

Pensieve header: Testing Zip 1,2,3 of BabyDoPeGDO. Continues pensieve://2020-03/.

## Startup

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\BabyDoPeGDO"];
<< Engine.m
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

## Utilities

```
(Alt) In[ ]:= HL[ε_] := Style[ε, Background → If[TrueQ@ε, Green, Red]];
```

Generic Perturbations:

```
(Alt) In[ ]:= GenericPerturbation[d_Integer, vars_List, gc_] := Total[Map[
  gcSequence@@# Times @@ (vars^#) &,
  Join @@ (Permutations /@ IntegerPartitions[d + Length@vars, {Length@vars}]) - 1
]];
GenericPerturbation[{m_}, vars_List, gc_] :=
  Prepend[0] [εSeries @@ Table[GenericPerturbation[2 d + 2, vars, gc], {d, m}]]
```

```
In[ ]:= GenericPerturbation[2, {x, y}, c]
```

```
Out[ ]:= y2 c0,2 + x y c1,1 + x2 c2,0
```

```
In[ ]:= GenericPerturbation[{2}, {x, y}, c]
```

```
Out[ ]:= εSeries[0, y4 c0,4 + x y3 c1,3 + x2 y2 c2,2 + x3 y c3,1 + x4 c4,0,
  y6 c0,6 + x y5 c1,5 + x2 y4 c2,4 + x3 y3 c3,3 + x4 y2 c4,2 + x5 y c5,1 + x6 c6,0]
```

## Preliminary Definitions

```
(Alt) In[ ]:= Unprotect[SeriesData];
Expand[sd_SeriesData] ^= MapAt[Expand, sd, 3];
Protect[SeriesData];
```

Act and Contract:

```
(Alt) In[ ]:= Evk@{F_, E[ω_, Q_, P_] | B_} := Module[{ε, d},
  ε = ω eQ+Sum[P[[d+1]] ed, {d, 0, Length[P]-1}];
  Expand[Total[
    CoefficientRules[Normal@Series[eB·F·B*/2, {ħ, 0, k}], B*] /.
    (ps_ → c_) ⇒ c D[ε, Sequence@@Thread[{B, ps}]
  ] + 0[ħ]k+1
];
Evk@<F_, ε_>B_ := Evk@{F, ε | B /. Alternatives@@B → 0;
Evk[B_, <F_, ε_>] := Evk@<Table[∂u,v F, {u, B*}, {v, B*}], ε>B;
```

$$\text{In[ ]:= } \left\{ \text{Ev}_2 @ \left[ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \mathbb{E}[1, xy, \text{eSeries}[0]] \right]_{\{x,y\}}, \text{Ev}_3 @ \left( \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \mathbb{E}[1, 3xy, \text{eSeries}[0]] \right)_{\{x,y\}} \right\}$$

$$\text{Out[ ]:= } \left\{ e^{xy} + (e^{xy} + e^{xy} xy) \hbar + \left( e^{xy} + 2 e^{xy} xy + \frac{1}{2} e^{xy} x^2 y^2 \right) \hbar^2 + 0[\hbar]^3, 1 + 3\hbar + 9\hbar^2 + 27\hbar^3 + 0[\hbar]^4 \right\}$$

## Testing Zip1

```
(Alt) In[ ]:= Zip1[<F_, ε_>B_] := Zip1B[<B*·F·B* / 2, ε>]
```

```
In[ ]:= {p = 3, B = {x}, F = ħ {{f}}; G = {{g}};
  ε = GenericPerturbation[{4}, B, c]; Z = <F, E[1, B·G·B / 2, ε]>B
  lhs = Evp@Z
  rhs = Evp[B, Zip1@Z];
  HL[lhs == rhs]
```

$$\text{Out[ ]:= } \left\{ 3, \{x\}, \left\{ \left\{ \{f \hbar\} \right\}, \mathbb{E} \left[ 1, \frac{g x^2}{2}, \text{eSeries}[0, x^4 c_4, x^6 c_6, x^8 c_8, x^{10} c_{10}] \right] \right\} \right\}_{\{x\}}$$

$$\text{Out[ ]:= } 1 + \frac{f g \hbar}{2} + \left( \frac{3 f^2 g^2}{8} + 3 f^2 \in c_4 \right) \hbar^2 + \left( \frac{5 f^3 g^3}{16} + \frac{15}{2} f^3 g \in c_4 + 15 f^3 \in^2 c_6 \right) \hbar^3 + 0[\hbar]^4$$

Out[ ]:= **True**

```
In[*]:= {p = 2, B = {x, y}, F = ħ ( f11 f12 ; f12 f22 ); G = ( g11 g12 ; g12 g22 ); ε = GenericPerturbation[{2}, B, c];
Z = <F, E[1, B.G.B / 2, ε]>B}
Timing[lhs = Evp@Z]
rhs = Evp[B, Zip1@Z];
HL[lhs == rhs]
```

```
Out[*]:= {2, {x, y}, { {ħ f11, ħ f12}, {ħ f12, ħ f22} }, E[1, 1/2 (x (x g11 + y g12) + y (x g12 + y g22)),
Series[0, y^4 c0,4 + x y^3 c1,3 + x^2 y^2 c2,2 + x^3 y c3,1 + x^4 c4,0,
y^6 c0,6 + x y^5 c1,5 + x^2 y^4 c2,4 + x^3 y^3 c3,3 + x^4 y^2 c4,2 + x^5 y c5,1 + x^6 c6,0]]}_{x,y}}
```

```
Out[*]:= {109.25,
1 + ( f11 g11 / 2 + f12 g12 + f22 g22 / 2 ) ħ + ( 3/8 f11^2 g11^2 + 3/2 f11 f12 g11 g12 + f12^2 g12^2 + 1/2 f11 f22 g12^2 + 1/2 f12^2 g11 g22 +
1/4 f11 f22 g11 g22 + 3/2 f12 f22 g12 g22 + 3/8 f22^2 g22^2 + 3 ε f22^2 c0,4 + 3 ε f12 f22 c1,3 +
2 ε f12^2 c2,2 + ε f11 f22 c2,2 + 3 ε f11 f12 c3,1 + 3 ε f11^2 c4,0 ) ħ^2 + O[ħ^3]}
```

```
Out[*]:= True
```

## Testing Zip2

```
(Alt) In[*]:= Zip2[<F_, ε_>B_] := Zip2B[<B*.F.B* / 2, ε>]
```

```
In[*]:= {n = 2, p = 2, B = Table[z_i, {i, n}],
Y = Table[y_i, {i, n}], F = ħ Table[f_{i0,1}.Sort[{i,j}], {i, n}, {j, n}],
ε = GenericPerturbation[{3}, B, c], Z = <F, E[1, Y.B, ε]>B}
```

```
Out[*]:= {2, 2, {z1, z2}, {y1, y2}, { {ħ f11, ħ f12}, {ħ f12, ħ f22} },
Series[0, z2^4 c0,4 + z1 z2^3 c1,3 + z1^2 z2^2 c2,2 + z1^3 z2 c3,1 + z1^4 c4,0,
z2^6 c0,6 + z1 z2^5 c1,5 + z1^2 z2^4 c2,4 + z1^3 z2^3 c3,3 + z1^4 z2^2 c4,2 + z1^5 z2 c5,1 + z1^6 c6,0,
z2^8 c0,8 + z1 z2^7 c1,7 + z1^2 z2^6 c2,6 + z1^3 z2^5 c3,5 + z1^4 z2^4 c4,4 + z1^5 z2^3 c5,3 + z1^6 z2^2 c6,2 + z1^7 z2 c7,1 + z1^8 c8,0],
{ {ħ f11, ħ f12}, {ħ f12, ħ f22} },
E[1, y1 z1 + y2 z2, Series[0, z2^4 c0,4 + z1 z2^3 c1,3 + z1^2 z2^2 c2,2 + z1^3 z2 c3,1 + z1^4 c4,0,
z2^6 c0,6 + z1 z2^5 c1,5 + z1^2 z2^4 c2,4 + z1^3 z2^3 c3,3 + z1^4 z2^2 c4,2 + z1^5 z2 c5,1 + z1^6 c6,0, z2^8 c0,8 + z1 z2^7 c1,7 +
z1^2 z2^6 c2,6 + z1^3 z2^5 c3,5 + z1^4 z2^4 c4,4 + z1^5 z2^3 c5,3 + z1^6 z2^2 c6,2 + z1^7 z2 c7,1 + z1^8 c8,0]]}_{z1,z2}}
```

```
In[ ]:= Timing[lhs = Evp@Z]
rhs = Evp[B, Zip2@Z];
HL[lhs == rhs]
```

$$\text{Out[ ]} = \left\{ 107.359, 1 + \left( \frac{1}{2} f_{11} y_1^2 + f_{12} y_1 y_2 + \frac{1}{2} f_{22} y_2^2 \right) \hbar + \right. \\ \left. \left( \frac{1}{8} f_{11}^2 y_1^4 + \frac{1}{2} f_{11} f_{12} y_1^3 y_2 + \frac{1}{2} f_{12}^2 y_1^2 y_2^2 + \frac{1}{4} f_{11} f_{22} y_1^2 y_2^2 + \frac{1}{2} f_{12} f_{22} y_1 y_2^3 + \frac{1}{8} f_{22}^2 y_2^4 + 3 \in f_{22}^2 c_{0,4} + \right. \right. \\ \left. \left. 3 \in f_{12} f_{22} c_{1,3} + 2 \in f_{12}^2 c_{2,2} + \in f_{11} f_{22} c_{2,2} + 3 \in f_{11} f_{12} c_{3,1} + 3 \in f_{11}^2 c_{4,0} \right) \hbar^2 + O[\hbar]^3 \right\}$$

```
Out[ ]:= True
```

## Testing Zip3

```
(Alt) In[ ]:= Zip3[⟨F_, ε_⟩_B] := Zip3_B[⟨B*.F.B*/2, ε⟩]
```

```
(Alt) In[ ]:= {n = 2, p = 2, $k = 2, B = Table[x_i, {i, n}], F = ħ Table[f_{10,1}.Sort[{i,j}], {i, n}, {j, n}],
P = GenericPerturbation[{$k}, B, c], Z = ⟨F, E[1, 0, P]⟩_B}
```

```
(Alt) Out[ ]:= {2, 2, 2, {x_1, x_2}, {{ħ f_{11}, ħ f_{12}}, {ħ f_{12}, ħ f_{22}}},
∈Series[0, x_2^4 c_{0,4} + x_1 x_2^3 c_{1,3} + x_1^2 x_2^2 c_{2,2} + x_1^3 x_2 c_{3,1} + x_1^4 c_{4,0},
x_2^6 c_{0,6} + x_1 x_2^5 c_{1,5} + x_1^2 x_2^4 c_{2,4} + x_1^3 x_2^3 c_{3,3} + x_1^4 x_2^2 c_{4,2} + x_1^5 x_2 c_{5,1} + x_1^6 c_{6,0}],
⟨{{ħ f_{11}, ħ f_{12}}, {ħ f_{12}, ħ f_{22}}}, E[1, 0, ∈Series[0, x_2^4 c_{0,4} + x_1 x_2^3 c_{1,3} + x_1^2 x_2^2 c_{2,2} + x_1^3 x_2 c_{3,1} +
x_1^4 c_{4,0}, x_2^6 c_{0,6} + x_1 x_2^5 c_{1,5} + x_1^2 x_2^4 c_{2,4} + x_1^3 x_2^3 c_{3,3} + x_1^4 x_2^2 c_{4,2} + x_1^5 x_2 c_{5,1} + x_1^6 c_{6,0}]]⟩_{x_1, x_2}
```

(Alt) In[ ]:= **ZZ = Zip3[Z][[3]]**

**lhs = Normal[Normal[Ev<sub>p</sub>@Z] + O[ε]<sup>k+1</sup>]**

**rhs = Normal[Series[e<sup>Sum[ZZ[[d+1]] ε<sup>d</sup>, {d,0,Length[ZZ]-1}</sup>], {ħ, 0, p}, {ε, 0, \$k}]]]**

**HL@Simplify[lhs == rhs]**

(Alt) Out[ ]:= Series[0, 3 ħ<sup>2</sup> f<sub>22</sub><sup>2</sup> c<sub>0,4</sub> + 3 ħ<sup>2</sup> f<sub>12</sub> f<sub>22</sub> c<sub>1,3</sub> + 2 ħ<sup>2</sup> f<sub>12</sub><sup>2</sup> c<sub>2,2</sub> + ħ<sup>2</sup> f<sub>11</sub> f<sub>22</sub> c<sub>2,2</sub> + 3 ħ<sup>2</sup> f<sub>11</sub> f<sub>12</sub> c<sub>3,1</sub> + 3 ħ<sup>2</sup> f<sub>11</sub><sup>2</sup> c<sub>4,0</sub>,  
 $\frac{1}{2}$  (96 ħ<sup>4</sup> f<sub>22</sub><sup>4</sup> c<sub>0,4</sub><sup>2</sup> + 30 ħ<sup>3</sup> f<sub>22</sub><sup>3</sup> c<sub>0,6</sub> + 192 ħ<sup>4</sup> f<sub>12</sub> f<sub>22</sub><sup>3</sup> c<sub>0,4</sub> c<sub>1,3</sub> + 81 ħ<sup>4</sup> f<sub>12</sub><sup>2</sup> f<sub>22</sub><sup>2</sup> c<sub>1,3</sub><sup>2</sup> + 15 ħ<sup>4</sup> f<sub>11</sub> f<sub>22</sub><sup>3</sup> c<sub>1,3</sub><sup>2</sup> +

30 ħ<sup>3</sup> f<sub>12</sub> f<sub>22</sub><sup>2</sup> c<sub>1,5</sub> + 168 ħ<sup>4</sup> f<sub>12</sub><sup>2</sup> f<sub>22</sub><sup>2</sup> c<sub>0,4</sub> c<sub>2,2</sub> + 24 ħ<sup>4</sup> f<sub>11</sub> f<sub>22</sub><sup>3</sup> c<sub>0,4</sub> c<sub>2,2</sub> + 108 ħ<sup>4</sup> f<sub>12</sub><sup>3</sup> f<sub>22</sub> c<sub>1,3</sub> c<sub>2,2</sub> +  
 84 ħ<sup>4</sup> f<sub>11</sub> f<sub>12</sub> f<sub>22</sub><sup>2</sup> c<sub>1,3</sub> c<sub>2,2</sub> + 20 ħ<sup>4</sup> f<sub>12</sub><sup>4</sup> c<sub>2,2</sub><sup>2</sup> + 68 ħ<sup>4</sup> f<sub>11</sub> f<sub>12</sub><sup>2</sup> f<sub>22</sub><sup>2</sup> c<sub>2,2</sub><sup>2</sup> + 8 ħ<sup>4</sup> f<sub>11</sub><sup>2</sup> f<sub>22</sub><sup>2</sup> c<sub>2,2</sub><sup>2</sup> +  
 24 ħ<sup>3</sup> f<sub>12</sub><sup>2</sup> f<sub>22</sub> c<sub>2,4</sub> + 6 ħ<sup>3</sup> f<sub>11</sub> f<sub>22</sub><sup>2</sup> c<sub>2,4</sub> + 120 ħ<sup>4</sup> f<sub>12</sub><sup>3</sup> f<sub>22</sub> c<sub>0,4</sub> c<sub>3,1</sub> + 72 ħ<sup>4</sup> f<sub>11</sub> f<sub>12</sub> f<sub>22</sub><sup>2</sup> c<sub>0,4</sub> c<sub>3,1</sub> +  
 48 ħ<sup>4</sup> f<sub>12</sub><sup>4</sup> c<sub>1,3</sub> c<sub>3,1</sub> + 126 ħ<sup>4</sup> f<sub>11</sub> f<sub>12</sub><sup>2</sup> f<sub>22</sub> c<sub>1,3</sub> c<sub>3,1</sub> + 18 ħ<sup>4</sup> f<sub>11</sub><sup>2</sup> f<sub>22</sub><sup>2</sup> c<sub>1,3</sub> c<sub>3,1</sub> + 108 ħ<sup>4</sup> f<sub>11</sub> f<sub>12</sub><sup>3</sup> c<sub>2,2</sub> c<sub>3,1</sub> +  
 84 ħ<sup>4</sup> f<sub>11</sub><sup>2</sup> f<sub>12</sub> f<sub>22</sub> c<sub>2,2</sub> c<sub>3,1</sub> + 81 ħ<sup>4</sup> f<sub>11</sub><sup>2</sup> f<sub>12</sub><sup>2</sup> c<sub>3,1</sub><sup>2</sup> + 15 ħ<sup>4</sup> f<sub>11</sub><sup>3</sup> f<sub>22</sub> c<sub>3,1</sub><sup>2</sup> + 12 ħ<sup>3</sup> f<sub>12</sub><sup>3</sup> c<sub>3,3</sub> +  
 18 ħ<sup>3</sup> f<sub>11</sub> f<sub>12</sub> f<sub>22</sub> c<sub>3,3</sub> + 48 ħ<sup>4</sup> f<sub>12</sub><sup>4</sup> c<sub>0,4</sub> c<sub>4,0</sub> + 144 ħ<sup>4</sup> f<sub>11</sub> f<sub>12</sub><sup>2</sup> f<sub>22</sub> c<sub>0,4</sub> c<sub>4,0</sub> + 120 ħ<sup>4</sup> f<sub>11</sub> f<sub>12</sub><sup>3</sup> c<sub>1,3</sub> c<sub>4,0</sub> +  
 72 ħ<sup>4</sup> f<sub>11</sub><sup>2</sup> f<sub>12</sub> f<sub>22</sub> c<sub>1,3</sub> c<sub>4,0</sub> + 168 ħ<sup>4</sup> f<sub>11</sub><sup>2</sup> f<sub>12</sub><sup>2</sup> c<sub>2,2</sub> c<sub>4,0</sub> + 24 ħ<sup>4</sup> f<sub>11</sub><sup>3</sup> f<sub>22</sub> c<sub>2,2</sub> c<sub>4,0</sub> + 192 ħ<sup>4</sup> f<sub>11</sub><sup>3</sup> f<sub>12</sub> c<sub>3,1</sub> c<sub>4,0</sub> +  
 96 ħ<sup>4</sup> f<sub>11</sub><sup>4</sup> c<sub>4,0</sub><sup>2</sup> + 24 ħ<sup>3</sup> f<sub>11</sub> f<sub>12</sub><sup>2</sup> c<sub>4,2</sub> + 6 ħ<sup>3</sup> f<sub>11</sub><sup>2</sup> f<sub>22</sub> c<sub>4,2</sub> + 30 ħ<sup>3</sup> f<sub>11</sub><sup>2</sup> f<sub>12</sub> c<sub>5,1</sub> + 30 ħ<sup>3</sup> f<sub>11</sub><sup>3</sup> c<sub>6,0</sub>) ]]

(Alt) Out[ ]:= 1 + ħ<sup>2</sup> (3 ∈ f<sub>22</sub><sup>2</sup> c<sub>0,4</sub> + 3 ∈ f<sub>12</sub> f<sub>22</sub> c<sub>1,3</sub> + 2 ∈ f<sub>12</sub><sup>2</sup> c<sub>2,2</sub> + ∈ f<sub>11</sub> f<sub>22</sub> c<sub>2,2</sub> + 3 ∈ f<sub>11</sub> f<sub>12</sub> c<sub>3,1</sub> + 3 ∈ f<sub>11</sub><sup>2</sup> c<sub>4,0</sub>)

(Alt) Out[ ]:= 1 + ∈ ħ<sup>2</sup> (3 f<sub>22</sub><sup>2</sup> c<sub>0,4</sub> + 3 f<sub>12</sub> f<sub>22</sub> c<sub>1,3</sub> + 2 f<sub>12</sub><sup>2</sup> c<sub>2,2</sub> + f<sub>11</sub> f<sub>22</sub> c<sub>2,2</sub> + 3 f<sub>11</sub> f<sub>12</sub> c<sub>3,1</sub> + 3 f<sub>11</sub><sup>2</sup> c<sub>4,0</sub>)

(Alt) Out[ ]:= **True**

(Alt) In[ ]:= {n = 2, p = 3, \$k = 3, B = Table[x<sub>i</sub>, {i, n}], F = ħ Table[f<sub>{10,1}.Sort[{i,j}]</sub>, {i, n}, {j, n}],  
 P = GenericPerturbation[{\$k}, B, c], Z = <F, E[1, 0, P]><sub>B</sub>}

(Alt) Out[ ]:= {2, 3, 3, {x<sub>1</sub>, x<sub>2</sub>}, {{ħ f<sub>11</sub>, ħ f<sub>12</sub>}, {ħ f<sub>12</sub>, ħ f<sub>22</sub>}},  
 ∈Series[0, x<sub>1</sub><sup>4</sup> c<sub>0,4</sub> + x<sub>1</sub> x<sub>2</sub><sup>3</sup> c<sub>1,3</sub> + x<sub>1</sub><sup>2</sup> x<sub>2</sub><sup>2</sup> c<sub>2,2</sub> + x<sub>1</sub><sup>3</sup> x<sub>2</sub> c<sub>3,1</sub> + x<sub>1</sub><sup>4</sup> c<sub>4,0</sub>,  
 x<sub>2</sub><sup>6</sup> c<sub>0,6</sub> + x<sub>1</sub> x<sub>2</sub><sup>5</sup> c<sub>1,5</sub> + x<sub>1</sub><sup>2</sup> x<sub>2</sub><sup>4</sup> c<sub>2,4</sub> + x<sub>1</sub><sup>3</sup> x<sub>2</sub><sup>3</sup> c<sub>3,3</sub> + x<sub>1</sub><sup>4</sup> x<sub>2</sub><sup>2</sup> c<sub>4,2</sub> + x<sub>1</sub><sup>5</sup> x<sub>2</sub> c<sub>5,1</sub> + x<sub>1</sub><sup>6</sup> c<sub>6,0</sub>,  
 x<sub>2</sub><sup>8</sup> c<sub>0,8</sub> + x<sub>1</sub> x<sub>2</sub><sup>7</sup> c<sub>1,7</sub> + x<sub>1</sub><sup>2</sup> x<sub>2</sub><sup>6</sup> c<sub>2,6</sub> + x<sub>1</sub><sup>3</sup> x<sub>2</sub><sup>5</sup> c<sub>3,5</sub> + x<sub>1</sub><sup>4</sup> x<sub>2</sub><sup>4</sup> c<sub>4,4</sub> + x<sub>1</sub><sup>5</sup> x<sub>2</sub><sup>3</sup> c<sub>5,3</sub> + x<sub>1</sub><sup>6</sup> x<sub>2</sub><sup>2</sup> c<sub>6,2</sub> + x<sub>1</sub><sup>7</sup> x<sub>2</sub> c<sub>7,1</sub> + x<sub>1</sub><sup>8</sup> c<sub>8,0</sub>],  
 {{ħ f<sub>11</sub>, ħ f<sub>12</sub>}, {ħ f<sub>12</sub>, ħ f<sub>22</sub>}}, E[1, 0, ∈Series[0, x<sub>1</sub><sup>4</sup> c<sub>0,4</sub> + x<sub>1</sub> x<sub>2</sub><sup>3</sup> c<sub>1,3</sub> + x<sub>1</sub><sup>2</sup> x<sub>2</sub><sup>2</sup> c<sub>2,2</sub> + x<sub>1</sub><sup>3</sup> x<sub>2</sub> c<sub>3,1</sub> +  
 x<sub>1</sub><sup>4</sup> c<sub>4,0</sub>, x<sub>2</sub><sup>6</sup> c<sub>0,6</sub> + x<sub>1</sub> x<sub>2</sub><sup>5</sup> c<sub>1,5</sub> + x<sub>1</sub><sup>2</sup> x<sub>2</sub><sup>4</sup> c<sub>2,4</sub> + x<sub>1</sub><sup>3</sup> x<sub>2</sub><sup>3</sup> c<sub>3,3</sub> + x<sub>1</sub><sup>4</sup> x<sub>2</sub><sup>2</sup> c<sub>4,2</sub> + x<sub>1</sub><sup>5</sup> x<sub>2</sub> c<sub>5,1</sub> + x<sub>1</sub><sup>6</sup> c<sub>6,0</sub>, x<sub>2</sub><sup>8</sup> c<sub>0,8</sub> +  
 x<sub>1</sub> x<sub>2</sub><sup>7</sup> c<sub>1,7</sub> + x<sub>1</sub><sup>2</sup> x<sub>2</sub><sup>6</sup> c<sub>2,6</sub> + x<sub>1</sub><sup>3</sup> x<sub>2</sub><sup>5</sup> c<sub>3,5</sub> + x<sub>1</sub><sup>4</sup> x<sub>2</sub><sup>4</sup> c<sub>4,4</sub> + x<sub>1</sub><sup>5</sup> x<sub>2</sub><sup>3</sup> c<sub>5,3</sub> + x<sub>1</sub><sup>6</sup> x<sub>2</sub><sup>2</sup> c<sub>6,2</sub> + x<sub>1</sub><sup>7</sup> x<sub>2</sub> c<sub>7,1</sub> + x<sub>1</sub><sup>8</sup> c<sub>8,0</sub>]]] }<sub>{x<sub>1</sub>, x<sub>2</sub>}</sub>

(Alt) In[ ]:= **ZZ = Zip3[Z][[3]]**

**lhs = Normal[Normal[Ev<sub>p</sub>@Z] + O[ε]<sup>k+1</sup>]**

**rhs = Normal[Series[e<sup>Sum[ZZ[[d+1]] ε<sup>d</sup>, {d,0,Length[ZZ]-1}</sup>], {ħ, 0, p}, {ε, 0, \$k}]]]**

**HL@Simplify[lhs == rhs]**

(Alt) Out[ ]:= Series[0, 3 ħ<sup>2</sup> f<sub>22</sub><sup>2</sup> c<sub>0,4</sub> + 3 ħ<sup>2</sup> f<sub>12</sub> f<sub>22</sub> c<sub>1,3</sub> + 2 ħ<sup>2</sup> f<sub>12</sub><sup>2</sup> c<sub>2,2</sub> + ħ<sup>2</sup> f<sub>11</sub> f<sub>22</sub> c<sub>2,2</sub> + 3 ħ<sup>2</sup> f<sub>11</sub> f<sub>12</sub> c<sub>3,1</sub> + 3 ħ<sup>2</sup> f<sub>11</sub><sup>2</sup> c<sub>4,0</sub>,  
 $\frac{1}{2}$  (96 ħ<sup>4</sup> f<sub>22</sub><sup>4</sup> c<sub>0,4</sub><sup>2</sup> + 30 ħ<sup>3</sup> f<sub>22</sub><sup>3</sup> c<sub>0,6</sub> + 192 ħ<sup>4</sup> f<sub>12</sub> f<sub>22</sub><sup>3</sup> c<sub>0,4</sub> c<sub>1,3</sub> + 81 ħ<sup>4</sup> f<sub>12</sub><sup>2</sup> f<sub>22</sub><sup>2</sup> c<sub>1,3</sub><sup>2</sup> + 15 ħ<sup>4</sup> f<sub>11</sub> f<sub>22</sub><sup>3</sup> c<sub>1,3</sub><sup>2</sup> +

30 ħ<sup>3</sup> f<sub>12</sub> f<sub>22</sub><sup>2</sup> c<sub>1,5</sub> + 168 ħ<sup>4</sup> f<sub>12</sub><sup>2</sup> f<sub>22</sub><sup>2</sup> c<sub>0,4</sub> c<sub>2,2</sub> + 24 ħ<sup>4</sup> f<sub>11</sub> f<sub>22</sub><sup>3</sup> c<sub>0,4</sub> c<sub>2,2</sub> + 108 ħ<sup>4</sup> f<sub>12</sub><sup>3</sup> f<sub>22</sub> c<sub>1,3</sub> c<sub>2,2</sub> +  
 84 ħ<sup>4</sup> f<sub>11</sub> f<sub>12</sub> f<sub>22</sub><sup>2</sup> c<sub>1,3</sub> c<sub>2,2</sub> + 20 ħ<sup>4</sup> f<sub>12</sub><sup>4</sup> c<sub>2,2</sub><sup>2</sup> + 68 ħ<sup>4</sup> f<sub>11</sub> f<sub>12</sub><sup>2</sup> f<sub>22</sub><sup>2</sup> c<sub>2,2</sub><sup>2</sup> + 8 ħ<sup>4</sup> f<sub>11</sub><sup>2</sup> f<sub>22</sub><sup>2</sup> c<sub>2,2</sub><sup>2</sup> +  
 24 ħ<sup>3</sup> f<sub>12</sub><sup>2</sup> f<sub>22</sub> c<sub>2,4</sub> + 6 ħ<sup>3</sup> f<sub>11</sub> f<sub>22</sub><sup>2</sup> c<sub>2,4</sub> + 120 ħ<sup>4</sup> f<sub>12</sub><sup>3</sup> f<sub>22</sub> c<sub>0,4</sub> c<sub>3,1</sub> + 72 ħ<sup>4</sup> f<sub>11</sub> f<sub>12</sub> f<sub>22</sub><sup>2</sup> c<sub>0,4</sub> c<sub>3,1</sub> +

$$\begin{aligned}
 & 48 \hbar^4 f_{12}^4 c_{1,3} c_{3,1} + 126 \hbar^4 f_{11} f_{12}^2 f_{22} c_{1,3} c_{3,1} + 18 \hbar^4 f_{11}^2 f_{22}^2 c_{1,3} c_{3,1} + 108 \hbar^4 f_{11} f_{12}^3 c_{2,2} c_{3,1} + \\
 & 84 \hbar^4 f_{11}^2 f_{12} f_{22} c_{2,2} c_{3,1} + 81 \hbar^4 f_{11}^2 f_{12}^2 c_{3,1}^2 + 15 \hbar^4 f_{11}^3 f_{22} c_{3,1}^2 + 12 \hbar^3 f_{12}^3 c_{3,3} + \\
 & 18 \hbar^3 f_{11} f_{12} f_{22} c_{3,3} + 48 \hbar^4 f_{12}^4 c_{0,4} c_{4,0} + 144 \hbar^4 f_{11} f_{12}^2 f_{22} c_{0,4} c_{4,0} + 120 \hbar^4 f_{11} f_{12}^3 c_{1,3} c_{4,0} + \\
 & 72 \hbar^4 f_{11}^2 f_{12} f_{22} c_{1,3} c_{4,0} + 168 \hbar^4 f_{11}^2 f_{12}^2 c_{2,2} c_{4,0} + 24 \hbar^4 f_{11}^3 f_{22} c_{2,2} c_{4,0} + 192 \hbar^4 f_{11}^3 f_{12} c_{3,1} c_{4,0} + \\
 & 96 \hbar^4 f_{11}^4 c_{4,0}^2 + 24 \hbar^3 f_{11} f_{12}^2 c_{4,2} + 6 \hbar^3 f_{11}^2 f_{22} c_{4,2} + 30 \hbar^3 f_{11}^2 f_{12} c_{5,1} + 30 \hbar^3 f_{11}^3 c_{6,0} \Big), \\
 & \frac{1}{3} (4752 \hbar^6 f_{22}^6 c_{0,4}^3 + 2700 \hbar^5 f_{22}^5 c_{0,4} c_{0,6} + 315 \hbar^4 f_{22}^4 c_{0,8} + 14256 \hbar^6 f_{12} f_{22}^5 c_{0,4}^2 c_{1,3} + \\
 & 2700 \hbar^5 f_{12} f_{22}^2 c_{0,6} c_{1,3} + 12906 \hbar^6 f_{12}^2 f_{22}^2 c_{0,4} c_{1,3}^2 + 1350 \hbar^6 f_{11} f_{22}^5 c_{0,4} c_{1,3}^2 + 3402 \hbar^6 f_{12}^3 f_{22}^2 c_{1,3}^3 + \\
 & 1350 \hbar^6 f_{11} f_{12} f_{22}^4 c_{1,3}^3 + 2700 \hbar^5 f_{12} f_{22}^4 c_{0,4} c_{1,5} + 2385 \hbar^5 f_{12}^2 f_{22}^3 c_{1,3} c_{1,5} + 315 \hbar^5 f_{11} f_{22}^4 c_{1,3} c_{1,5} + \\
 & 315 \hbar^4 f_{12} f_{22}^3 c_{1,7} + 13104 \hbar^6 f_{12}^2 f_{22}^2 c_{0,4}^2 c_{2,2} + 1152 \hbar^6 f_{11} f_{22}^5 c_{0,4}^2 c_{2,2} + 2430 \hbar^5 f_{12}^2 f_{22}^2 c_{0,6} c_{2,2} + \\
 & 270 \hbar^5 f_{11} f_{22}^2 c_{0,6} c_{2,2} + 20808 \hbar^6 f_{12}^3 f_{22}^3 c_{0,4} c_{1,3} c_{2,2} + 7704 \hbar^6 f_{11} f_{12} f_{22}^4 c_{0,4} c_{1,3} c_{2,2} + \\
 & 6804 \hbar^6 f_{12}^4 f_{22}^2 c_{1,3}^2 c_{2,2} + 7002 \hbar^6 f_{11} f_{12}^2 f_{22}^2 c_{1,3}^2 c_{2,2} + 450 \hbar^6 f_{11}^2 f_{22}^4 c_{1,3}^2 c_{2,2} + \\
 & 1800 \hbar^5 f_{12}^3 f_{22}^2 c_{1,5} c_{2,2} + 900 \hbar^5 f_{11} f_{12} f_{22}^3 c_{1,5} c_{2,2} + 6948 \hbar^6 f_{12}^4 f_{22}^2 c_{0,4} c_{2,2}^2 + \\
 & 6912 \hbar^6 f_{11} f_{12}^2 f_{22}^3 c_{0,4} c_{2,2}^2 + 396 \hbar^6 f_{11}^2 f_{22}^4 c_{0,4} c_{2,2}^2 + 3348 \hbar^6 f_{12}^5 f_{22} c_{1,3} c_{2,2}^2 + \\
 & 8712 \hbar^6 f_{11} f_{12}^3 f_{22}^2 c_{1,3} c_{2,2}^2 + 2196 \hbar^6 f_{11}^2 f_{12} f_{22}^2 c_{1,3} c_{2,2}^2 + 296 \hbar^6 f_{12}^6 c_{2,2}^3 + 2460 \hbar^6 f_{11} f_{12}^4 f_{22} c_{2,2}^3 + \\
 & 1896 \hbar^6 f_{11}^2 f_{12}^2 f_{22}^2 c_{2,2}^3 + 100 \hbar^6 f_{11}^3 f_{22}^3 c_{2,2}^3 + 2412 \hbar^5 f_{12}^2 f_{22}^3 c_{0,4} c_{2,4} + 288 \hbar^5 f_{11} f_{22}^4 c_{0,4} c_{2,4} + \\
 & 1782 \hbar^5 f_{12}^3 f_{22}^2 c_{1,3} c_{2,4} + 918 \hbar^5 f_{11} f_{12} f_{22}^3 c_{1,3} c_{2,4} + 1008 \hbar^5 f_{12}^4 f_{22} c_{2,2} c_{2,4} + \\
 & 1566 \hbar^5 f_{11} f_{12}^2 f_{22}^2 c_{2,2} c_{2,4} + 126 \hbar^5 f_{11}^2 f_{22}^2 c_{2,2} c_{2,4} + 270 \hbar^4 f_{12}^2 f_{22}^2 c_{2,6} + 45 \hbar^4 f_{11} f_{22}^2 c_{2,6} + \\
 & 10800 \hbar^6 f_{12}^3 f_{22}^2 c_{0,4}^2 c_{3,1} + 3456 \hbar^6 f_{11} f_{12} f_{22}^4 c_{0,4}^2 c_{3,1} + 1890 \hbar^5 f_{12}^3 f_{22}^2 c_{0,6} c_{3,1} + \\
 & 810 \hbar^5 f_{11} f_{12} f_{22}^3 c_{0,6} c_{3,1} + 14364 \hbar^6 f_{12}^4 f_{22}^2 c_{0,4} c_{1,3} c_{3,1} + 13284 \hbar^6 f_{11} f_{12}^2 f_{22}^3 c_{0,4} c_{1,3} c_{3,1} + \\
 & 864 \hbar^6 f_{11}^2 f_{22}^4 c_{0,4} c_{1,3} c_{3,1} + 3564 \hbar^6 f_{12}^5 f_{22} c_{1,3} c_{3,1} + 8478 \hbar^6 f_{11} f_{12}^3 f_{22}^2 c_{1,3}^2 c_{3,1} + \\
 & 2214 \hbar^6 f_{11}^2 f_{12} f_{22}^2 c_{1,3}^2 c_{3,1} + 1080 \hbar^5 f_{12}^4 f_{22} c_{1,5} c_{3,1} + 1485 \hbar^5 f_{11} f_{12}^2 f_{22}^2 c_{1,5} c_{3,1} + \\
 & 135 \hbar^5 f_{11}^2 f_{22}^3 c_{1,5} c_{3,1} + 7200 \hbar^6 f_{12}^5 f_{22} c_{0,4} c_{2,2} c_{3,1} + 17208 \hbar^6 f_{11} f_{12}^3 f_{22}^2 c_{0,4} c_{2,2} c_{3,1} + \\
 & 4104 \hbar^6 f_{11}^2 f_{12} f_{22}^3 c_{0,4} c_{2,2} c_{3,1} + 2016 \hbar^6 f_{12}^6 c_{1,3} c_{2,2} c_{3,1} + 14724 \hbar^6 f_{11} f_{12}^4 f_{22} c_{1,3} c_{2,2} c_{3,1} + \\
 & 11124 \hbar^6 f_{11}^2 f_{12}^2 f_{22}^2 c_{1,3} c_{2,2} c_{3,1} + 648 \hbar^6 f_{11}^3 f_{22}^2 c_{1,3} c_{2,2} c_{3,1} + 3348 \hbar^6 f_{11} f_{12}^2 c_{2,2}^2 c_{3,1} + \\
 & 8712 \hbar^6 f_{11}^2 f_{12}^3 f_{22} c_{2,2}^2 c_{3,1} + 2196 \hbar^6 f_{11}^3 f_{12} f_{22}^2 c_{2,2}^2 c_{3,1} + 360 \hbar^5 f_{12}^5 c_{2,4} c_{3,1} + \\
 & 1692 \hbar^5 f_{11} f_{12}^3 f_{22} c_{2,4} c_{3,1} + 648 \hbar^5 f_{11}^2 f_{12} f_{22}^2 c_{2,4} c_{3,1} + 1080 \hbar^6 f_{12}^6 c_{0,4} c_{3,1}^2 + \\
 & 7560 \hbar^6 f_{11} f_{12}^4 f_{22} c_{0,4} c_{3,1}^2 + 5346 \hbar^6 f_{11}^2 f_{12}^2 f_{22}^2 c_{0,4} c_{3,1}^2 + 270 \hbar^6 f_{11}^3 f_{22}^3 c_{0,4} c_{3,1}^2 + \\
 & 3564 \hbar^6 f_{11} f_{12}^5 c_{1,3} c_{3,1}^2 + 8478 \hbar^6 f_{11}^2 f_{12}^3 f_{22} c_{1,3} c_{3,1}^2 + 2214 \hbar^6 f_{11}^3 f_{12} f_{22}^2 c_{1,3} c_{3,1}^2 + \\
 & 6804 \hbar^6 f_{11}^2 f_{12}^4 c_{2,2} c_{3,1}^2 + 7002 \hbar^6 f_{11}^3 f_{12}^2 f_{22} c_{2,2} c_{3,1}^2 + 450 \hbar^6 f_{11}^4 f_{22}^2 c_{2,2} c_{3,1}^2 + \\
 & 3402 \hbar^6 f_{11}^3 f_{12}^3 c_{3,1}^3 + 1350 \hbar^6 f_{11} f_{12} f_{22} c_{3,1}^3 + 1836 \hbar^5 f_{12}^3 f_{22}^2 c_{0,4} c_{3,3} + 864 \hbar^5 f_{11} f_{12} f_{22}^3 c_{0,4} c_{3,3} + \\
 & 1026 \hbar^5 f_{12}^4 f_{22} c_{1,3} c_{3,3} + 1539 \hbar^5 f_{11} f_{12}^2 f_{22}^2 c_{1,3} c_{3,3} + 135 \hbar^5 f_{11}^2 f_{22}^3 c_{1,3} c_{3,3} + \\
 & 324 \hbar^5 f_{12}^5 c_{2,2} c_{3,3} + 1728 \hbar^5 f_{11} f_{12}^3 f_{22} c_{2,2} c_{3,3} + 648 \hbar^5 f_{11}^2 f_{12} f_{22}^2 c_{2,2} c_{3,3} + \\
 & 1026 \hbar^5 f_{11} f_{12}^4 c_{3,1} c_{3,3} + 1539 \hbar^5 f_{11}^2 f_{12}^2 f_{22} c_{3,1} c_{3,3} + 135 \hbar^5 f_{11}^3 f_{22}^2 c_{3,1} c_{3,3} + \\
 & 180 \hbar^4 f_{12}^4 f_{22} c_{3,5} + 135 \hbar^4 f_{11} f_{12} f_{22}^2 c_{3,5} + 7344 \hbar^6 f_{12}^4 f_{22}^2 c_{0,4}^2 c_{4,0} + 6912 \hbar^6 f_{11} f_{12}^2 f_{22}^3 c_{0,4}^2 c_{4,0} + \\
 & 1080 \hbar^5 f_{12}^4 f_{22} c_{0,6} c_{4,0} + 1620 \hbar^5 f_{11} f_{12}^2 f_{22}^2 c_{0,6} c_{4,0} + 7344 \hbar^6 f_{12}^5 f_{22} c_{0,4} c_{1,3} c_{4,0} + \\
 & 17712 \hbar^6 f_{11} f_{12}^3 f_{22}^2 c_{0,4} c_{1,3} c_{4,0} + 3456 \hbar^6 f_{11}^2 f_{12} f_{22}^3 c_{0,4} c_{1,3} c_{4,0} + 1080 \hbar^6 f_{12}^6 c_{1,3}^2 c_{4,0} + \\
 & 7560 \hbar^6 f_{11} f_{12}^4 f_{22} c_{1,3}^2 c_{4,0} + 5346 \hbar^6 f_{11}^2 f_{12}^2 f_{22}^2 c_{1,3}^2 c_{4,0} + 270 \hbar^6 f_{11}^3 f_{22}^3 c_{1,3}^2 c_{4,0} + \\
 & 360 \hbar^5 f_{12}^5 c_{1,5} c_{4,0} + 1800 \hbar^5 f_{11} f_{12}^3 f_{22} c_{1,5} c_{4,0} + 540 \hbar^5 f_{11}^2 f_{12} f_{22}^2 c_{1,5} c_{4,0} + \\
 & 2016 \hbar^6 f_{12}^6 c_{0,4} c_{2,2} c_{4,0} + 15696 \hbar^6 f_{11} f_{12}^4 f_{22} c_{0,4} c_{2,2} c_{4,0} + 10368 \hbar^6 f_{11}^2 f_{12}^2 f_{22}^2 c_{0,4} c_{2,2} c_{4,0} + \\
 & 432 \hbar^6 f_{11}^3 f_{22}^3 c_{0,4} c_{2,2} c_{4,0} + 7200 \hbar^6 f_{11} f_{12}^5 c_{1,3} c_{2,2} c_{4,0} + 17208 \hbar^6 f_{11}^2 f_{12}^3 f_{22} c_{1,3} c_{2,2} c_{4,0} + \\
 & 4104 \hbar^6 f_{11}^3 f_{12} f_{22}^2 c_{1,3} c_{2,2} c_{4,0} + 6948 \hbar^6 f_{11}^2 f_{12}^4 c_{2,2}^2 c_{4,0} + 6912 \hbar^6 f_{11}^3 f_{12}^2 f_{22} c_{2,2}^2 c_{4,0} +
 \end{aligned}$$

$$\begin{aligned}
 & 396 \hbar^6 f_{11}^4 f_{22}^2 c_{2,2}^2 c_{4,0} + 1080 \hbar^5 f_{11} f_{12}^4 c_{2,4} c_{4,0} + 1512 \hbar^5 f_{11}^2 f_{12}^2 f_{22} c_{2,4} c_{4,0} + \\
 & 108 \hbar^5 f_{11}^3 f_{22}^2 c_{2,4} c_{4,0} + 7344 \hbar^6 f_{11} f_{12}^5 c_{0,4} c_{3,1} c_{4,0} + 17712 \hbar^6 f_{11}^2 f_{12}^3 f_{22} c_{0,4} c_{3,1} c_{4,0} + \\
 & 3456 \hbar^6 f_{11}^3 f_{12} f_{22}^2 c_{0,4} c_{3,1} c_{4,0} + 14364 \hbar^6 f_{11}^2 f_{12}^4 c_{1,3} c_{3,1} c_{4,0} + 13284 \hbar^6 f_{11}^2 f_{12}^2 f_{22} c_{1,3} c_{3,1} c_{4,0} + \\
 & 864 \hbar^6 f_{11}^4 f_{22}^2 c_{1,3} c_{3,1} c_{4,0} + 20808 \hbar^6 f_{11}^3 f_{12}^3 c_{2,2} c_{3,1} c_{4,0} + 7704 \hbar^6 f_{11}^4 f_{12} f_{22} c_{2,2} c_{3,1} c_{4,0} + \\
 & 12906 \hbar^6 f_{11}^4 f_{12}^2 c_{3,1}^2 c_{4,0} + 1350 \hbar^6 f_{11}^5 f_{22} c_{3,1}^2 c_{4,0} + 1836 \hbar^5 f_{11}^2 f_{12}^3 c_{3,3} c_{4,0} + \\
 & 864 \hbar^5 f_{11}^3 f_{12} f_{22} c_{3,3} c_{4,0} + 7344 \hbar^6 f_{11}^2 f_{12}^4 c_{0,4} c_{4,0}^2 + 6912 \hbar^6 f_{11}^3 f_{12}^2 f_{22} c_{0,4} c_{4,0}^2 + \\
 & 10800 \hbar^6 f_{11}^3 f_{12}^2 c_{1,3} c_{4,0}^2 + 3456 \hbar^6 f_{11}^4 f_{12} f_{22} c_{1,3} c_{4,0}^2 + 13104 \hbar^6 f_{11}^2 f_{12}^2 c_{2,2} c_{4,0}^2 + \\
 & 1152 \hbar^6 f_{11}^5 f_{22} c_{2,2} c_{4,0}^2 + 14256 \hbar^6 f_{11}^5 f_{12} c_{3,1} c_{4,0}^2 + 4752 \hbar^6 f_{11}^6 c_{4,0}^3 + 1080 \hbar^5 f_{12}^4 f_{22} c_{0,4} c_{4,2} + \\
 & 1512 \hbar^5 f_{11} f_{12}^2 f_{22}^2 c_{0,4} c_{4,2} + 108 \hbar^5 f_{11}^2 f_{22}^3 c_{0,4} c_{4,2} + 360 \hbar^5 f_{12}^5 c_{1,3} c_{4,2} + \\
 & 1692 \hbar^5 f_{11} f_{12}^3 f_{22} c_{1,3} c_{4,2} + 648 \hbar^5 f_{11}^2 f_{12}^2 c_{1,3} c_{4,2} + 1008 \hbar^5 f_{11} f_{12}^4 c_{2,2} c_{4,2} + \\
 & 1566 \hbar^5 f_{11}^2 f_{12}^2 f_{22} c_{2,2} c_{4,2} + 126 \hbar^5 f_{11}^3 f_{22}^2 c_{2,2} c_{4,2} + 1782 \hbar^5 f_{11}^2 f_{12}^3 c_{3,1} c_{4,2} + \\
 & 918 \hbar^5 f_{11}^3 f_{12} f_{22} c_{3,1} c_{4,2} + 2412 \hbar^5 f_{11}^3 f_{12}^2 c_{4,0} c_{4,2} + 288 \hbar^5 f_{11}^4 f_{22} c_{4,0} c_{4,2} + \\
 & 72 \hbar^4 f_{12}^4 c_{4,4} + 216 \hbar^4 f_{11} f_{12}^2 f_{22} c_{4,4} + 27 \hbar^4 f_{11}^2 f_{22}^2 c_{4,4} + 360 \hbar^5 f_{12}^5 c_{0,4} c_{5,1} + \\
 & 1800 \hbar^5 f_{11} f_{12}^3 f_{22} c_{0,4} c_{5,1} + 540 \hbar^5 f_{11}^2 f_{12} f_{22}^2 c_{0,4} c_{5,1} + 1080 \hbar^5 f_{11} f_{12}^4 c_{1,3} c_{5,1} + \\
 & 1485 \hbar^5 f_{11}^2 f_{12}^2 f_{22} c_{1,3} c_{5,1} + 135 \hbar^5 f_{11}^3 f_{22}^2 c_{1,3} c_{5,1} + 1800 \hbar^5 f_{11}^2 f_{12}^3 c_{2,2} c_{5,1} + \\
 & 900 \hbar^5 f_{11}^3 f_{12} f_{22} c_{2,2} c_{5,1} + 2385 \hbar^5 f_{11}^3 f_{12}^2 c_{3,1} c_{5,1} + 315 \hbar^5 f_{11}^4 f_{22} c_{3,1} c_{5,1} + \\
 & 2700 \hbar^5 f_{11}^4 f_{12} c_{4,0} c_{5,1} + 180 \hbar^4 f_{11} f_{12}^3 c_{5,3} + 135 \hbar^4 f_{11}^2 f_{12} f_{22} c_{5,3} + 1080 \hbar^5 f_{11} f_{12}^4 c_{0,4} c_{6,0} + \\
 & 1620 \hbar^5 f_{11}^2 f_{12}^2 f_{22} c_{0,4} c_{6,0} + 1890 \hbar^5 f_{11}^2 f_{12}^3 c_{1,3} c_{6,0} + 810 \hbar^5 f_{11}^3 f_{12} f_{22} c_{1,3} c_{6,0} + \\
 & 2430 \hbar^5 f_{11}^3 f_{12}^2 c_{2,2} c_{6,0} + 270 \hbar^5 f_{11}^4 f_{22} c_{2,2} c_{6,0} + 2700 \hbar^5 f_{11}^4 f_{12} c_{3,1} c_{6,0} + 2700 \hbar^5 f_{11}^5 c_{4,0} c_{6,0} + \\
 & 270 \hbar^4 f_{11}^2 f_{12}^2 c_{6,2} + 45 \hbar^4 f_{11}^3 f_{22} c_{6,2} + 315 \hbar^4 f_{11}^3 f_{12} c_{7,1} + 315 \hbar^4 f_{11}^4 c_{8,0} ) ]
 \end{aligned}$$

$$\begin{aligned}
 \text{(Alt) Out[*]= } & 1 + \hbar^2 ( 3 \in f_{22}^2 c_{0,4} + 3 \in f_{12} f_{22} c_{1,3} + 2 \in f_{12}^2 c_{2,2} + \in f_{11} f_{22} c_{2,2} + 3 \in f_{11} f_{12} c_{3,1} + 3 \in f_{11}^2 c_{4,0} ) + \\
 & \hbar^3 ( 15 \in^2 f_{22}^3 c_{0,6} + 15 \in^2 f_{12} f_{22}^2 c_{1,5} + 12 \in^2 f_{12}^2 f_{22} c_{2,4} + 3 \in^2 f_{11} f_{22}^2 c_{2,4} + 6 \in^2 f_{12}^3 c_{3,3} + \\
 & 9 \in^2 f_{11} f_{12} f_{22} c_{3,3} + 12 \in^2 f_{11} f_{12}^2 c_{4,2} + 3 \in^2 f_{11}^2 f_{22} c_{4,2} + 15 \in^2 f_{11}^2 f_{12} c_{5,1} + 15 \in^2 f_{11}^3 c_{6,0} )
 \end{aligned}$$

$$\begin{aligned}
 \text{(Alt) Out[*]= } & 1 + \in \hbar^2 ( 3 f_{22}^2 c_{0,4} + 3 f_{12} f_{22} c_{1,3} + 2 f_{12}^2 c_{2,2} + f_{11} f_{22} c_{2,2} + 3 f_{11} f_{12} c_{3,1} + 3 f_{11}^2 c_{4,0} ) + \\
 & 3 \in^2 \hbar^3 ( 5 f_{22}^3 c_{0,6} + 5 f_{12} f_{22}^2 c_{1,5} + 4 f_{12}^2 f_{22} c_{2,4} + f_{11} f_{22}^2 c_{2,4} + 2 f_{12}^3 c_{3,3} + \\
 & 3 f_{11} f_{12} f_{22} c_{3,3} + 4 f_{11} f_{12}^2 c_{4,2} + f_{11}^2 f_{22} c_{4,2} + 5 f_{11}^2 f_{12} c_{5,1} + 5 f_{11}^3 c_{6,0} )
 \end{aligned}$$

(Alt) Out[\*]= True

```

(Alt) In[*]= {n = 2, p = 4, $k = 3, B = Table[xi, {i, n}], F = h Table[f_{10,1}.Sort[{i,j}], {i, n}, {j, n}],
  P = GenericPerturbation[{$k}, B, c], Z = <F, E[1, 0, P]>_B}
ZZ = Zip3[Z][[3]]
lhs = Normal[Normal[Ev_p@Z] + O[epsilon]^{k+1}]
rhs = Normal[Series[e^{Sum[ZZ[[d+1]] e^d, {d, 0, Length[ZZ]-1}], {h, 0, p}, {epsilon, 0, $k}]]]
HL@Simplify[lhs == rhs]
    
```

$$\begin{aligned}
 (\text{Alt}) \text{Out}[*]= & \{2, 4, 3, \{x_1, x_2\}, \{\{\hbar f_{11}, \hbar f_{12}\}, \{\hbar f_{12}, \hbar f_{22}\}\}, \\
 & \in \text{Series} \left[ 0, x_2^4 c_{0,4} + x_1 x_2^3 c_{1,3} + x_1^2 x_2^2 c_{2,2} + x_1^3 x_2 c_{3,1} + x_1^4 c_{4,0}, \right. \\
 & x_2^6 c_{0,6} + x_1 x_2^5 c_{1,5} + x_1^2 x_2^4 c_{2,4} + x_1^3 x_2^3 c_{3,3} + x_1^4 x_2^2 c_{4,2} + x_1^5 x_2 c_{5,1} + x_1^6 c_{6,0}, \\
 & \left. x_2^8 c_{0,8} + x_1 x_2^7 c_{1,7} + x_1^2 x_2^6 c_{2,6} + x_1^3 x_2^5 c_{3,5} + x_1^4 x_2^4 c_{4,4} + x_1^5 x_2^3 c_{5,3} + x_1^6 x_2^2 c_{6,2} + x_1^7 x_2 c_{7,1} + x_1^8 c_{8,0} \right], \\
 & \left\{ \left\{ \{\hbar f_{11}, \hbar f_{12}\}, \{\hbar f_{12}, \hbar f_{22}\} \right\}, \mathbb{E} \left[ 1, 0, \in \text{Series} \left[ 0, x_2^4 c_{0,4} + x_1 x_2^3 c_{1,3} + x_1^2 x_2^2 c_{2,2} + x_1^3 x_2 c_{3,1} + \right. \right. \right. \\
 & \left. \left. \left. x_1^4 c_{4,0}, x_2^6 c_{0,6} + x_1 x_2^5 c_{1,5} + x_1^2 x_2^4 c_{2,4} + x_1^3 x_2^3 c_{3,3} + x_1^4 x_2^2 c_{4,2} + x_1^5 x_2 c_{5,1} + x_1^6 c_{6,0}, x_2^8 c_{0,8} + \right. \right. \right. \\
 & \left. \left. \left. x_1 x_2^7 c_{1,7} + x_1^2 x_2^6 c_{2,6} + x_1^3 x_2^5 c_{3,5} + x_1^4 x_2^4 c_{4,4} + x_1^5 x_2^3 c_{5,3} + x_1^6 x_2^2 c_{6,2} + x_1^7 x_2 c_{7,1} + x_1^8 c_{8,0} \right] \right] \right\}_{\{x_1, x_2\}}
 \end{aligned}$$

$$(\text{Alt}) \text{Out}[*]= \in \text{Series} \left[ 0, 3 \hbar^2 f_{22}^2 c_{0,4} + 3 \hbar^2 f_{12} f_{22} c_{1,3} + 2 \hbar^2 f_{12}^2 c_{2,2} + \hbar^2 f_{11} f_{22} c_{2,2} + 3 \hbar^2 f_{11} f_{12} c_{3,1} + 3 \hbar^2 f_{11}^2 c_{4,0}, \right.$$

$$\frac{1}{2} \left( 96 \hbar^4 f_{22}^4 c_{0,4}^2 + 30 \hbar^3 f_{22}^3 c_{0,6} + 192 \hbar^4 f_{12} f_{22}^3 c_{0,4} c_{1,3} + 81 \hbar^4 f_{12}^2 f_{22}^2 c_{1,3}^2 + 15 \hbar^4 f_{11} f_{22}^3 c_{1,3}^2 + \right.$$

$$\begin{aligned}
 & 30 \hbar^3 f_{12} f_{22}^2 c_{1,5} + 168 \hbar^4 f_{12}^2 f_{22}^2 c_{0,4} c_{2,2} + 24 \hbar^4 f_{11} f_{22}^3 c_{0,4} c_{2,2} + 108 \hbar^4 f_{12}^3 f_{22} c_{1,3} c_{2,2} + \\
 & 84 \hbar^4 f_{11} f_{12} f_{22}^2 c_{1,3} c_{2,2} + 20 \hbar^4 f_{12}^4 c_{2,2}^2 + 68 \hbar^4 f_{11} f_{12}^2 f_{22} c_{2,2}^2 + 8 \hbar^4 f_{11}^2 f_{22}^2 c_{2,2}^2 + \\
 & 24 \hbar^3 f_{12}^2 f_{22} c_{2,4} + 6 \hbar^3 f_{11} f_{22}^2 c_{2,4} + 120 \hbar^4 f_{12}^3 f_{22} c_{0,4} c_{3,1} + 72 \hbar^4 f_{11} f_{12} f_{22}^2 c_{0,4} c_{3,1} + \\
 & 48 \hbar^4 f_{12}^4 c_{1,3} c_{3,1} + 126 \hbar^4 f_{11} f_{12}^2 f_{22} c_{1,3} c_{3,1} + 18 \hbar^4 f_{11}^2 f_{22}^2 c_{1,3} c_{3,1} + 108 \hbar^4 f_{11} f_{12}^3 c_{2,2} c_{3,1} + \\
 & 84 \hbar^4 f_{11}^2 f_{12} f_{22} c_{2,2} c_{3,1} + 81 \hbar^4 f_{11}^2 f_{12}^2 c_{3,1}^2 + 15 \hbar^4 f_{11}^3 f_{22} c_{3,1}^2 + 12 \hbar^3 f_{12}^3 c_{3,3} + \\
 & 18 \hbar^3 f_{11} f_{12} f_{22} c_{3,3} + 48 \hbar^4 f_{12}^4 c_{0,4} c_{4,0} + 144 \hbar^4 f_{11} f_{12}^2 f_{22} c_{0,4} c_{4,0} + 120 \hbar^4 f_{11} f_{12}^3 c_{1,3} c_{4,0} + \\
 & 72 \hbar^4 f_{11}^2 f_{12} f_{22} c_{1,3} c_{4,0} + 168 \hbar^4 f_{11}^2 f_{12}^2 c_{2,2} c_{4,0} + 24 \hbar^4 f_{11}^3 f_{22} c_{2,2} c_{4,0} + 192 \hbar^4 f_{11}^3 f_{12} c_{3,1} c_{4,0} + \\
 & 96 \hbar^4 f_{11}^4 c_{4,0}^2 + 24 \hbar^3 f_{11} f_{12}^2 c_{4,2} + 6 \hbar^3 f_{11}^2 f_{22} c_{4,2} + 30 \hbar^3 f_{11}^2 f_{12} c_{5,1} + 30 \hbar^3 f_{11}^3 c_{6,0} \left. \right),
 \end{aligned}$$

$$\frac{1}{3} \left( 4752 \hbar^6 f_{22}^6 c_{0,4}^3 + 2700 \hbar^5 f_{22}^5 c_{0,4} c_{0,6} + 315 \hbar^4 f_{22}^4 c_{0,8} + 14256 \hbar^6 f_{12} f_{22}^5 c_{0,4}^2 c_{1,3} + \right.$$

$$\begin{aligned}
 & 2700 \hbar^5 f_{12} f_{22}^4 c_{0,6} c_{1,3} + 12906 \hbar^6 f_{12}^2 f_{22}^4 c_{0,4} c_{1,3}^2 + 1350 \hbar^6 f_{11} f_{22}^5 c_{0,4} c_{1,3}^2 + 3402 \hbar^6 f_{12}^3 f_{22}^3 c_{1,3}^3 + \\
 & 1350 \hbar^6 f_{11} f_{12} f_{22}^4 c_{1,3}^3 + 2700 \hbar^5 f_{12} f_{22}^4 c_{0,4} c_{1,5} + 2385 \hbar^5 f_{12}^2 f_{22}^3 c_{1,3} c_{1,5} + 315 \hbar^5 f_{11} f_{22}^4 c_{1,3} c_{1,5} + \\
 & 315 \hbar^4 f_{12} f_{22}^3 c_{1,7} + 13104 \hbar^6 f_{12}^2 f_{22}^4 c_{0,4} c_{2,2} + 1152 \hbar^6 f_{11} f_{22}^5 c_{0,4} c_{2,2} + 2430 \hbar^5 f_{12}^2 f_{22}^3 c_{0,6} c_{2,2} + \\
 & 270 \hbar^5 f_{11} f_{22}^4 c_{0,6} c_{2,2} + 20808 \hbar^6 f_{12}^3 f_{22}^3 c_{0,4} c_{1,3} c_{2,2} + 7704 \hbar^6 f_{11} f_{12} f_{22}^4 c_{0,4} c_{1,3} c_{2,2} + \\
 & 6804 \hbar^6 f_{12}^4 f_{22}^2 c_{1,3}^2 c_{2,2} + 7002 \hbar^6 f_{11} f_{12}^2 f_{22}^3 c_{1,3}^2 c_{2,2} + 450 \hbar^6 f_{11}^2 f_{22}^4 c_{1,3}^2 c_{2,2} + \\
 & 1800 \hbar^5 f_{12}^3 f_{22}^2 c_{1,5} c_{2,2} + 900 \hbar^5 f_{11} f_{12} f_{22}^3 c_{1,5} c_{2,2} + 6948 \hbar^6 f_{12}^4 f_{22}^2 c_{0,4} c_{2,2}^2 + \\
 & 6912 \hbar^6 f_{11} f_{12}^2 f_{22}^3 c_{0,4} c_{2,2}^2 + 396 \hbar^6 f_{11}^2 f_{22}^4 c_{0,4} c_{2,2}^2 + 3348 \hbar^6 f_{12}^5 f_{22} c_{1,3} c_{2,2}^2 + \\
 & 8712 \hbar^6 f_{11} f_{12}^3 f_{22}^2 c_{1,3} c_{2,2}^2 + 2196 \hbar^6 f_{11}^2 f_{12} f_{22}^2 c_{1,3} c_{2,2}^2 + 296 \hbar^6 f_{12}^6 c_{2,2}^2 + 2460 \hbar^6 f_{11} f_{12}^4 f_{22} c_{2,2}^3 + \\
 & 1896 \hbar^6 f_{11}^2 f_{12}^2 f_{22}^2 c_{2,2}^3 + 100 \hbar^6 f_{11}^3 f_{22}^2 c_{2,2}^3 + 2412 \hbar^5 f_{12}^2 f_{22}^3 c_{0,4} c_{2,4} + 288 \hbar^5 f_{11} f_{22}^4 c_{0,4} c_{2,4} + \\
 & 1782 \hbar^5 f_{12}^3 f_{22}^2 c_{1,3} c_{2,4} + 918 \hbar^5 f_{11} f_{12} f_{22}^3 c_{1,3} c_{2,4} + 1008 \hbar^5 f_{12}^4 f_{22} c_{2,2} c_{2,4} + \\
 & 1566 \hbar^5 f_{11} f_{12}^2 f_{22}^2 c_{2,2} c_{2,4} + 126 \hbar^5 f_{11}^2 f_{22}^3 c_{2,2} c_{2,4} + 270 \hbar^4 f_{12}^2 f_{22}^2 c_{2,6} + 45 \hbar^4 f_{11} f_{22}^3 c_{2,6} + \\
 & 10800 \hbar^6 f_{12}^3 f_{22}^2 c_{0,4}^2 c_{3,1} + 3456 \hbar^6 f_{11} f_{12} f_{22}^4 c_{0,4}^2 c_{3,1} + 1890 \hbar^5 f_{12}^2 f_{22}^2 c_{0,6} c_{3,1} + \\
 & 810 \hbar^5 f_{11} f_{12} f_{22}^3 c_{0,6} c_{3,1} + 14364 \hbar^6 f_{12}^4 f_{22}^2 c_{0,4} c_{1,3} c_{3,1} + 13284 \hbar^6 f_{11} f_{12}^2 f_{22}^3 c_{0,4} c_{1,3} c_{3,1} + \\
 & 864 \hbar^6 f_{11}^2 f_{22}^4 c_{0,4} c_{1,3} c_{3,1} + 3564 \hbar^6 f_{12}^5 f_{22} c_{1,3}^2 c_{3,1} + 8478 \hbar^6 f_{11} f_{12}^3 f_{22}^2 c_{1,3}^2 c_{3,1} + \\
 & 2214 \hbar^6 f_{11}^2 f_{12} f_{22}^3 c_{1,3}^2 c_{3,1} + 1080 \hbar^5 f_{12}^4 f_{22} c_{1,5} c_{3,1} + 1485 \hbar^5 f_{11} f_{12}^2 f_{22}^2 c_{1,5} c_{3,1} + \\
 & 135 \hbar^5 f_{11}^2 f_{22}^3 c_{1,5} c_{3,1} + 7200 \hbar^6 f_{12}^5 f_{22} c_{0,4} c_{2,2} c_{3,1} + 17208 \hbar^6 f_{11} f_{12}^3 f_{22}^2 c_{0,4} c_{2,2} c_{3,1} + \\
 & 4104 \hbar^6 f_{11}^2 f_{12} f_{22}^3 c_{0,4} c_{2,2} c_{3,1} + 2016 \hbar^6 f_{12}^6 c_{1,3} c_{2,2} c_{3,1} + 14724 \hbar^6 f_{11} f_{12}^4 f_{22} c_{1,3} c_{2,2} c_{3,1} + \\
 & 11124 \hbar^6 f_{11}^2 f_{12}^2 f_{22}^2 c_{1,3} c_{2,2} c_{3,1} + 648 \hbar^6 f_{11}^3 f_{22}^3 c_{1,3} c_{2,2} c_{3,1} + 3348 \hbar^6 f_{11} f_{12}^5 c_{2,2}^2 c_{3,1} + \\
 & 8712 \hbar^6 f_{11}^2 f_{12}^3 f_{22}^2 c_{2,2}^2 c_{3,1} + 2196 \hbar^6 f_{11}^2 f_{12}^2 f_{22}^2 c_{2,2}^2 c_{3,1} + 360 \hbar^5 f_{12}^5 c_{2,4} c_{3,1} + \\
 & 1692 \hbar^5 f_{11} f_{12}^3 f_{22} c_{2,4} c_{3,1} + 648 \hbar^5 f_{11}^2 f_{12}^2 f_{22}^2 c_{2,4} c_{3,1} + 1080 \hbar^6 f_{12}^6 c_{0,4} c_{3,1}^2 + \\
 & 7560 \hbar^6 f_{11} f_{12}^4 f_{22} c_{0,4} c_{3,1}^2 + 5346 \hbar^6 f_{11}^2 f_{12}^2 f_{22}^2 c_{0,4} c_{3,1}^2 + 270 \hbar^6 f_{11}^3 f_{22}^3 c_{0,4} c_{3,1}^2 +
 \end{aligned}$$



$$\begin{aligned}
 & 3564 \hbar^6 f_{11} f_{12}^5 c_{1,3} c_{3,1}^2 + 8478 \hbar^6 f_{11}^2 f_{12}^3 f_{22} c_{1,3} c_{3,1}^2 + 2214 \hbar^6 f_{11}^3 f_{12} f_{22}^2 c_{1,3} c_{3,1}^2 + \\
 & 6804 \hbar^6 f_{11}^2 f_{12}^4 c_{2,2} c_{3,1}^2 + 7002 \hbar^6 f_{11}^3 f_{12}^2 f_{22} c_{2,2} c_{3,1}^2 + 450 \hbar^6 f_{11}^4 f_{22}^2 c_{2,2} c_{3,1}^2 + \\
 & 3402 \hbar^6 f_{11}^3 f_{12}^2 c_{3,1}^3 + 1350 \hbar^6 f_{11}^4 f_{12} f_{22} c_{3,1}^3 + 1836 \hbar^5 f_{12}^3 f_{22}^2 c_{0,4} c_{3,3} + 864 \hbar^5 f_{11} f_{12} f_{22}^2 c_{0,4} c_{3,3} + \\
 & 1026 \hbar^5 f_{12}^4 f_{22} c_{1,3} c_{3,3} + 1539 \hbar^5 f_{11} f_{12}^2 f_{22}^2 c_{1,3} c_{3,3} + 135 \hbar^5 f_{11}^2 f_{22}^3 c_{1,3} c_{3,3} + \\
 & 324 \hbar^5 f_{12}^5 c_{2,2} c_{3,3} + 1728 \hbar^5 f_{11} f_{12}^3 f_{22} c_{2,2} c_{3,3} + 648 \hbar^5 f_{11}^2 f_{12} f_{22}^2 c_{2,2} c_{3,3} + \\
 & 1026 \hbar^5 f_{11} f_{12}^4 c_{3,1} c_{3,3} + 1539 \hbar^5 f_{11}^2 f_{12}^2 f_{22} c_{3,1} c_{3,3} + 135 \hbar^5 f_{11}^3 f_{22}^3 c_{3,1} c_{3,3} + \\
 & 180 \hbar^4 f_{12}^3 f_{22} c_{3,5} + 135 \hbar^4 f_{11} f_{12} f_{22}^2 c_{3,5} + 7344 \hbar^6 f_{12}^4 f_{22}^2 c_{0,4}^2 c_{4,0} + 6912 \hbar^6 f_{11} f_{12}^2 f_{22}^2 c_{0,4}^2 c_{4,0} + \\
 & 1080 \hbar^5 f_{12}^4 f_{22} c_{0,6} c_{4,0} + 1620 \hbar^5 f_{11} f_{12}^2 f_{22}^2 c_{0,6} c_{4,0} + 7344 \hbar^6 f_{12}^5 f_{22} c_{0,4} c_{1,3} c_{4,0} + \\
 & 17712 \hbar^6 f_{11} f_{12}^3 f_{22}^2 c_{0,4} c_{1,3} c_{4,0} + 3456 \hbar^6 f_{11}^2 f_{12} f_{22}^3 c_{0,4} c_{1,3} c_{4,0} + 1080 \hbar^6 f_{12}^6 c_{1,3}^2 c_{4,0} + \\
 & 7560 \hbar^6 f_{11} f_{12}^4 f_{22} c_{1,3}^2 c_{4,0} + 5346 \hbar^6 f_{11}^2 f_{12}^2 f_{22}^2 c_{1,3}^2 c_{4,0} + 270 \hbar^6 f_{11}^3 f_{22}^3 c_{1,3}^2 c_{4,0} + \\
 & 360 \hbar^5 f_{12}^5 c_{1,5} c_{4,0} + 1800 \hbar^5 f_{11} f_{12}^3 f_{22} c_{1,5} c_{4,0} + 540 \hbar^5 f_{11}^2 f_{12} f_{22}^2 c_{1,5} c_{4,0} + \\
 & 2016 \hbar^6 f_{12}^6 c_{0,4} c_{2,2} c_{4,0} + 15696 \hbar^6 f_{11} f_{12}^4 f_{22} c_{0,4} c_{2,2} c_{4,0} + 10368 \hbar^6 f_{11}^2 f_{12}^2 f_{22}^2 c_{0,4} c_{2,2} c_{4,0} + \\
 & 432 \hbar^6 f_{11}^3 f_{22}^3 c_{0,4} c_{2,2} c_{4,0} + 7200 \hbar^6 f_{11} f_{12}^5 c_{1,3} c_{2,2} c_{4,0} + 17208 \hbar^6 f_{11}^2 f_{12}^3 f_{22} c_{1,3} c_{2,2} c_{4,0} + \\
 & 4104 \hbar^6 f_{11}^3 f_{12} f_{22}^2 c_{1,3} c_{2,2} c_{4,0} + 6948 \hbar^6 f_{11}^4 f_{12}^2 c_{2,2}^2 c_{4,0} + 6912 \hbar^6 f_{11}^3 f_{12}^2 f_{22} c_{2,2}^2 c_{4,0} + \\
 & 396 \hbar^6 f_{11}^4 f_{22}^2 c_{2,2}^2 c_{4,0} + 1080 \hbar^5 f_{11} f_{12}^4 c_{2,4} c_{4,0} + 1512 \hbar^5 f_{11}^2 f_{12}^2 f_{22} c_{2,4} c_{4,0} + \\
 & 108 \hbar^5 f_{11}^3 f_{22}^2 c_{2,4} c_{4,0} + 7344 \hbar^6 f_{11} f_{12}^5 c_{0,4} c_{3,1} c_{4,0} + 17712 \hbar^6 f_{11}^2 f_{12}^3 f_{22} c_{0,4} c_{3,1} c_{4,0} + \\
 & 3456 \hbar^6 f_{11}^3 f_{12} f_{22}^2 c_{0,4} c_{3,1} c_{4,0} + 14364 \hbar^6 f_{11}^2 f_{12}^2 c_{1,3} c_{3,1} c_{4,0} + 13284 \hbar^6 f_{11}^3 f_{12}^2 f_{22} c_{1,3} c_{3,1} c_{4,0} + \\
 & 864 \hbar^6 f_{11}^4 f_{22}^2 c_{1,3} c_{3,1} c_{4,0} + 20808 \hbar^6 f_{11}^3 f_{12}^2 c_{2,2} c_{3,1} c_{4,0} + 7704 \hbar^6 f_{11}^4 f_{12} f_{22} c_{2,2} c_{3,1} c_{4,0} + \\
 & 12906 \hbar^6 f_{11}^4 f_{12}^2 c_{3,1}^2 c_{4,0} + 1350 \hbar^6 f_{11}^5 f_{22} c_{3,1}^2 c_{4,0} + 1836 \hbar^5 f_{11}^2 f_{12}^3 c_{3,3} c_{4,0} + \\
 & 864 \hbar^5 f_{11}^3 f_{12} f_{22} c_{3,3} c_{4,0} + 7344 \hbar^6 f_{11}^2 f_{12}^4 c_{0,4} c_{4,0}^2 + 6912 \hbar^6 f_{11}^3 f_{12}^2 f_{22} c_{0,4} c_{4,0}^2 + \\
 & 10800 \hbar^6 f_{11}^3 f_{12}^3 c_{1,3} c_{4,0}^2 + 3456 \hbar^6 f_{11}^4 f_{12} f_{22} c_{1,3} c_{4,0}^2 + 13104 \hbar^6 f_{11}^4 f_{12}^2 c_{2,2} c_{4,0}^2 + \\
 & 1152 \hbar^6 f_{11}^5 f_{22} c_{2,2} c_{4,0}^2 + 14256 \hbar^6 f_{11}^5 f_{12} c_{3,1} c_{4,0}^2 + 4752 \hbar^6 f_{11}^6 c_{4,0}^3 + 1080 \hbar^5 f_{12}^4 f_{22} c_{0,4} c_{4,2} + \\
 & 1512 \hbar^5 f_{11} f_{12}^2 f_{22}^2 c_{0,4} c_{4,2} + 108 \hbar^5 f_{11}^2 f_{22}^3 c_{0,4} c_{4,2} + 360 \hbar^5 f_{12}^5 c_{1,3} c_{4,2} + \\
 & 1692 \hbar^5 f_{11} f_{12}^3 f_{22} c_{1,3} c_{4,2} + 648 \hbar^5 f_{11}^2 f_{12} f_{22}^2 c_{1,3} c_{4,2} + 1008 \hbar^5 f_{11} f_{12}^4 c_{2,2} c_{4,2} + \\
 & 1566 \hbar^5 f_{11}^2 f_{12}^2 f_{22} c_{2,2} c_{4,2} + 126 \hbar^5 f_{11}^3 f_{22}^2 c_{2,2} c_{4,2} + 1782 \hbar^5 f_{11}^2 f_{12}^3 c_{3,1} c_{4,2} + \\
 & 918 \hbar^5 f_{11}^3 f_{12} f_{22} c_{3,1} c_{4,2} + 2412 \hbar^5 f_{11}^3 f_{12}^2 c_{4,0} c_{4,2} + 288 \hbar^5 f_{11}^4 f_{22} c_{4,0} c_{4,2} + \\
 & 72 \hbar^4 f_{12}^4 c_{4,4} + 216 \hbar^4 f_{11} f_{12}^2 f_{22} c_{4,4} + 27 \hbar^4 f_{11}^2 f_{22}^2 c_{4,4} + 360 \hbar^5 f_{12}^5 c_{0,4} c_{5,1} + \\
 & 1800 \hbar^5 f_{11} f_{12}^3 f_{22} c_{0,4} c_{5,1} + 540 \hbar^5 f_{11}^2 f_{12} f_{22}^2 c_{0,4} c_{5,1} + 1080 \hbar^5 f_{11} f_{12}^4 c_{1,3} c_{5,1} + \\
 & 1485 \hbar^5 f_{11}^2 f_{12}^2 f_{22} c_{1,3} c_{5,1} + 135 \hbar^5 f_{11}^3 f_{22}^2 c_{1,3} c_{5,1} + 1800 \hbar^5 f_{11}^2 f_{12}^3 c_{2,2} c_{5,1} + \\
 & 900 \hbar^5 f_{11}^3 f_{12} f_{22} c_{2,2} c_{5,1} + 2385 \hbar^5 f_{11}^3 f_{12}^2 c_{3,1} c_{5,1} + 315 \hbar^5 f_{11}^4 f_{22} c_{3,1} c_{5,1} + \\
 & 2700 \hbar^5 f_{11}^4 f_{12} c_{4,0} c_{5,1} + 180 \hbar^4 f_{11} f_{12}^3 c_{5,3} + 135 \hbar^4 f_{11}^2 f_{12} f_{22} c_{5,3} + 1080 \hbar^5 f_{11} f_{12}^4 c_{0,4} c_{6,0} + \\
 & 1620 \hbar^5 f_{11}^2 f_{12}^2 f_{22} c_{0,4} c_{6,0} + 1890 \hbar^5 f_{11}^2 f_{12}^3 c_{1,3} c_{6,0} + 810 \hbar^5 f_{11}^3 f_{12} f_{22} c_{1,3} c_{6,0} + \\
 & 2430 \hbar^5 f_{11}^3 f_{12}^2 c_{2,2} c_{6,0} + 270 \hbar^5 f_{11}^4 f_{22} c_{2,2} c_{6,0} + 2700 \hbar^5 f_{11}^4 f_{12} c_{3,1} c_{6,0} + 2700 \hbar^5 f_{11}^5 c_{4,0} c_{6,0} + \\
 & 270 \hbar^4 f_{11}^2 f_{12}^2 c_{6,2} + 45 \hbar^4 f_{11}^3 f_{22} c_{6,2} + 315 \hbar^4 f_{11}^3 f_{12} c_{7,1} + 315 \hbar^4 f_{11}^4 c_{8,0} ) ]
 \end{aligned}$$

```

(Alt) In[ ]:= {n = 1, p = 4, $k = 3, B = Table[xi, {i, n}], F = h Table[f_{i0,1}.Sort[{i,j}], {i, n}, {j, n}],
  P = GenericPerturbation[{$k}, B, c], Z = <F, E[1, 0, P]>_B}
ZZ = Zip3[Z][[3]]
lhs = Normal[Normal[Ev_p@Z] + O[epsilon]^{k+1}]
rhs = Normal[Series[e^{Sum[ZZ[[d+1]] e^d, {d,0,Length[ZZ]-1}], {h, 0, p}, {epsilon, 0, $k}]]]
HL@Simplify[lhs == rhs]

```

```
(Alt) Out[*]= {1, 4, 3, {x1}, {{h f11}}, Series[0, c4 x1^4, c6 x1^6, c8 x1^8],
  <{{h f11}}, E[1, 0, Series[0, c4 x1^4, c6 x1^6, c8 x1^8]]>_{x1}
```

```
(Alt) Out[*]= Series[0, 3 h^2 c4 f11^2, 15 h^3 c6 f11^3 + 48 h^4 c4^2 f11^4, 105 h^4 c8 f11^4 + 900 h^5 c4 c6 f11^5 + 1584 h^6 c4^3 f11^6]
```

```
(Alt) Out[*]= 1 + 3 E h^2 c4 f11^2 + 15 E^2 h^3 c6 f11^3 + h^4 (105/2 E^2 c4^2 f11^4 + 105 E^3 c8 f11^4)
```

```
(Alt) Out[*]= 1 + 3 E h^2 c4 f11^2 + 15 E^2 h^3 c6 f11^3 + h^4 (105/2 E^2 c4^2 f11^4 + 105 E^3 c8 f11^4)
```

```
(Alt) Out[*]= True
```

```
(Alt) In[*]= {n = 1, p = 5, $k = 4, B = Table[x1, {i, n}], F = h Table[f_{10,1}.Sort[{i,j}], {i, n}, {j, n}],
  P = GenericPerturbation[{$k}, B, c], Z = <F, E[1, 0, P]>_B}
```

```
ZZ = Zip3[Z] [[3]]
```

```
lhs = Normal[Normal[Ev_p@Z] + O[epsilon]^{k+1}]
```

```
rhs = Normal[Series[e^{Sum[ZZ[[d+1]] epsilon^d, {d,0,Length[ZZ]-1}], {h, 0, p}, {epsilon, 0, $k}]]]
```

```
HL@Simplify[lhs == rhs]
```

```
(Alt) Out[*]= {1, 5, 4, {x1}, {{h f11}}, Series[0, c4 x1^4, c6 x1^6, c8 x1^8, c10 x1^10],
  <{{h f11}}, E[1, 0, Series[0, c4 x1^4, c6 x1^6, c8 x1^8, c10 x1^10]]>_{x1}
```

```
(Alt) Out[*]= Series[0, 3 h^2 c4 f11^2, 15 h^3 c6 f11^3 + 48 h^4 c4^2 f11^4, 105 h^4 c8 f11^4 + 900 h^5 c4 c6 f11^5 + 1584 h^6 c4^3 f11^6,
  945 h^5 c10 f11^5 + 5085 h^6 c6^2 f11^6 + 10080 h^6 c4 c8 f11^6 + 64080 h^7 c4^2 c6 f11^7 + 78336 h^8 c4^4 f11^8]
```

```
(Alt) Out[*]= 1 + 3 E h^2 c4 f11^2 + 15 E^2 h^3 c6 f11^3 + h^4 (105/2 E^2 c4^2 f11^4 + 105 E^3 c8 f11^4) + h^5 (945 E^3 c4 c6 f11^5 + 945 E^4 c10 f11^5)
```

```
(Alt) Out[*]= 1 + 3 E h^2 c4 f11^2 + 15 E^2 h^3 c6 f11^3 + h^4 (105/2 E^2 c4^2 f11^4 + 105 E^3 c8 f11^4) + h^5 (945 E^3 c4 c6 f11^5 + 945 E^4 c10 f11^5)
```

```
(Alt) Out[*]= True
```