In[∘]:= $\{\eta, \xi, y, x\}^*$

Out[∘]= $\{y, x, \eta, \xi\}$

In[∘]:= $m_{i,j\to k}$

$lhs = m_{1,2\to 1}$ // $m_{1,3\to 1}$

$rhs = m_{2,3\to 2}$ // $m_{1,2\to 1}$

$lhs \equiv rhs$

Out[∘]= $\mathbb{E}_{\{i,j\}\to\{k\}}\left[1, y_k\left(\eta_i+\eta_j\right)-\eta_j\,\xi_i+x_k\left(\xi_i+\xi_j\right), \epsilon\text{Series}[0, 0]\right]$

Out[∘]= $\mathbb{E}_{\{1,2,3\}\to\{1\}}[1, y_1\,\eta_1+y_1\,\eta_2+y_1\,\eta_3+x_1\,\xi_1-\eta_2\,\xi_1-\eta_3\,\xi_1+x_1\,\xi_2-\eta_3\,\xi_2+x_1\,\xi_3, \epsilon\text{Series}[0, 0]]$

Out[∘]= $\mathbb{E}_{\{1,2,3\}\to\{1\}}[1, y_1\,\eta_1+y_1\,\eta_2+y_1\,\eta_3+x_1\,\xi_1-\eta_2\,\xi_1-\eta_3\,\xi_1+x_1\,\xi_2-\eta_3\,\xi_2+x_1\,\xi_3, \epsilon\text{Series}[0, 0]]$

Out[∘]= True

In[∘]:= $R_{1,2}\,\overline{R}_{3,4}$

Out[∘]= $$\mathbb{E}_{\{\}\to\{1,2,3,4\}}\Big[1, (-1+T)\,x_2\,(y_1-y_2)+\left(-1+\frac{1}{T}\right)x_4\,(y_3-y_4), \epsilon\text{Series}\Big[0, -\frac{1}{2}(1-T)\,x_2^2\,y_1^2+x_1\,x_2\,y_1\,y_2+\frac{1}{2}(1-3T)\,x_2^2\,y_1\,y_2-\frac{(-1+T)\,x_3\,x_4\,y_3^2}{T^2}-\frac{(1-T)\,x_4^2\,y_3^2}{2T^3}-\frac{x_3\,x_4\,y_3\,y_4}{T^2}-\frac{(-1-T)\,x_4^2\,y_3\,y_4}{2T^3}\Big]\Big]$$

In[∘]:= $\left(R_{1,2}\,\overline{R}_{3,4}\right)$ // $m_{1,3\to 1}$

Out[∘]= $$\mathbb{E}_{\{\}\to\{1,2,4\}}\Big[1, (-1+T)\,x_2\,y_1+\frac{(1-T)\,x_4\,y_1}{T}+(1-T)\,x_2\,y_2+\frac{(-1+T)\,x_4\,y_4}{T}, \epsilon\text{Series}\Big[0, \frac{1}{2}(-1+T)\,x_2^2\,y_1^2+\frac{(1-T)\,x_1\,x_4\,y_1^2}{T^2}+\frac{(-1+T)\,x_4^2\,y_1^2}{2T^3}+x_1\,x_2\,y_1\,y_2+\frac{1}{2}(1-3T)\,x_2^2\,y_1\,y_2+\frac{(-1+T)\,x_2\,x_4\,y_1\,y_2}{T}-\frac{x_1\,x_4\,y_1\,y_4}{T^2}+\frac{(1+T)\,x_4^2\,y_1\,y_4}{2T^3}\Big]\Big]$$

In[∘]:= {Z[0], Z[1], Z[2]} // Column

Out[∘]= $$\epsilon\text{Series}\Big[0, \frac{1}{2}(1-3T)\,x_2^2\,y_2\,(y_1+y_{\$[1]})+x_2\left(x_1-\frac{(1-T)\,x_4}{T}+x_{\$[1]}\right)y_2\,(y_1+y_{\$[1]})-\frac{1}{2}(1-T)\,x_2^2\,(y_1+y_{\$[1]})^2-\frac{(-1-T)\,x_4^2\,y_4\,(y_1+y_{\$[3]})}{2T^3}-\frac{x_4\,(x_1+x_{\$[3]})\,y_4\,(y_1+y_{\$[3]})}{T^2}-\frac{(1-T)\,x_4^2\,(y_1+y_{\$[3]})^2}{2T^3}-\frac{(-1+T)\,x_4\,(x_1+x_{\$[3]})\,(y_1+y_{\$[3]})^2}{T^2}\Big]$$

$\epsilon\text{Series}[0, 0]$

$\epsilon\text{Series}[0, 0]$

In[∘]:= $m_{2,4\to 2}$

Out[∘]= $\mathbb{E}_{\{2,4\}\to\{2\}}[1, y_2\,(\eta_2+\eta_4)-\eta_4\,\xi_2+x_2\,(\xi_2+\xi_4), \epsilon\text{Series}[0, 0]]$

In[∘]:= $R_{1,2}\,\overline{R}_{3,4}$ // $m_{1,3\to 1}$ // $m_{2,4\to 2}$ // CF

Out[∘]= $\mathbb{E}_{\{\}\to\{1,2\}}[1, 0, \epsilon\text{Series}[0, 0]]$

In[∘]:= **? Z**

Out[∘]=

Symbol

Global`Z

Definitions

$Z[0] = \epsilon\text{Series}\left[0, -\frac{1}{2}(1-T)\left(x_2 + \frac{(1-T)\,x_2}{T} + x_{\$[2]}\right)^2 y_1^2 - \frac{(-1+T)\,x_1\,(x_2+x_{\$[4]})\,y_1^2}{T^2} - \frac{(1-T)\,(x_2+x_{\$[4]})^2\,y_1^2}{2\,T^3} + \frac{1}{2}\,(1-3\,T\right.$

$Z[1] = \epsilon\text{Series}[0, 0]$

$Z[2] = \epsilon\text{Series}[0, 0]$

Z[rvk_RVK] := Module[{todo, n, rots, ζ, done, st, cx, ζ1, i, j, k, k1, k2, k3}, {todo, rots} = L

Z[K_] := Z[RVK[K]]

Full Name Global`Z

In[∘]:= **CF@Z[Knot[3, 1]]**

Out[∘]= $\mathbb{E}_{\{\}\to\{0\}}\left[\frac{T}{1-T+T^2}, 0, \epsilon\text{Series}\left[0, \frac{-2+3\,T-2\,T^2+T^3}{T-2\,T^2+3\,T^3-2\,T^4+T^5}\right]\right]$

In[∘]:= $\text{Factor}\left[\frac{-2+3\,T-2\,T^2+T^3}{T-2\,T^2+3\,T^3-2\,T^4+T^5}\right]$

Out[∘]= $\frac{(-1+T)\,(2-T+T^2)}{T\,(1-T+T^2)^2}$

In[∘]:= $\frac{T-2\,T^2+2\,T^3}{\sqrt{(1-3\,T+5\,T^2-4\,T^3+2\,T^4)^2}}$ // PowerExpand

In[∘]:= $\frac{T-2\,T^2+2\,T^3}{1-3\,T+5\,T^2-4\,T^3+2\,T^4}$ // Factor

Out[∘]= $\frac{T}{1-T+T^2}$