

DETAILED PROPOSAL: EXPANSIONS, LIE ALGEBRAS, AND INVARIANTS.

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ORGANIZING COMMITTEE AND SCIENTIFIC COMMITTEE

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EXECUTIVE SUMMARY

We hope to bring together a number of experts working on “expansions” and a number of experts working on “invariants” in the hope that the two groups will learn from each other and influence each other. “Expansions” are solutions of a certain type of intricate equations within graded spaces often associated with free Lie algebras; they include Drinfel’d associators, solutions of the Kashiwara-Vergne equations, solutions of various deformation quantization problems, and more. By “invariants” we refer to quantum-algebra-inspired invariants of various objects within low dimensional topology; these are often associated with various semi-simple Lie algebras. The two subjects were born together in the early days of quantum group theory, but have to a large extent evolved separately. **We believe there is much to gain by bringing the two together again.**

SCIENTIFIC DESCRIPTION

A bit more about the topic.

The 1st-year-calculus “Taylor Expansion” is a structure-preserving map which carries the algebra of smooth functions to its “associated graded” algebra, the algebra of power series. Yet expansions are vastly more general than that: they make sense whenever an algebraic structure carries a “filtration”, and questions around their existence and uniqueness are often deep and interesting. This is especially true for structures that arise in low-dimensional topology: sets of knots and manifolds, groups of braids, categories of tangles, circuit algebras of virtually knotted objects, and the like.

The set of (formal linear combinations of) knots, for example, can be filtered using the “Vassiliev filtration”, which is generated by iterated differences between overcrossings and undercrossings. An “expansion” in this case amounts to a “universal finite type invariant”, and there are at least

three constructions of such expansions, with each one revealing a connection between knot theory and another deep part of mathematics: The “Kontsevich Integral” construction is closely related to the Drinfel’d-Kohno theorem connecting monodromies of certain differential equations and quantum groups. The “configuration space integrals” construction highlights the relationship between knot theory and the Chern-Simons-Witten topological quantum field theory, and the “associators” construction relates knot theory with associators, free Lie algebras, multi-zeta values, and the Grothendieck-Teichmüller group.

In a similar manner “expansions” for certain 2-dimensional knotted objects in 4-space are related to the BF quantum field theory and via the work of Alekseev and Torossian and of Bar-Natan and Dancso to the Kashiwara-Vergne problem of Lie theory. Likewise, expansions for various classes of “virtual knots” are related to quantization of algebras with a classical Yang-Baxter element and to deformation quantization of Lie bialgebras. There are plenty of further examples.

Typically, the target spaces of expansions are spaces of “chord diagrams” and related spaces of “linear combinations of diagrams modulo local relations”. These spaces can often be interpreted as spaces of “universal formulas within Lie algebras of a certain class”, and thus the existence of expansions predicts the existence of topological invariants associated with specific Lie algebras (and representations thereof).

Yet it is often very hard to work with the universal formulas provided by expansions and to specialize them to the case of specific Lie algebras and representations. Practical constructions of pairings between topological objects and finite dimensional Lie objects usually involve the theory of quantum groups, which carries a completely different flavor. Perhaps the first thing that comes to mind when thinking about quantum groups is “semi-simple”, while the first thing that comes to mind when thinking about expansions is “free”.

Hence there is a large body of results about “invariants”, usually arising from quantum algebra. And there is a large body of results about “expansions”, often centered around associators and free Lie algebras. These two subjects are clearly deeply related, yet they interact less than they should.

An example for a fruitful interaction is the study of “Rozansky polynomials”, or “higher Melvin-Morton polynomials”. From the “quantum group” perspective these are certain “sections” of the coloured Jones polynomial that Rozansky studied some 20 years ago (and his student Overbay tabulated about 5 years ago). From the “expansions” perspective they can be viewed as the “two-loop” Kontsevich integral, or as a “limited co-bracket” quotient of an expansion for virtual knots. The former perspective led to the original definition and the first computations. The latter perspective leads to the Rozansky’s realization that these invariants ought to have topological interpretations as analogues of the Casson invariant for knot complements, and to the realization (work in progress by B-N and VDV) that these invariants can be computed in polynomial time using non-semi-simple techniques (and hence they are the first poly-time computable knot polynomials since the Alexander polynomial of 1928!) and that they can be extended to tangles

(hence opening the field to several potential topological applications, such as genus and ribbon detection).

We expect the interplay between “expansions” and “invariants” to go much deeper.

A bit more about our plans.

While we are guided by the “ideological statements” above, the reality of mathematical research is that excellent researchers tend to follow their own instincts, not ours. The most we can do is to invite the best researchers whose work fits within our agenda and hope to create an atmosphere conducive to the pursuit of that agenda. We believe this will be best achieved by inviting a relatively small number of people to share their knowledge in depth, rather than a larger number who would each be giving a 1-hour conference-style talk.

Hence we propose to invite 8-10 experts, listed below, who would each be encouraged to invite a partner for a “research in pairs”-style stay of around two weeks. We will expect each such expert or pair of experts to mostly concentrate on their own research yet to also give a short “mini-course” of 2-3 two-hour-long evening lectures or “Russian seminars” to seed interaction with us (the organizers) and the other groups. This will form the core of our activity.

We hope to also attract about an equal number of junior participants. We expect that some of these will be the graduate and post-doctoral students of our experts and some may contact us as this activity is advertised. Some 1-hour lectures will be given by the junior participants.

We plan to tape all of our formal activities (mini-courses and lectures) and make these videos available on the web in a timely manner, along with lecture notes, handouts, programs and every other material that will be made available by our speakers. Our experience is that such diligent record-keeping enables a “multiplier effect” as excellent lectures are viewed again and again by both the people who were present and by those who were not able to make it.

PUBLICATIONS

As indicated, we plan to diligently keep a record of our activity on the web; we consider it an integral part of the activity itself, and we will encourage our participants to contribute to this record. If enough material of the right form will accumulate we will be delighted to see it published as a CRM publication.

PARTICIPANTS

We will choose our core 8-10 invitees mostly from within the people in the list below. We have already reached out to the people marked with a (*) and confirmed their tentative interest.

(named removed)

LOCAL EXPERTISE

There is relatively little activity in Montreal in the subjects of our proposed workshop. Yet we can expect some interest from the following people, and perhaps more: *(names removed)*.

TIMELINE, RELATION WITH OTHER ACTIVITIES

We hope to hold our activity in the summer of 2019, sometime between May and August. The activity itself is “uniform” and does not lend itself to an internal subdivision.

PROJECTED BUDGET, FUNDING SOURCES

At about \$150 per day for accommodations for an average of 14 days, plus a little bit extra for travel assistance, we expect that the cost per senior participant will be around \$2,500. Assuming a budget of \$40K, this will allow us to invite about 8 “pairs”.

This is not enough. We seek your (CRM) advice on how to obtain additional funds that will allow us to support younger participants, the organizers, and to cover incidental expenses.