

Pensieve header: Time 6165.57, continues 201225-2. No CF in encapsulation.

In[*]:= **Date** []

Out[*]:= {2020, 12, 26, 7, 32, 56.5466739}

```
In[*]:= SetDirectory ["C:\\drorbn\\AcademicPensieve\\Projects\\BabyDoPeGDO"];
Once [ << KnotTheory` ];
Once [ Get @ "../Profile/Profile.m" ];
<< Objects.m
<< KT.m
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

Engine

```
In[*]:= CCF [  $\mathcal{E}$  ] := PPCCF @ ExpandDenominator @ ExpandNumerator @ Together [  $\mathcal{E}$  ];
(*CoefficientCanonical Form *)
CF [  $\mathcal{E}$  _List ] := CF /@  $\mathcal{E}$ ;
CF [  $\mathcal{E}$  _eSeries ] := CF /@  $\mathcal{E}$ ;
CF [  $\mathcal{E}$  ] := PPCF @ Module [
  { vs = Cases [  $\mathcal{E}$ , ( y | x |  $\eta$  |  $\xi$  )_,  $\infty$  ]  $\cup$  { y | x |  $\eta$  |  $\xi$  } },
  Total [ ( CCF [ # [2] ] ( Times @@ vs#[1] ) ) & /@ CoefficientRules [ Expand [  $\mathcal{E}$  ], vs ]
];
(*CF[ $\mathcal{E}$ ] := PPCF@CCF[ $\mathcal{E}$ ];*)
CF [  $\mathcal{E}$  _E ] := CF /@  $\mathcal{E}$ ;
CF [ Esp [  $\mathcal{E}$  _ ] ] := CF /@ Esp [  $\mathcal{E}$  ];
```

```
In[*]:= eSeries /: S1 _eSeries  $\equiv$  S2 _eSeries :=
  Length [ S1 ] == Length [ S2 ]  $\wedge$  Inner [ CF [ #1 ] == CF [ #2 ] &, S1, S2, And ];
eSeries [ 0 ] := eSeries @@ Table [ 0, { k, 1 } ];
eSeries /: S1 _eSeries + S2 _eSeries :=
  eSeries @@ Table [ S1 [ k ] + S2 [ k ], { k, Min [ Length @ S1, Length @ S2 ] };
eSeries /: S1 _eSeries * S2 _eSeries := eSeries @@
  Table [ Sum [ S1 [ j + 1 ] * S2 [ k - j + 1 ], { j, 0, k }, { k, 0, Min [ Length @ S1, Length @ S2 ] - 1 } ];
eSeries /: c _ * S _eSeries := ( c # ) & /@ S;
eSeries /:  $\partial_{v_s}$  _ S _eSeries := ( s  $\mapsto$   $\partial_{v_s}$  s ) /@ S;
```

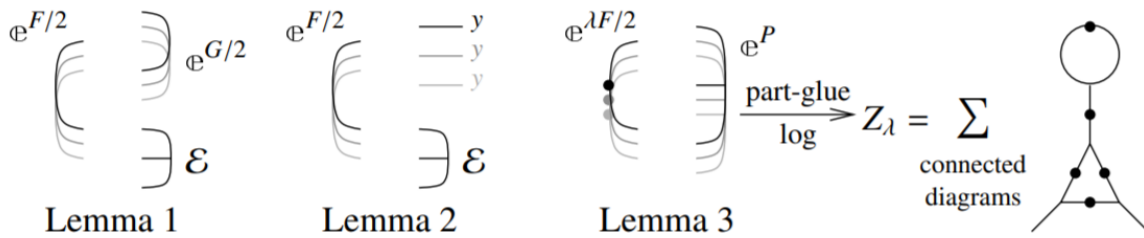
Variables and their duals:

```
In[*]:= { y*, x*,  $\eta$ *,  $\xi$ * } = {  $\eta$ ,  $\xi$ , y, x };
(vs _List)* := ( v  $\mapsto$  v* ) /@ vs;
(u _i)* := (u*)i;
```

E operations:

```
In[*]:=
E /: E[ω1_, Q1_, P1_] ≡ E[ω2_, Q2_, P2_] := CF[ω1 == ω2] ∧ CF[Q1 == Q2] ∧ (P1 ≡ P2);
E /: E[ω1_, Q1_, P1_] × E[ω2_, Q2_, P2_] := E[ω1 ω2, Q1 + Q2, P1 + P2];
Ed1→r1[E1s___] ≡ Ed2→r2[E2s___] ^:= (d1 == d2) ∧ (r1 == r2) ∧ (E[E1s] ≡ E[E2s]);
Ed1→r1[E1s___] Ed2→r2[E2s___] ^:= E(d1∪d2)→(r1∪r2) @@ (E[E1s] × E[E2s]);
Edr[Es___]$k := Edr @@ E[Es]$k;
```

```
In[*]:=
Ed1→r1[E1s___] // Ed2→r2[E2s___] := Module[{is = r1 ∩ d2, lvs},
  lvs = Flatten@Table[{x$ei, y$ei}, {i, is}];
  E(d1∪Complement[d2,is])→(r2∪Complement[r1,is]) @@ (Ziplvs∪lvs*[lvs*.lvs, Times[
    E[E1s] /. Table[(v : x | y)i → v$ei, {i, is}],
    E[E2s] /. Table[(v : ξ | η)i → v$ei, {i, is}]
  ]])
]
```



```
In[*]:=
Zipvs[F_, E_] := ⟨F, E⟩ // Zip1vs // Zip2vs // Zip3vs;
Zipvs[F_, E_] := ⟨F, E⟩ // Zip1vs // EZip23vs;
```

Getting rid of the quadratic.

Lemma 1. With convergences left to the reader,

$$\left\langle F : \mathcal{E} \otimes \mathbb{E}^{\frac{1}{2} \sum_{i,j \in B} G_{ij} z_i z_j} \right\rangle_B = \det(1 - GF)^{-1/2} \left\langle F(1 - GF)^{-1} : \mathcal{E} \right\rangle_B$$

```
In[*]:=
Zip1_{ } = Identity;
Zip1vs @ ⟨F_, E[ω_, Q_, P_]⟩ := PPZip1 @ Module[{I, F, G, u, v},
  I = IdentityMatrix@Length@vs;
  F = Table[∂u,v F, {u, vs*}, {v, vs*}];
  G = Table[∂u,v Q, {u, vs}, {v, vs}];
  ⟨CF[vs*.F.Inverse[I - G.F].vs* / 2],
  E[CF@PowerExpand@Factor[ω Det[I - G.F]^{-1/2}, CF[Q - vs.G.vs / 2], P]]
]
```

Getting rid of linear terms.

Lemma 2. $\left\langle F : \mathcal{E} \otimes \mathbb{E}^{\sum_{i \in B} y_i z_i} \right\rangle_B = \mathbb{E}^{\frac{1}{2} \sum_{i,j \in B} F_{ij} y_i y_j} \left\langle F : \mathcal{E} \Big|_{z_B \rightarrow z_B + F y_B} \right\rangle_B$.

```

In[ ]:= Zip2_{ } = Identity;
Zip2_{vs_} @ < F_, E[ω_, Q_, P_] > := PPZip2 @ Module[{F, Y, u, v},
  F = Table[∂_{u,v} F, {u, vs*}, {v, vs*}];
  Y = Table[∂_v Q, {v, vs}];
  CF /@ < F_, E[ω, Q - Y.v + Y.F.Y / 2, P /. Thread[v → vs + F.Y]] >
]

```

Dealing with Feynman diagrams.

Lemma 3. With an extra variable λ , $Z_\lambda := \log[\lambda F: \mathbb{C}^P]_B$ satisfies and is determined by the following PDE / IVP:

$$Z_0 = P \quad \text{and} \quad \partial_\lambda Z_\lambda = \frac{1}{2} \sum_{i,j \in B} F_{ij} (\partial_{z_i} \partial_{z_j} Z_\lambda + (\partial_{z_i} Z_\lambda)(\partial_{z_j} Z_\lambda)).$$

Note that the power m of λ is at most $k - 1 + \frac{2k+2}{2} = 2k$. We write $Z_\lambda = \sum Z[m] \lambda^m$.

```

In[ ]:= Zip3_{vs_} @ < F_, E[ω_, Q_, P_] > := PPZip3 @ Module[{F, Z, u, v, m, j},
  F[u_, v_] := F[u, v] = ∂_{u*,v*} F;
  Z[j_, v_] := Z[j, v] = ∂_v Z[j];
  Z[0] = P;
  For[m = 0, m < 2 $k, ++m,
    Z[m + 1] = CF [
      1
      / (2 (m + 1))
      Sum[F[u, v] (∂_{u,v} Z[m] + Sum[Z[j, u] * Z[m - j, v], {j, 0, m}]), {u, vs}, {v, vs}]
    ];
  E[ω, Q, CF[Sum[Z[m], {m, 0, 2 $k}]] /. Table[v → 0, {v, vs}]]
]

```

```

In[ ]:= EZip23_{vs_} @ < F_, E[ω_, Q_, P_] > := PPEZip23 @ Module [
  {nP, nF, nQ, j = 0, ps, c, t, rr = {(*release rules*)}},
  nP = Total [
    CoefficientRules[#, vs] /.
    (ps_ → c_) ⇒ (AppendTo[rr, t[++j] → c]; t[j] (Times @@ vs^{ps}))
  ] & /@ P;
  nQ = Total [CoefficientRules[Q, vs] /.
    (ps_ → c_) ⇒ (AppendTo[rr, t[++j] → c]; t[j] (Times @@ vs^{ps}))];
  nF = Total [CoefficientRules[F, vs*] /. (ps_ → c_) ⇒
    (AppendTo[rr, t[++j] → c]; t[j] (Times @@ (vs*)^{ps}))];
  CF [Expand[<nF, E[ω, nQ, nP]> // Zip2_{vs_} // Zip3_{vs_}] /. rr]
]

```

Profile

```
In[ ]:= BeginProfile[];
        PopupWindow[Button["Show Profile Monitor"],
                    Dynamic[PrintProfile[], UpdateInterval -> 3, TrackedSymbols -> {}]]
```

Out[]:=

\$k = 1

```
In[ ]:= NewBit[K_] := Module[{Alex = Alexander[K][T]},
    T^3  $\frac{\text{Alex}^2}{T-1}$  Z[K][[3, 2]] // Factor]
```

```
In[ ]:= $k = 1; NewBit /@ AllKnots[{3, 5}]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[]:= $\left\{ 2 - T + T^2, (1 + T) (1 - 3 T + T^2), \frac{4 - 3 T + 5 T^2 - 3 T^3 + 3 T^4 - T^5 + T^6}{T^2}, 9 - 11 T + 7 T^2 - T^3 \right\}$

```
In[ ]:= (*Two knots with equal Alexander, new bit does not agree*)
        Alexander[Knot[6, 1]] == Alexander[Knot[9, 46]]
        $k = 1; Timing[NewBit[Knot[6, 1]] == NewBit[Knot[9, 46]]]
```

Out[]:= True

Out[]:= $\{ 17.3125, 5 - 11 T - T^2 + 3 T^3 == 7 - 21 T + 9 T^2 + T^3 \}$

```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 34.674
  ( 24) 0/ 0.031 above CF
  ( 237) 1.485/ 32.160 above EZip23
  ( 237) 0.945/ 2.483 above Zip1
Zip3: called 237 times, time in 16.223/17.606
  ( 237) 16.220/ 17.610 under EZip23
  ( 1422) 0.548/ 1.383 above CF
CCF: called 10873 times, time in 8.048/8.048
  ( 10873) 8.048/ 8.048 under CF
CF: called 4290 times, time in 7.381/15.429
  ( 948) 5.172/ 9.311 under EZip23
  ( 24) 0/ 0.031 under ProfileRoot
  ( 711) 0.660/ 1.538 under Zip1
  ( 1185) 1.001/ 3.166 under Zip2
  ( 1422) 0.548/ 1.383 under Zip3
  ( 10873) 8.048/ 8.048 above CCF
EZip23: called 237 times, time in 1.485/32.16
  ( 237) 1.485/ 32.160 under ProfileRoot
  ( 948) 5.172/ 9.311 above CF
  ( 237) 0.592/ 3.758 above Zip2
  ( 237) 16.220/ 17.610 above Zip3
Zip1: called 237 times, time in 0.945/2.483
  ( 237) 0.945/ 2.483 under ProfileRoot
  ( 711) 0.660/ 1.538 above CF
Zip2: called 237 times, time in 0.592/3.758
  ( 237) 0.592/ 3.758 under EZip23
  ( 1185) 1.001/ 3.166 above CF
```

```
In[ ]:= $k = 1; equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};  
Length@Union[Z /@ equiv]
```

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

```
Out[ ]:= 1
```

```
In[ ]:= $k = 1; equiv =  
{Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};  
Length@Union[Z /@ equiv]
```

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

```
Out[ ]:= 1
```

```
In[ ]:= PrintProfile[]
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 174.319
( 44) 0.016/ 0.063 above CF
( 652) 9.034/ 167.210 above EZip23
( 652) 2.523/ 7.049 above Zip1
CF: called 11780 times, time in 64.643/119.859
( 2608) 58.506/ 103.420 under EZip23
( 44) 0.016/ 0.063 under ProfileRoot
( 1956) 2.176/ 4.526 under Zip1
( 3260) 2.381/ 8.248 under Zip2
( 3912) 1.564/ 3.604 under Zip3
( 32286) 55.216/ 55.216 above CCF
CCF: called 32286 times, time in 55.216/55.216
( 32286) 55.216/ 55.216 under CF
Zip3: called 652 times, time in 41.248/44.852
( 652) 41.248/ 44.852 under EZip23
( 3912) 1.564/ 3.604 above CF
EZip23: called 652 times, time in 9.034/167.207
( 652) 9.034/ 167.210 under ProfileRoot
( 2608) 58.506/ 103.420 above CF
( 652) 1.655/ 9.903 above Zip2
( 652) 41.248/ 44.852 above Zip3
Zip1: called 652 times, time in 2.523/7.049
( 652) 2.523/ 7.049 under ProfileRoot
( 1956) 2.176/ 4.526 above CF
Zip2: called 652 times, time in 1.655/9.903
( 652) 1.655/ 9.903 under EZip23
( 3260) 2.381/ 8.248 above CF
```

$\$k = 2$

```
In[ ]:= $k = 2; equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};
Length@Union[Z /@ equiv]
```

```
Out[ ]:= 2
```

```
In[ ]:= PrintProfile [ ]
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 1409.44
( 54) 0.047/ 0.126 above CF
( 813) 33.712/ 1400.210 above EZip23
( 813) 3.279/ 9.112 above Zip1
CF: called 16459 times, time in 770.443/1291.79
( 3413) 753.173/ 1202.300 under EZip23
( 54) 0.047/ 0.126 under ProfileRoot
( 2439) 2.872/ 5.833 under Zip1
( 4226) 5.091/ 22.044 under Zip2
( 6327) 9.260/ 61.484 under Zip3
( 52601) 521.347/ 521.347 above CCF
CCF: called 52601 times, time in 521.347/521.347
( 52601) 521.347/ 521.347 under CF
Zip3: called 813 times, time in 78.646/140.13
( 813) 78.646/ 140.130 under EZip23
( 6327) 9.260/ 61.484 above CF
EZip23: called 813 times, time in 33.712/1400.21
( 813) 33.712/ 1400.210 under ProfileRoot
( 3413) 753.173/ 1202.300 above CF
( 813) 2.017/ 24.061 above Zip2
( 813) 78.646/ 140.130 above Zip3
Zip1: called 813 times, time in 3.279/9.112
( 813) 3.279/ 9.112 under ProfileRoot
( 2439) 2.872/ 5.833 above CF
Zip2: called 813 times, time in 2.017/24.061
( 813) 2.017/ 24.061 under EZip23
( 4226) 5.091/ 22.044 above CF
```

```
In[ ]:= $k = 2; equiv =
{Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};
Length@Union[Z /@ equiv]
```

```
Out[ ]:= 3
```

```
In[ ]:= PrintProfile [ ]
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 4561.99
( 69) 0.063/ 0.158 above CF
( 1071) 87.811/ 4550.251 above EZip23
( 1071) 4.201/ 11.585 above Zip1
CF: called 23956 times, time in 2624.06/4327.56
( 4703) 2586.867/ 4099.847 under EZip23
( 69) 0.063/ 0.158 under ProfileRoot
( 3213) 3.705/ 7.384 under Zip1
( 5774) 9.552/ 47.168 under Zip2
( 10197) 23.872/ 173.001 under Zip3
( 87339) 1703.499/ 1703.499 above CCF
CCF: called 87339 times, time in 1703.5/1703.5
( 87339) 1703.499/ 1703.499 under CF
Zip3: called 1071 times, time in 139.818/312.819
( 1071) 139.818/ 312.819 under EZip23
( 10197) 23.872/ 173.001 above CF
EZip23: called 1071 times, time in 87.811/4550.25
( 1071) 87.811/ 4550.251 under ProfileRoot
( 4703) 2586.867/ 4099.847 above CF
( 1071) 2.606/ 49.774 above Zip2
( 1071) 139.818/ 312.819 above Zip3
Zip1: called 1071 times, time in 4.201/11.585
( 1071) 4.201/ 11.585 under ProfileRoot
( 3213) 3.705/ 7.384 above CF
Zip2: called 1071 times, time in 2.606/49.774
( 1071) 2.606/ 49.774 under EZip23
( 5774) 9.552/ 47.168 above CF
```

```
In[ ]:= Date [ ]
```

```
Out[ ]:= {2020, 12, 26, 8, 51, 50.1910008}
```

```
In[ ]:= $k = 2; equiv = {Knot[12, NonAlternating, 60],  

    Knot[12, NonAlternating, 61], Knot[12, NonAlternating, 219]};  

Length@Union[Z /@equiv]
```

```
Out[ ]:= 1
```



```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 6165.57
( 84) 0.093/ 0.235 above CF
( 1329) 129.547/ 6151.049 above EZip23
( 1329) 5.298/ 14.286 above Zip1
CF: called 31453 times, time in 3481.66/5831.09
( 5993) 3427.491/ 5490.177 under EZip23
( 84) 0.093/ 0.235 under ProfileRoot
( 3987) 4.548/ 8.988 under Zip1
( 7322) 13.601/ 68.950 under Zip2
( 14067) 35.927/ 262.735 under Zip3
( 117202) 2349.425/ 2349.425 above CCF
CCF: called 117202 times, time in 2349.42/2349.42
( 117202) 2349.425/ 2349.425 under CF
Zip3: called 1329 times, time in 196.517/459.252
( 1329) 196.517/ 459.252 under EZip23
( 14067) 35.927/ 262.735 above CF
EZip23: called 1329 times, time in 129.547/6151.05
( 1329) 129.547/ 6151.049 under ProfileRoot
( 5993) 3427.491/ 5490.177 above CF
( 1329) 3.123/ 72.073 above Zip2
( 1329) 196.517/ 459.252 above Zip3
Zip1: called 1329 times, time in 5.298/14.286
( 1329) 5.298/ 14.286 under ProfileRoot
( 3987) 4.548/ 8.988 above CF
Zip2: called 1329 times, time in 3.123/72.073
( 1329) 3.123/ 72.073 under EZip23
( 7322) 13.601/ 68.950 above CF
```

```
In[ ]:= Date []
```

```
Out[ ]:= {2020, 12, 26, 9, 19, 27.0080616}
```