\{ 164.172, \mathbb{E}_{\{\} \rightarrow \{\}} \left[\frac{1}{2} \times \left(-4 t \hbar - \text{Log} \left[\left(\frac{1}{T^3} - \frac{2}{T^2} + \frac{2}{T} \right)^2 \right] - \text{Log} \left[\left(1 + \frac{T}{1-2T+2T^2} - \frac{T^2}{1-2T+2T^2} \right)^2 \right] \right) \right], \right. \\
 & \frac{a(-2\hbar + 2T^2\hbar)}{1-T+T^2} + \frac{-2\hbar + 3T\hbar - 2T^2\hbar + T^3\hbar}{1-2T+3T^2-2T^3+T^4} + \frac{xy(-2\hbar^2 - 2T\hbar^2)}{1-T+T^2}, \frac{a^2(2T\hbar^2 - 8T^2\hbar^2 + 2T^3\hbar^2)}{1-2T+3T^2-2T^3+T^4} + \\
 & \frac{a(2T\hbar^2 - 14T^2\hbar^2 + 12T^3\hbar^2 - 6T^4\hbar^2 + 2T^5\hbar^2)}{1-3T+6T^2-7T^3+6T^4-3T^5+T^6} + \frac{T\hbar^2 - 11T^2\hbar^2 + 16T^3\hbar^2 - 12T^4\hbar^2 + 8T^5\hbar^2 - 3T^6\hbar^2 + T^7\hbar^2}{2-8T+20T^2-32T^3+38T^4-32T^5+20T^6-8T^7+2T^8} + \\
 & \left. \frac{axy(8T\hbar^3 - 8T^2\hbar^3 - 4T^3\hbar^3)}{1-2T+3T^2-2T^3+T^4} + \frac{xy(-2\hbar^3 - 2T^2\hbar^3 - 6T^3\hbar^3 + 2T^5\hbar^3)}{1-3T+6T^2-7T^3+6T^4-3T^5+T^6} + \frac{x^2y^2(\hbar^4 + 5T\hbar^4 + T^2\hbar^4)}{1-2T+3T^2-2T^3+T^4} \right\}
 \end{aligned}$$

(Alt) In[]:= **PrintProfile** []

(Alt) Out[]:= ProfileRoot is root. Profiled time: 233.484

```
( 3) 0.578/ 233.110 above Z
( 1) 0.219/ 0.219 above Boot
( 2) 0.015/ 0.015 above CF
( 1) 0/ 0.047 above EZip3
( 3) 0/ 0 above RVK
( 1) 0/ 0.016 above Zip1
( 1) 0/ 0.031 above Zip2
```



```

( 1) 0.016/ 0.047 above Zip3
CCF: called 36901 times, time in 122.686/122.686
( 36901) 122.690/ 122.690 under CF
CF: called 41349 times, time in 83.328/206.014
( 424) 2.124/ 4.908 under Z
( 202) 0.202/ 0.685 under Boot
( 501) 25.033/ 64.418 under EZip3
( 2) 0.015/ 0.015 under ProfileRoot
( 304) 0.640/ 1.690 under Zip1
( 1002) 8.161/ 21.563 under Zip2
( 38914) 47.153/ 112.740 under Zip3
( 36901) 122.690/ 122.690 above CCF
Zip3: called 304 times, time in 15.672/128.407
( 101) 6.083/ 77.808 under Z
( 50) 2.547/ 6.208 under Boot
( 152) 7.026/ 44.344 under EZip3
( 1) 0.016/ 0.047 under ProfileRoot
( 38914) 47.153/ 112.740 above CF
Zip1: called 152 times, time in 3.922/5.612
( 101) 2.110/ 3.328 under Z
( 50) 1.812/ 2.268 under Boot
( 1) 0/ 0.016 under ProfileRoot
( 304) 0.640/ 1.690 above CF
Zip2: called 304 times, time in 3.654/25.217
( 101) 1.534/ 16.161 under Z
( 50) 0.716/ 1.448 under Boot
( 152) 1.404/ 7.577 under EZip3
( 1) 0/ 0.031 under ProfileRoot
( 1002) 8.161/ 21.563 above CF
EZip3: called 152 times, time in 3.177/119.516
( 101) 2.944/ 113.910 under Z
( 50) 0.233/ 5.564 under Boot
( 1) 0/ 0.047 under ProfileRoot
( 501) 25.033/ 64.418 above CF
( 152) 1.404/ 7.577 above Zip2
( 152) 7.026/ 44.344 above Zip3
Z: called 3 times, time in 0.578/233.109
( 3) 0.578/ 233.110 under ProfileRoot
( 12) 0.061/ 16.421 above Boot
( 424) 2.124/ 4.908 above CF
( 101) 2.944/ 113.910 above EZip3
( 101) 2.110/ 3.328 above Zip1
( 101) 1.534/ 16.161 above Zip2
( 101) 6.083/ 77.808 above Zip3
Boot: called 48 times, time in 0.467/41.996
( 12) 0.061/ 16.421 under Z
( 35) 0.187/ 25.356 under Boot
( 1) 0.219/ 0.219 under ProfileRoot
( 35) 0.187/ 25.356 above Boot
( 202) 0.202/ 0.685 above CF
( 50) 0.233/ 5.564 above EZip3
( 50) 1.812/ 2.268 above Zip1

```

(50) 0.716/ 1.448 above Zip2

(50) 2.547/ 6.208 above Zip3

RVK: called 3 times, time in 0./0.

(3) 0/ 0 under ProfileRoot

(Alt) In[]:= **Timing@Block**[{**\$k = 2**}, **Z[Knot**[8, 17]]]

$$\begin{aligned}
 \text{(Alt) Out[]} = & \left\{ 1716.23, \mathbb{E}_{\{\} \rightarrow \{\}} \left[\frac{1}{2} \times \left(-2 t \hbar - \text{Log} \left[\left(-1 - \frac{1}{T^4} + \frac{4}{T^3} - \frac{6}{T^2} + \frac{5}{T} \right)^2 \right] - \right. \right. \\
 & \text{Log} \left[\left(1 + \frac{T}{1 - 4 T + 6 T^2 - 5 T^3 + T^4} - \frac{2 T^2}{1 - 4 T + 6 T^2 - 5 T^3 + T^4} + \frac{T^3}{1 - 4 T + 6 T^2 - 5 T^3 + T^4} \right)^2 \right] - \\
 & \text{Log} \left[\left(1 - \frac{T}{1 - 3 T + 4 T^2 - 4 T^3 + T^4} + \frac{4 T^2}{1 - 3 T + 4 T^2 - 4 T^3 + T^4} - \frac{7 T^3}{1 - 3 T + 4 T^2 - 4 T^3 + T^4} + \right. \right. \\
 & \left. \left. \frac{7 T^4}{1 - 3 T + 4 T^2 - 4 T^3 + T^4} - \frac{4 T^5}{1 - 3 T + 4 T^2 - 4 T^3 + T^4} + \frac{T^6}{1 - 3 T + 4 T^2 - 4 T^3 + T^4} \right)^2 \right] \right], \\
 & -3 \hbar + 8 T \hbar - 8 T^2 \hbar + 8 T^4 \hbar - 8 T^5 \hbar + 3 T^6 \hbar \quad a \left(-6 \hbar + 16 T \hbar - 16 T^2 \hbar + 16 T^4 \hbar - 16 T^5 \hbar + 6 T^6 \hbar \right) \\
 & \frac{1 - 4 T + 8 T^2 - 11 T^3 + 8 T^4 - 4 T^5 + T^6}{1 - 4 T + 8 T^2 - 11 T^3 + 8 T^4 - 4 T^5 + T^6} + \frac{a \left(-6 \hbar + 16 T \hbar - 16 T^2 \hbar + 16 T^4 \hbar - 16 T^5 \hbar + 6 T^6 \hbar \right)}{1 - 4 T + 8 T^2 - 11 T^3 + 8 T^4 - 4 T^5 + T^6} \\
 & \frac{x y \left(-6 \hbar^2 + 10 T \hbar^2 - 6 T^2 \hbar^2 - 6 T^3 \hbar^2 + 10 T^4 \hbar^2 - 6 T^5 \hbar^2 \right)}{1 - 4 T + 8 T^2 - 11 T^3 + 8 T^4 - 4 T^5 + T^6}, \\
 & \left(a \left(8 T \hbar^2 - 64 T^2 \hbar^2 + 262 T^3 \hbar^2 - 608 T^4 \hbar^2 + 952 T^5 \hbar^2 - 1096 T^6 \hbar^2 + 952 T^7 \hbar^2 - 608 T^8 \hbar^2 + 262 T^9 \hbar^2 - 64 T^{10} \hbar^2 + \right. \right. \\
 & \left. \left. 8 T^{11} \hbar^2 \right) \right) / \left(1 - 8 T + 32 T^2 - 86 T^3 + 168 T^4 - 248 T^5 + 283 T^6 - 248 T^7 + 168 T^8 - 86 T^9 + 32 T^{10} - 8 T^{11} + T^{12} \right) + \\
 & \left(a^2 \left(8 T \hbar^2 - 64 T^2 \hbar^2 + 262 T^3 \hbar^2 - 608 T^4 \hbar^2 + 952 T^5 \hbar^2 - 1096 T^6 \hbar^2 + 952 T^7 \hbar^2 - 608 T^8 \hbar^2 + 262 T^9 \hbar^2 - 64 T^{10} \hbar^2 + \right. \right. \\
 & \left. \left. 8 T^{11} \hbar^2 \right) \right) / \left(1 - 8 T + 32 T^2 - 86 T^3 + 168 T^4 - 248 T^5 + 283 T^6 - 248 T^7 + 168 T^8 - 86 T^9 + 32 T^{10} - 8 T^{11} + T^{12} \right) + \\
 & \left(4 T \hbar^2 - 50 T^2 \hbar^2 + 307 T^3 \hbar^2 - 1160 T^4 \hbar^2 + 3062 T^5 \hbar^2 - 6127 T^6 \hbar^2 + 9760 T^7 \hbar^2 - 12754 T^8 \hbar^2 + 13916 T^9 \hbar^2 - \right. \\
 & \left. 12754 T^{10} \hbar^2 + 9760 T^{11} \hbar^2 - 6127 T^{12} \hbar^2 + 3062 T^{13} \hbar^2 - 1160 T^{14} \hbar^2 + 307 T^{15} \hbar^2 - 50 T^{16} \hbar^2 + 4 T^{17} \hbar^2 \right) / \\
 & \left(2 - 24 T + 144 T^2 - 578 T^3 + 1728 T^4 - 4056 T^5 + 7708 T^6 - 12072 T^7 + 15744 T^8 - 17194 T^9 + \right. \\
 & \left. 15744 T^{10} - 12072 T^{11} + 7708 T^{12} - 4056 T^{13} + 1728 T^{14} - 578 T^{15} + 144 T^{16} - 24 T^{17} + 2 T^{18} \right) + \\
 & \left(a x y \left(28 T \hbar^3 - 168 T^2 \hbar^3 + 544 T^3 \hbar^3 - 1000 T^4 \hbar^3 + 1248 T^5 \hbar^3 - 1096 T^6 \hbar^3 + \right. \right. \\
 & \left. \left. 656 T^7 \hbar^3 - 216 T^8 \hbar^3 - 20 T^9 \hbar^3 + 40 T^{10} \hbar^3 - 12 T^{11} \hbar^3 \right) \right) / \\
 & \left(1 - 8 T + 32 T^2 - 86 T^3 + 168 T^4 - 248 T^5 + 283 T^6 - 248 T^7 + 168 T^8 - 86 T^9 + 32 T^{10} - 8 T^{11} + T^{12} \right) + \\
 & \left(x y \left(-18 \hbar^3 + 78 T \hbar^3 - 146 T^2 \hbar^3 + 110 T^3 \hbar^3 + 78 T^4 \hbar^3 - 274 T^5 \hbar^3 + \right. \right. \\
 & \left. \left. 274 T^6 \hbar^3 - 78 T^7 \hbar^3 - 110 T^8 \hbar^3 + 146 T^9 \hbar^3 - 78 T^{10} \hbar^3 + 18 T^{11} \hbar^3 \right) \right) / \\
 & \left(1 - 8 T + 32 T^2 - 86 T^3 + 168 T^4 - 248 T^5 + 283 T^6 - 248 T^7 + 168 T^8 - 86 T^9 + 32 T^{10} - 8 T^{11} + T^{12} \right) + \\
 & \left(x^2 y^2 \left(3 \hbar^4 - 37 T^2 \hbar^4 + 153 T^3 \hbar^4 - 261 T^4 \hbar^4 + 325 T^5 \hbar^4 - 261 T^6 \hbar^4 + 153 T^7 \hbar^4 - 37 T^8 \hbar^4 + 3 T^{10} \hbar^4 \right) \right) / \\
 & \left. \left. \left(1 - 8 T + 32 T^2 - 86 T^3 + 168 T^4 - 248 T^5 + 283 T^6 - 248 T^7 + 168 T^8 - 86 T^9 + 32 T^{10} - 8 T^{11} + T^{12} \right) \right] \right\}
 \end{aligned}$$

(Alt) In[]:= **PrintProfile**[]

(Alt) Out[]:= ProfileRoot is root. Profiled time: 1949.72

(4) 1.439/ 1949.340 above Z

(1) 0.219/ 0.219 above Boot

(2) 0.015/ 0.015 above CF

(1) 0/ 0.047 above EZip3

(4) 0/ 0 above RVK

(1) 0/ 0.016 above Zip1

(1) 0/ 0.031 above Zip2

(1) 0.016/ 0.047 above Zip3

CCF: called 75053 times, time in 1015.35/1015.35

(75053) 1015.350/ 1015.350 under CF

CF: called 57038 times, time in 871.023/1886.37

(745) 10.701/ 23.216 under Z

(220) 0.233/ 0.780 under Boot

(745) 405.395/ 878.573 under EZip3

(2) 0.015/ 0.015 under ProfileRoot

(426) 0.842/ 2.267 under Zip1

(1490) 51.850/ 118.873 under Zip2

(53410) 401.987/ 862.646 under Zip3

(75053) 1015.350/ 1015.350 above CCF

Zip3: called 426 times, time in 36.966/899.612

(158) 18.219/ 763.903 under Z

(54) 2.689/ 6.536 under Boot

(213) 16.042/ 129.126 under EZip3

(1) 0.016/ 0.047 under ProfileRoot

(53410) 401.987/ 862.646 above CF

EZip3: called 213 times, time in 13.35/1037.16

(158) 13.102/ 1031.220 under Z

(54) 0.248/ 5.892 under Boot

(1) 0/ 0.047 under ProfileRoot

(745) 405.395/ 878.573 above CF

(213) 1.946/ 16.107 above Zip2

(213) 16.042/ 129.126 above Zip3

Zip2: called 426 times, time in 5.905/124.778

(158) 3.196/ 107.098 under Z

(54) 0.763/ 1.542 under Boot

(213) 1.946/ 16.107 under EZip3

(1) 0/ 0.031 under ProfileRoot

(1490) 51.850/ 118.873 above CF

Zip1: called 213 times, time in 5.221/7.488

(158) 3.348/ 5.096 under Z

(54) 1.873/ 2.376 under Boot

(1) 0/ 0.016 under ProfileRoot

(426) 0.842/ 2.267 above CF

Z: called 4 times, time in 1.439/1949.34

(4) 1.439/ 1949.340 under ProfileRoot

(14) 0.061/ 17.374 above Boot

(745) 10.701/ 23.216 above CF

(158) 13.102/ 1031.220 above EZip3

(158) 3.348/ 5.096 above Zip1

(158) 3.196/ 107.098 above Zip2

(158) 18.219/ 763.903 above Zip3

Boot: called 52 times, time in 0.467/43.512

(14) 0.061/ 17.374 under Z

(37) 0.187/ 25.919 under Boot

(1) 0.219/ 0.219 under ProfileRoot

(37) 0.187/ 25.919 above Boot

(220) 0.233/ 0.780 above CF

(54) 0.248/ 5.892 above EZip3

(54) 1.873/ 2.376 above Zip1

(54) 0.763/ 1.542 above Zip2

(54) 2.689/ 6.536 above Zip3

RVK: called 4 times, time in 0./0.

(4) 0/ 0 under ProfileRoot

(Alt) In[]:= **Timing@Block**[{**\$k = 3**}, **Z[Knot**[**3, 1**]]]

$$\begin{aligned}
 & \left\{ 5163.2, E_{\{\}} \rightarrow \{\emptyset\} \left[\frac{1}{2} \times \left(-4 t \hbar - \text{Log} \left[\left(\frac{1}{T^3} - \frac{2}{T^2} + \frac{2}{T} \right)^2 \right] - \text{Log} \left[\left(1 + \frac{T}{1 - 2T + 2T^2} - \frac{T^2}{1 - 2T + 2T^2} \right)^2 \right] \right) \right], \right. \\
 & \frac{a \left(-2 \hbar + 2 T^2 \hbar \right)}{1 - T + T^2} + \frac{-2 \hbar + 3 T \hbar - 2 T^2 \hbar + T^3 \hbar}{1 - 2 T + 3 T^2 - 2 T^3 + T^4} + \frac{x y \left(-2 \hbar^2 - 2 T \hbar^2 \right)}{1 - T + T^2}, \frac{a^2 \left(2 T \hbar^2 - 8 T^2 \hbar^2 + 2 T^3 \hbar^2 \right)}{1 - 2 T + 3 T^2 - 2 T^3 + T^4} + \\
 & \frac{a \left(2 T \hbar^2 - 14 T^2 \hbar^2 + 12 T^3 \hbar^2 - 6 T^4 \hbar^2 + 2 T^5 \hbar^2 \right)}{1 - 3 T + 6 T^2 - 7 T^3 + 6 T^4 - 3 T^5 + T^6} + \frac{T \hbar^2 - 11 T^2 \hbar^2 + 16 T^3 \hbar^2 - 12 T^4 \hbar^2 + 8 T^5 \hbar^2 - 3 T^6 \hbar^2 + T^7 \hbar^2}{2 - 8 T + 20 T^2 - 32 T^3 + 38 T^4 - 32 T^5 + 20 T^6 - 8 T^7 + 2 T^8} + \\
 & \frac{a x y \left(8 T \hbar^3 - 8 T^2 \hbar^3 - 4 T^3 \hbar^3 \right)}{1 - 2 T + 3 T^2 - 2 T^3 + T^4} + \frac{x y \left(-2 \hbar^3 - 2 T^2 \hbar^3 - 6 T^3 \hbar^3 + 2 T^5 \hbar^3 \right)}{1 - 3 T + 6 T^2 - 7 T^3 + 6 T^4 - 3 T^5 + T^6} + \frac{x^2 y^2 \left(\hbar^4 + 5 T \hbar^4 + T^2 \hbar^4 \right)}{1 - 2 T + 3 T^2 - 2 T^3 + T^4}, \\
 & \frac{a^3 \left(-4 T \hbar^3 + 28 T^2 \hbar^3 - 28 T^4 \hbar^3 + 4 T^5 \hbar^3 \right)}{3 - 9 T + 18 T^2 - 21 T^3 + 18 T^4 - 9 T^5 + 3 T^6} + \frac{a^2 \left(-2 T \hbar^3 + 24 T^2 \hbar^3 - 12 T^3 \hbar^3 - 32 T^4 \hbar^3 + 20 T^5 \hbar^3 - 8 T^6 \hbar^3 + 2 T^7 \hbar^3 \right)}{1 - 4 T + 10 T^2 - 16 T^3 + 19 T^4 - 16 T^5 + 10 T^6 - 4 T^7 + T^8} + \\
 & \frac{a \left(-T \hbar^3 + 19 T^2 \hbar^3 - 19 T^3 \hbar^3 - 34 T^4 \hbar^3 + 40 T^5 \hbar^3 - 22 T^6 \hbar^3 + 11 T^7 \hbar^3 - 3 T^8 \hbar^3 + T^9 \hbar^3 \right)}{1 - 5 T + 15 T^2 - 30 T^3 + 45 T^4 - 51 T^5 + 45 T^6 - 30 T^7 + 15 T^8 - 5 T^9 + T^{10}} + \\
 & \frac{\left(-T \hbar^3 + 29 T^2 \hbar^3 - 43 T^3 \hbar^3 - 71 T^4 \hbar^3 + 131 T^5 \hbar^3 - 84 T^6 \hbar^3 + 53 T^7 \hbar^3 - 23 T^8 \hbar^3 + 11 T^9 \hbar^3 - 3 T^{10} \hbar^3 + T^{11} \hbar^3 \right)}{\left(6 - 36 T + 126 T^2 - 300 T^3 + 540 T^4 - 756 T^5 + 846 T^6 - 756 T^7 + 540 T^8 - 300 T^9 + 126 T^{10} - 36 T^{11} + 6 T^{12} \right) +} \\
 & \frac{a^2 x y \left(-8 T \hbar^4 + 8 T^2 \hbar^4 + 36 T^3 \hbar^4 - 20 T^4 \hbar^4 - 4 T^5 \hbar^4 \right)}{1 - 3 T + 6 T^2 - 7 T^3 + 6 T^4 - 3 T^5 + T^6} + \\
 & \frac{a x y \left(12 T \hbar^4 - 16 T^2 \hbar^4 + 40 T^3 \hbar^4 - 16 T^4 \hbar^4 - 56 T^5 \hbar^4 + 8 T^6 \hbar^4 + 4 T^7 \hbar^4 \right)}{1 - 4 T + 10 T^2 - 16 T^3 + 19 T^4 - 16 T^5 + 10 T^6 - 4 T^7 + T^8} + \\
 & \frac{x y \left(-4 \hbar^4 + 3 T \hbar^4 - 6 T^2 \hbar^4 - 9 T^3 \hbar^4 - 15 T^4 \hbar^4 - 63 T^5 \hbar^4 - 9 T^6 \hbar^4 + 42 T^7 \hbar^4 + 3 T^8 \hbar^4 - 4 T^9 \hbar^4 \right)}{3 - 15 T + 45 T^2 - 90 T^3 + 135 T^4 - 153 T^5 + 135 T^6 - 90 T^7 + 45 T^8 - 15 T^9 + 3 T^{10}} + \\
 & \frac{a x^2 y^2 \left(-14 T \hbar^5 - 6 T^2 \hbar^5 + 30 T^3 \hbar^5 + 4 T^4 \hbar^5 \right)}{1 - 3 T + 6 T^2 - 7 T^3 + 6 T^4 - 3 T^5 + T^6} + \\
 & \frac{x^2 y^2 \left(2 \hbar^5 + 23 T \hbar^5 - 10 T^2 \hbar^5 + 11 T^3 \hbar^5 + 42 T^4 \hbar^5 - 29 T^5 \hbar^5 - 8 T^6 \hbar^5 \right)}{1 - 4 T + 10 T^2 - 16 T^3 + 19 T^4 - 16 T^5 + 10 T^6 - 4 T^7 + T^8} + \\
 & \left. \frac{x^3 y^3 \left(-2 \hbar^6 - 24 T \hbar^6 - 24 T^2 \hbar^6 - 2 T^3 \hbar^6 \right)}{3 - 9 T + 18 T^2 - 21 T^3 + 18 T^4 - 9 T^5 + 3 T^6} \right\}
 \end{aligned}$$

(Alt) In[]:= **PrintProfile**[[]]

(Alt) Out[]:= ProfileRoot is root. Profiled time: 7112.92

(5) 2.001/ 7112.546 above Z

(1) 0.219/ 0.219 above Boot

(2) 0.015/ 0.015 above CF

(1) 0/ 0.047 above EZip3

(5) 0/ 0 above RVK

(1) 0/ 0.016 above Zip1

(1) 0/ 0.031 above Zip2

(1) 0.016/ 0.047 above Zip3

CCF: called 119154 times, time in 4179.14/4179.14

(119154) 4179.141/ 4179.141 under CF

CF: called 71965 times, time in 2558.99/6738.14

(913) 13.217/ 28.779 under Z
 (372) 0.514/ 1.589 under Boot
 (970) 876.916/ 1771.275 under EZip3
 (2) 0.015/ 0.015 under ProfileRoot
 (516) 1.031/ 2.722 under Zip1
 (1940) 80.550/ 195.897 under Zip2
 (67252) 1586.751/ 4737.858 under Zip3
 (119154) 4179.141/ 4179.141 above CCF

Zip3: called 516 times, time in 249.015/4986.87

(180) 31.949/ 2177.216 under Z
 (77) 4.762/ 11.017 under Boot
 (258) 212.288/ 2798.593 under EZip3
 (1) 0.016/ 0.047 under ProfileRoot
 (67252) 1586.751/ 4737.858 above CF

EZip3: called 258 times, time in 108.725/4718.62

(180) 108.258/ 4707.606 under Z
 (77) 0.467/ 10.968 under Boot
 (1) 0/ 0.047 under ProfileRoot
 (970) 876.916/ 1771.275 above CF
 (258) 2.474/ 40.028 above Zip2
 (258) 212.288/ 2798.593 above Zip3

Zip2: called 516 times, time in 7.7/203.597

(180) 4.119/ 160.709 under Z
 (77) 1.107/ 2.829 under Boot
 (258) 2.474/ 40.028 under EZip3
 (1) 0/ 0.031 under ProfileRoot
 (1940) 80.550/ 195.897 above CF

Zip1: called 258 times, time in 6.61/9.332

(180) 3.941/ 5.940 under Z
 (77) 2.669/ 3.376 under Boot
 (1) 0/ 0.016 under ProfileRoot
 (516) 1.031/ 2.722 above CF

Z: called 5 times, time in 2.001/7112.55

(5) 2.001/ 7112.546 under ProfileRoot
 (19) 0.109/ 30.295 above Boot
 (913) 13.217/ 28.779 above CF
 (180) 108.258/ 4707.606 above EZip3
 (180) 3.941/ 5.940 above Zip1
 (180) 4.119/ 160.709 above Zip2
 (180) 31.949/ 2177.216 above Zip3

Boot: called 72 times, time in 0.735/75.604

(19) 0.109/ 30.295 under Z
 (52) 0.407/ 45.090 under Boot
 (1) 0.219/ 0.219 under ProfileRoot
 (52) 0.407/ 45.090 above Boot
 (372) 0.514/ 1.589 above CF
 (77) 0.467/ 10.968 above EZip3
 (77) 2.669/ 3.376 above Zip1
 (77) 1.107/ 2.829 above Zip2
 (77) 4.762/ 11.017 above Zip3

RVK: called 5 times, time in 0./0.

(5) 0/ 0 under ProfileRoot

(Alt) In[]:= **Timing@Block**[{**\$k = 3**}, **Z[Knot**[**8, 17**]]]

In[]:= **PrintProfile** []