April 8, 2024

Dror Bar-Natan
University of Toronto
Mathematics (St. George Campus)

Applicant Category: Established Researcher
Program: Discovery Grants Program
Evaluation Group: Mathematics and Statistics - 1508
Title: *Knot Theory as an Excuse*

We regret to inform you that the Natural Sciences and Engineering Research Council of Canada (NSERC) has not approved funding for the above-mentioned application.

Dr. Marc Fortin, ICD.D
Vice-President, Research Grants and Scholarships
Natural Sciences and Engineering Research Council of Canada

cc: Research Grants Officer, University of Toronto
Financial Officer, University of Toronto

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Message from the Evaluation Group

Name	Bar-Natan, Dror
Institution	University of Toronto
Application ID	RGPIN-2024-04086
Application title	Knot Theory as an Excuse
Type of grant	Discovery Grant
Evaluation group	1508 – Mathematics and Statistics

The evaluation group (EG) rated your application as follows, and in some cases has included additional feedback on certain criteria.

Excellence of the researcher	**Very Strong**

The evaluation group assessed your contributions to the natural sciences and engineering (NSE) based on a number of elements, including knowledge, expertise and experience, stature in the field as well as the quality, impact, and importance of the research accomplishments over the active research period evaluated.

Merit of the proposal	**Strong**

The evaluation group assessed a number of components such as (i) originality and innovation of the proposed research; (ii) significance and expected contributions to research; (iii) clarity and scope of objectives; (iv) clarity and appropriateness of methodology; (v) feasibility of the research plan; (vi) extent to which the scope of the proposal addresses all relevant issues; (vii) consideration of sex, gender and diversity in the research design, if applicable; and (viii) appropriateness of, and justification for, the budget.

While the short-term goals were clearly identified, and the underlying problems described, the committee did not find enough technical details about the methodology used to deal with these problems. It was also unclear how the long-term goal will be approached, and what the possible impact on the area will be.

Training of highly qualified personnel	**Moderate**

The evaluation group assessed a number of components such as (i) the quality and impact of past training of highly qualified personnel (HQP) over the active research period evaluated; (ii) the quality, suitability and clarity of the planned training of HQP; and (iii) the consideration of equity, diversity and inclusion in past and future training.

Details of how the training of HQP would be integrated into the research program were not sufficiently described. A more detailed explanation of the projects that HQP will be involved in, and the associated learning outcomes, would have been useful. Challenges or barriers experienced in the field of research or institutional environments involved in this program were not described.

Additional comments:

A message from NSERC

The following comments are intended to provide you with information about the context of the competition and the comments from the external reviews.

In this year's competition, the evaluation groups (EGs) evaluated over 3,100 Discovery Grant (DG) applications. These applications were assessed according to the guidelines found in NSERC's Peer Review Manual (PRM). EG members provide quality assessment on applications assigned to them. When the peer review process is complete, NSERC takes into consideration the merit of applications and the available budget. Due to budget limitations, not all meritorious applications were funded.

The 2024 DG competition meetings were held by videoconference again this year. NSERC continues to make every effort to maintain a peer review process of the highest quality, including fairness and consistency between EGs and across competition years.

Since the 2020 competition, the assessment of contributions to training HQP has included the requirement to consider equity, diversity and inclusion in the HQP training plan. The assessment of equity, diversity and inclusion focuses on the description of the existing challenges or barriers to inclusion and advancement of underrepresented groups in the NSE. In addition, the assessment includes the planned approach to promoting participation of a diverse group of HQP, taking into account equity and inclusion in recruitment practices, mentorship approaches and initiatives aimed at ensuring an inclusive research and training environment and trainee growth.

For each application, NSERC invites individuals with expertise to carry out an external review and submit a report. Those selected include a mix of individuals suggested by you and others selected by EG members. This selection process is described in section 3.1.5 of the PRM. While NSERC requests several reviews for each application, the number of reports received (normally 1–3) depends on the participation of these individuals. Instructions for the external review of applications can be found on NSERC's website.

NSERC includes the external review reports as part of your feedback package because they may contain information that will help in developing future research plans. These reports are considered your personal information as defined by section 3 of the *Privacy Act*. As such, NSERC shares them proactively, while protecting the names and identifiers of the individuals providing external reviews.

You might find inconsistencies between external review reports and the feedback and/or ratings of the EG. Please keep in mind that external review reports are only one of the elements that contribute to the EG's assessment. Individuals providing external reviews do not have the same comparative view as EG members who participate in a variety of calibration activities in preparation for the assessment process and review on average up to 40 applications per year.

External Reviewer Report - Application for a Grant
Rapport de l'évaluatrice ou de l'évaluateur externe - Demande de subvention

Name of applicant / Nom de la candidate ou du candidat:	**Application no. / N° de la demande:**
Bar-Natan, Dror	RGPIN-2024-04086
Type of grant / Type de subvention: RGPIN	

Confidence Level / Niveau de confiance

I rate my ability to assess this particular application as / J'estime que mon niveau de compétence pour évaluer la demande est :

High / Elevée:	Satisfactory / Satisfaisante:	Low / Faible:
X		

EVALUATE EACH ITEM BELOW. EXPLAIN YOUR ASSESSMENT AND WHAT LEADS YOU TO YOUR CONCLUSIONS. ONLY REFER TO INFORMATION THAT IS PROVIDED IN THE APPLICATION. / ÉVALUEZ CHACUN DES CRITÈRES CI-DESSOUS. JUSTIFIEZ VOTRE RÉPONSE ET INDIQUEZ LES RAISONS QUI ONT MENÉ AUX CONCLUSIONS QUE VOUS AVEZ TIRÉES. FIEZ-VOUS UNIQUEMENT À L'INFORMATION PRÉSENTÉE DANS LA DEMANDE.

Merit of the Proposal: The proposal must clearly present a **program** of research in the natural sciences and engineering. The proposed program of research must have long-term goals, rather than a single short-term project or collection of projects.

Mérite de la proposition : La proposition doit présenter clairement **un programme** de recherche en sciences naturelles ou en génie. Le programme proposé doit prévoir des objectifs de recherche à long terme plutôt qu'un seul projet à court terme ou une série de projets.

Describe both the strengths and weaknesses of the proposal related to: / Décrivez les points forts et les points faibles de la demande en ce qui concerne les aspects suivants :

The originality, innovation of the proposal, and the extent to which it suggests and explores novel concepts and lines of inquiry; / Originalité et caractère novateur de la proposition, et mesure dans laquelle elle propose des pistes de recherche et des concepts nouveaux

Il est proposé d'approfondir ici l'étude de certains invariants topologiques des noeuds, que le candidat a très récemment construits dans une série de travaux avec R. van der Veen (professeur associé à l'Université de Groningen). L'importance et l'originalité de ces invariants résident dans le fait que, tout en étant équivalents à certaines familles d'invariants quantiques (notamment les "polynômes de Jones coloriés" des noeuds), ils sont construits sous forme d'intégrales gaussiennes ce qui, à la différence des invariants quantiques "habituels", les rend facilement calculables en un temps polynomial. Les invariants de Bar Natan & van der Veen viennent en famille, indexée par un certain "epsilon". Pour "epsilon=0", on retrouve un invariant très

Translation #1 below

classique (découvert, il y a près d'un siècle, par le mathématicien américain J. W. Alexander) et qui est très bien compris désormais. Le candidat se propose de concentrer ses recherches sur le cran suivant ("epsilon=1") : quoique l'invariant correspondant (noté "rho_1" dans le document) jouisse d'une formule très simple (si simple que le candidat a même pu nous la présenter dans le document en une seule page !), il a un très fort pouvoir de distinction pour les noeuds. Cette approche "computationelle" des invariants quantiques est d'une très grande originalité, et il est donc important de la poursuivre. A ma connaissance, actuellement, seul le candidat et ses collaborateurs travaillent sur ces pistes qu'il vient d'ouvrir en collaboration avec R. van der Veen.

The significance of and expected contributions to research, and potential for impact; / Importance des travaux, contributions prévues à la recherche et l'incidence possible

Voici l'enjeu principal, tel que je le vois, pour ce projet de recherche : donner une interprétation topologique aux formules définissant ce nouvel invariant "rho_1" (avant d'envisager ensuite des valeurs plus élevées du paramètre "epsilon"). Comme le candidat l'explique fort bien dans son document, il s'agit à la fois de s'inspirer de ce qu'on sait faire pour le polynôme d'Alexander (cas "epsilon=0") tout en le transcendant (car les formules pour le cas "epsilon=1" sont très différentes de celles qu'on connait pour Alexander.) Si on parvient à donner à "rho_1" une telle interprétation topologique, alors on pourra envisager des applications topologiques qui viendront améliorer fortement celles du polynôme d'Alexander (e.g. donner des conditions nécessaires pour les noeuds fibrés ou les noeuds "slice", donner des bornes pour le genre des noeuds, etc.). Il serait formidable de découvrir ainsi de nouvelles applications des invariants quantiques à des problèmes classiques de la topologie de petite dimension.

Translation #2 below

The clarity and scope of the short and long-term objectives, methodology and feasibility. / Clarté et portée des objectifs à court et à long terme du programme de recherche, méthodologie et faisabilité

La finalité du programme de recherche est clairement formulée au fil du document, même si le candidat n'a pas "égrené" cette finalité en "objectifs" à "court", "moyen" ou "long" termes. J'estime que la faisabilité est élevée, eu égard à l'expérience et aux réussites passées du candidat en la matière. En effet, le candidat a déjà par le passé mené un travail similaire d'analyse très poussée pour d'autres invariants topologique des noeuds, qui (en un sens) étaient encore plus compliqués (tels que l'intégrale de Kontsevich ou l'homologie de Khovanov). La méthodologie proposée est elle-même originale pour le domaine concerné : en effet, fidèle à ses habitudes, le candidat entend procéder à une implémentation systématique de ses constructions mathématiques sous forme informatique (principalement des "packages" Mathematica). Cette approche computationnelle permettra au candidat, ses colloborateurs et nombreux lecteurs, de "tester" / "expérimenter" à volonté les résultats obtenus.

Translation #3 below

The integration of equity, diversity and inclusion considerations in the research process (e.g. the research questions, design of the study, methodology, analysis, interpretation, and dissemination of results), where relevant. Consult Equity, diversity and inclusion considerations at each stage of the research process for more information.

La prise en compte de l'équité, de la diversité et de l'inclusion dans le processus de recherche (p. ex., questions de recherche; conception de l'étude; méthodologie; analyse, interprétation et diffusion des résultats), s'il y a lieu. Pour plus de détails, consultez les considérations en matière d'équité, de diversité et d'inclusion à chaque étape du processus de recherche

Le programme de recherche ne me semble soulever aucune question d'équité, de diversité ou d'inclusion. A noter que le programme prévoit plusieurs contrats doctoraux et stages de licence ou master (ainsi qu'un contrat post-doctoral), permettant ainsi d'impliquer de nombreux étudiants dans ce processus de recherche. J'imagine donc que c'est la réglementation de l'Université de Toronto qui s'appliquera pour ces

Translation #4 below

recrutements.

All Translations by Google Translate.

#1: It is proposed to deepen here the study of certain topological invariants of the nodes, which the candidate has very recently constructed in a series of works with R. van der Veen (associate professor at the University of Groningen). The importance and originality of these invariants lie in the fact that, while being equivalent to certain families of quantum invariants (in particular the "colored Jones polynomials" of the knots), they are constructed in the form of Gaussian integrals which, unlike quantum invariants "usual", makes them easily calculable in polynomial time. The invariants of Bar Natan & van der Veen come in family, indexed by a certain "epsilon". For "epsilon=0", we find a very classic invariant (discovered almost a century ago by the American mathematician J. W. Alexander) and which is very well understood now. The candidate proposes to concentrate his research on the following notch ("epsilon=1"): although the corresponding invariant (noted "rho_1" in the document) has a very simple (so simple that the candidate was even able to present it to us in the document on a single page!), he a very strong power of distinction for the nodes. This “computational” approach to invariants quantum research is very original, and it is therefore important to pursue it. To my knowledge, currently, only the candidate and his collaborators are working on these avenues that he has just opened in collaboration with R. van der Veen.

#2: Here is the main issue, as I see it, for this research project: providing a topological interpretation to the formulas defining this new invariant "rho_1" (before then considering higher values of the parameter "epsilon"). As the candidate explains very well in his document, it is a question of both draw inspiration from what we know how to do for the Alexander polynomial (case "epsilon=0") while transcending it (because the formulas for the case "epsilon=1" are very different from those we know for Alexander.) If we manages to give "rho_1" such a topological interpretation, then we can consider applications topologies which will greatly improve those of the Alexander polynomial (e.g. giving conditions necessary for fiber nodes or "slice" nodes, give limits for the type of nodes, etc.). It would be great to discover new applications of quantum invariants to classic problems of small-dimensional topology.

#3: The purpose of the research program is clearly formulated throughout the document, even if the candidate has not not “divided” this purpose into “short”, “medium” or “long” term “objectives”. I believe that the feasibility is high, taking into account the candidate's experience and past success in this area. In fact, the candidate has already in the past carried out similar very in-depth analysis work for other topological invariants of knots, which (in a sense) were even more complicated (such as the Kontsevich integral or the homology by Khovanov). The proposed methodology is itself original for the field concerned: in fact, faithful to his habits, the candidate intends to proceed with a systematic implementation of his mathematical constructions in computer form (mainly Mathematica “packages”). This computational approach will allow the candidate, his collaborators and numerous readers, to "test" / “experiment” at will with the results obtained.

#4: The research program does not seem to me to raise any issues of equity, diversity or inclusion. Note that the program provides for several doctoral contracts and bachelor's or master's internships (as well as a post-doctoral contract), thus making it possible to involve numerous students in this process of research. So I imagine that the University of Toronto regulations will apply to these recruitments.