

Pensieve header: Time 6036.26, continues 201226. No Expand in CF.

In[\*]:= **Date** []

Out[\*]:= {2020, 12, 26, 9, 47, 25.6076768}

```
In[*]:= SetDirectory ["C:\\drorbn\\AcademicPensieve\\Projects\\BabyDoPeGDO"];
Once [<< KnotTheory`];
Once [Get@"../Profile/Profile.m"];
<< Objects.m
<< KT.m
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

## Engine

```
In[*]:= CCF [ $\mathcal{E}$ ] := PPCCF@ExpandDenominator@ExpandNumerator@Together [ $\mathcal{E}$ ];
(*CoefficientCanonical Form *)
CF [ $\mathcal{E}$ _List] := CF /@  $\mathcal{E}$ ;
CF [ $\mathcal{E}$ _eSeries] := CF /@  $\mathcal{E}$ ;
CF [ $\mathcal{E}$ ] := PPCF@Module [
  {vs = Cases [ $\mathcal{E}$ , (y | x |  $\eta$  |  $\xi$ )_,  $\infty$ ]  $\cup$  {y | x |  $\eta$  |  $\xi$ }},
  Total [(CCF [#[[2]]] (Times @@ vs#[[1]])) & /@ CoefficientRules [ $\mathcal{E}$ , vs]]
];
(*CF[ $\mathcal{E}$ ] := PPCF@CCF[ $\mathcal{E}$ ];*)
CF [ $\mathcal{E}$ _E] := CF /@  $\mathcal{E}$ ;
CF [Esp___ [ $\mathcal{E}$ S___]] := CF /@ Esp [ $\mathcal{E}$ S];
```

```
eSeries /: S1_eSeries  $\equiv$  S2_eSeries :=
  Length[S1] == Length[S2]  $\wedge$  Inner[CF [#1] == CF [#2] &, S1, S2, And];
eSeries[0] /; $k > 0 := eSeries @@ Table[0, $k + 1];
eSeries /: S1_eSeries + S2_eSeries :=
  eSeries @@ Table[S1[[k]] + S2[[k]], {k, Min[Length@S1, Length@S2]}];
eSeries /: S1_eSeries * S2_eSeries := eSeries @@
  Table[Sum[S1[[j + 1]] * S2[[k - j + 1]], {j, 0, k}], {k, 0, Min[Length@S1, Length@S2] - 1}];
eSeries /: c_ * S_eSeries := (c #) & /@ S;
eSeries /:  $\partial_{v_s}$ ___ S_eSeries := (s  $\mapsto$   $\partial_{v_s}$  s) /@ S;
```

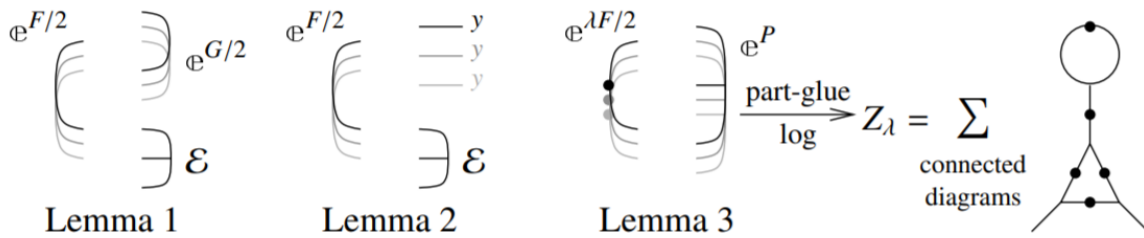
Variables and their duals:

```
In[*]:= {y*, x*,  $\eta$ *,  $\xi$ *} = { $\eta$ ,  $\xi$ , y, x};
(vs_List)* := (v  $\mapsto$  v*) /@ vs;
(u_i)* := (u*)i;
```

E operations:

```
In[*]:=
E /: E[\omega1_, Q1_, P1_] == E[\omega2_, Q2_, P2_] := CF[\omega1 == \omega2] ^ CF[Q1 == Q2] ^ (P1 == P2);
E /: E[\omega1_, Q1_, P1_] E[\omega2_, Q2_, P2_] := E[\omega1 \omega2, Q1 + Q2, P1 + P2];
E_{d1 -> r1}[\mathcal{E}1s] == E_{d2 -> r2}[\mathcal{E}2s] ^:= (d1 == d2) ^ (r1 == r2) ^ (E[\mathcal{E}1s] == E[\mathcal{E}2s]);
E_{d1 -> r1}[\mathcal{E}1s] E_{d2 -> r2}[\mathcal{E}2s] ^:= E_{(d1 \cup d2) -> (r1 \cup r2)} @ @ (E[\mathcal{E}1s] E[\mathcal{E}2s]);
E_{dr}[\mathcal{E}S]_{\$k} := E_{dr} @ @ E[\mathcal{E}S]_{\$k};
```

```
In[*]:=
E_{d1 -> r1}[\mathcal{E}1s] // E_{d2 -> r2}[\mathcal{E}2s] := Module[{is = r1 \cap d2, lvs},
  lvs = Flatten@Table[{x_{\$ei}, y_{\$ei}}, {i, is}];
  E_{(d1 \cup Complement[d2, is]) -> (r2 \cup Complement[r1, is])} @ @ (Zip_{lvs \cup lvs} [lvs*.lvs, Times[
    E[\mathcal{E}1s] /. Table[(v : x | y)_i -> v_{\$ei}, {i, is}],
    E[\mathcal{E}2s] /. Table[(v : \xi | \eta)_i -> v_{\$ei}, {i, is}]
  ]])
]
```



```
In[*]:=
Zip_{vs}[\mathcal{F}_-, \mathcal{E}_-] := \langle \mathcal{F}_-, \mathcal{E}_- \rangle // Zip1_{vs} // Zip2_{vs} // Zip3_{vs};
Zip_{vs}[\mathcal{F}_-, \mathcal{E}_-] := \langle \mathcal{F}_-, \mathcal{E}_- \rangle // Zip1_{vs} // EZip23_{vs};
```

Getting rid of the quadratic.

**Lemma 1.** With convergences left to the reader,

$$\left\langle F : \mathcal{E} \mathbb{E}^{\frac{1}{2} \sum_{i,j \in B} G_{ij} z_i z_j} \right\rangle_B = \det(1 - GF)^{-1/2} \left\langle F(1 - GF)^{-1} : \mathcal{E} \right\rangle_B$$

```
In[*]:=
Zip1_{\{}} = Identity;
Zip1_{vs}[\langle \mathcal{F}_-, E[\omega_-, Q_-, P_-] \rangle] := PP_{Zip1} @ Module[{I, F, G, u, v},
  I = IdentityMatrix@Length@vs;
  F = Table[\partial_{u,v} \mathcal{F}, {u, vs*}, {v, vs*}];
  G = Table[\partial_{u,v} Q, {u, vs}, {v, vs}];
  \langle CF[vs*.F.Inverse[I - G.F].vs* / 2],
  E[CF @ PowerExpand @ Factor[\omega Det[I - G.F]^{-1/2}, CF[Q - vs.G.vs / 2], P]]
]
```

Getting rid of linear terms.

**Lemma 2.**  $\left\langle F : \mathcal{E} \mathbb{E}^{\sum_{i \in B} y_i z_i} \right\rangle_B = \mathbb{E}^{\frac{1}{2} \sum_{i,j \in B} F_{ij} y_i y_j} \left\langle F : \mathcal{E} |_{z_B \rightarrow z_B + F y_B} \right\rangle_B$ .

```

In[ ]:= Zip2_{ } = Identity;
Zip2_{vs_} @ <F_, E[ω_, Q_, P_] > := PPZip2@Module[{F, Y, u, v},
  F = Table[∂_{u,v} F, {u, vs*}, {v, vs*}];
  Y = Table[∂_v Q, {v, vs}];
  CF /@ <F_, E[ω, Q - Y.v + Y.F.Y / 2, P /. Thread[v → vs + F.Y]] >
]

```

Dealing with Feynman diagrams.

**Lemma 3.** With an extra variable  $\lambda$ ,  $Z_\lambda := \log[\lambda F: \mathbb{C}^P]_B$  satisfies and is determined by the following PDE / IVP:

$$Z_0 = P \quad \text{and} \quad \partial_\lambda Z_\lambda = \frac{1}{2} \sum_{i,j \in B} F_{ij} (\partial_{z_i} \partial_{z_j} Z_\lambda + (\partial_{z_i} Z_\lambda)(\partial_{z_j} Z_\lambda)).$$

Note that the power  $m$  of  $\lambda$  is at most  $k - 1 + \frac{2k+2}{2} = 2k$ . We write  $Z_\lambda = \sum Z[m] \lambda^m$ .

```

In[ ]:= Zip3_{vs_} @ <F_, E[ω_, Q_, P_] > := PPZip3@Module[{F, Z, u, v, m, j},
  F[u_, v_] := F[u, v] = ∂_{u*,v*} F;
  Z[j_, v_] := Z[j, v] = ∂_v Z[j];
  Z[0] = P;
  For[m = 0, m < 2 $k, ++m,
    Z[m + 1] = CF[
      1 / (2 (m + 1))
      Sum[F[u, v] (∂_{u,v} Z[m] + Sum[Z[j, u] * Z[m - j, v], {j, 0, m}]), {u, vs}, {v, vs}]
    ];
  E[ω, Q, CF[Sum[Z[m], {m, 0, 2 $k}]] /. Table[v → 0, {v, vs}]]
]

```

```

In[ ]:= EZip23_{vs_} @ <F_, E[ω_, Q_, P_] > := PPEZip23@Module[
  {nP, nF, nQ, j = 0, ps, c, t, rr = {(*release rules*)}},
  nP = Total[
    CoefficientRules[#, vs] /.
    (ps_ → c_) => (AppendTo[rr, t[++] → c]; t[j] (Times @@ vs^ps))
  ] & /@ P;
  nQ = Total[CoefficientRules[Q, vs] /.
    (ps_ → c_) => (AppendTo[rr, t[++] → c]; t[j] (Times @@ vs^ps))];
  nF = Total[CoefficientRules[F, vs*] /. (ps_ → c_) =>
    (AppendTo[rr, t[++] → c]; t[j] (Times @@ (vs*)^ps))];
  CF[Expand[<nF, E[ω, nQ, nP] > // Zip2_{vs_} // Zip3_{vs_}] /. rr]
]

```

## Profile

```
In[ ]:= BeginProfile[];
        PopupWindow[Button["Show Profile Monitor"],
                    Dynamic[PrintProfile[], UpdateInterval -> 3, TrackedSymbols -> {}]]
```

Out[ ]:=

## \$k = 1

```
In[ ]:= NewBit[K_] := Module[{Alex = Alexander[K][T]},
    T^3  $\frac{\text{Alex}^2}{T-1}$  Z[K][[3, 2]] // Factor]
```

```
In[ ]:= $k = 1; NewBit /@ AllKnots[{3, 5}]
```

**KnotTheory:** Loading precomputed data in PD4Knots`.

Out[ ]:=  $\left\{ 2 - T + T^2, (1 + T) (1 - 3 T + T^2), \frac{4 - 3 T + 5 T^2 - 3 T^3 + 3 T^4 - T^5 + T^6}{T^2}, 9 - 11 T + 7 T^2 - T^3 \right\}$

```
In[ ]:= (*Two knots with equal Alexander, new bit does not agree*)
        Alexander[Knot[6, 1]] == Alexander[Knot[9, 46]]
        $k = 1; Timing[NewBit[Knot[6, 1]] == NewBit[Knot[9, 46]]]
```

Out[ ]:= True

Out[ ]:=  $\{16.3906, 5 - 11 T - T^2 + 3 T^3 == 7 - 21 T + 9 T^2 + T^3\}$

```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 34.218
  ( 24) 0.015/ 0.031 above CF
  ( 237) 1.655/ 31.610 above EZip23
  ( 237) 1.056/ 2.573 above Zip1
Zip3: called 237 times, time in 16.575/17.911
  ( 237) 16.580/ 17.910 under EZip23
  ( 1422) 0.603/ 1.336 above CF
CCF: called 10873 times, time in 7.686/7.686
  ( 10873) 7.686/ 7.686 under CF
CF: called 4290 times, time in 6.604/14.29
  ( 948) 4.405/ 8.553 under EZip23
  ( 24) 0.015/ 0.031 under ProfileRoot
  ( 711) 0.855/ 1.517 under Zip1
  ( 1185) 0.726/ 2.853 under Zip2
  ( 1422) 0.603/ 1.336 under Zip3
  ( 10873) 7.686/ 7.686 above CCF
EZip23: called 237 times, time in 1.655/31.614
  ( 237) 1.655/ 31.610 under ProfileRoot
  ( 948) 4.405/ 8.553 above CF
  ( 237) 0.642/ 3.495 above Zip2
  ( 237) 16.580/ 17.910 above Zip3
Zip1: called 237 times, time in 1.056/2.573
  ( 237) 1.056/ 2.573 under ProfileRoot
  ( 711) 0.855/ 1.517 above CF
Zip2: called 237 times, time in 0.642/3.495
  ( 237) 0.642/ 3.495 under EZip23
  ( 1185) 0.726/ 2.853 above CF
```

```
In[ ]:= $k = 1; equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};  
Length@Union[Z /@equiv]
```

**KnotTheory**: Loading precomputed data in KnotTheory/12N.dts.

**KnotTheory**: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

```
Out[ ]:= 1
```

```
In[ ]:= $k = 1; equiv =  
{Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};  
Length@Union[Z /@equiv]
```

**KnotTheory**: Loading precomputed data in KnotTheory/12A.dts.

```
Out[ ]:= 1
```

```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 168.653
( 44) 0.015/ 0.063 above CF
( 652) 8.979/ 161.430 above EZip23
( 652) 2.882/ 7.157 above Zip1
CF: called 11780 times, time in 59.009/113.314
( 2608) 53.087/ 97.580 under EZip23
( 44) 0.015/ 0.063 under ProfileRoot
( 1956) 2.113/ 4.275 under Zip1
( 3260) 2.291/ 7.787 under Zip2
( 3912) 1.503/ 3.609 under Zip3
( 32286) 54.305/ 54.305 above CCF
CCF: called 32286 times, time in 54.305/54.305
( 32286) 54.305/ 54.305 under CF
Zip3: called 652 times, time in 41.941/45.55
( 652) 41.941/ 45.550 under EZip23
( 3912) 1.503/ 3.609 above CF
EZip23: called 652 times, time in 8.979/161.433
( 652) 8.979/ 161.430 under ProfileRoot
( 2608) 53.087/ 97.580 above CF
( 652) 1.537/ 9.324 above Zip2
( 652) 41.941/ 45.550 above Zip3
Zip1: called 652 times, time in 2.882/7.157
( 652) 2.882/ 7.157 under ProfileRoot
( 1956) 2.113/ 4.275 above CF
Zip2: called 652 times, time in 1.537/9.324
( 652) 1.537/ 9.324 under EZip23
( 3260) 2.291/ 7.787 above CF
```

**\$k = 2**

```
In[ ]:= $k = 2; equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};  
Length@Union[Z /@ equiv]
```

```
Out[ ]:= 2
```

```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 1309.36
( 54) 0.046/ 0.125 above CF
( 813) 31.967/ 1300.340 above EZip23
( 813) 3.441/ 8.895 above Zip1
CF: called 16459 times, time in 706.715/1195.78
( 3413) 690.727/ 1112.820 under EZip23
( 54) 0.046/ 0.125 under ProfileRoot
( 2439) 2.711/ 5.454 under Zip1
( 4226) 4.619/ 21.240 under Zip2
( 6327) 8.612/ 56.150 under Zip3
( 52601) 489.069/ 489.069 above CCF
CCF: called 52601 times, time in 489.069/489.069
( 52601) 489.069/ 489.069 under CF
Zip3: called 813 times, time in 76.427/132.577
( 813) 76.427/ 132.577 under EZip23
( 6327) 8.612/ 56.150 above CF
EZip23: called 813 times, time in 31.967/1300.34
( 813) 31.967/ 1300.340 under ProfileRoot
( 3413) 690.727/ 1112.820 above CF
( 813) 1.740/ 22.980 above Zip2
( 813) 76.427/ 132.577 above Zip3
Zip1: called 813 times, time in 3.441/8.895
( 813) 3.441/ 8.895 under ProfileRoot
( 2439) 2.711/ 5.454 above CF
Zip2: called 813 times, time in 1.74/22.98
( 813) 1.740/ 22.980 under EZip23
( 4226) 4.619/ 21.240 above CF
```

```
In[ ]:= $k = 2; equiv =
{Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};
Length@Union[Z /@ equiv]
```

```
Out[ ]:= 3
```

```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 4461.51
( 69) 0.062/ 0.173 above CF
( 1071) 86.556/ 4449.912 above EZip23
( 1071) 4.336/ 11.428 above Zip1
CF: called 23956 times, time in 2545.08/4229.75
( 4703) 2509.595/ 4006.888 under EZip23
( 69) 0.062/ 0.173 under ProfileRoot
( 3213) 3.441/ 7.092 under Zip1
( 5774) 9.061/ 46.875 under Zip2
( 10197) 22.919/ 168.722 under Zip3
( 87339) 1684.672/ 1684.672 above CCF
CCF: called 87339 times, time in 1684.67/1684.67
( 87339) 1684.672/ 1684.672 under CF
Zip3: called 1071 times, time in 138.574/307.296
( 1071) 138.574/ 307.296 under EZip23
( 10197) 22.919/ 168.722 above CF
EZip23: called 1071 times, time in 86.556/4449.91
( 1071) 86.556/ 4449.912 under ProfileRoot
( 4703) 2509.595/ 4006.888 above CF
( 1071) 2.297/ 49.172 above Zip2
( 1071) 138.574/ 307.296 above Zip3
Zip1: called 1071 times, time in 4.336/11.428
( 1071) 4.336/ 11.428 under ProfileRoot
( 3213) 3.441/ 7.092 above CF
Zip2: called 1071 times, time in 2.297/49.172
( 1071) 2.297/ 49.172 under EZip23
( 5774) 9.061/ 46.875 above CF
```

```
In[ ]:= Date []
```

```
Out[ ]:= {2020, 12, 26, 11, 4, 39.1598921}
```

```
In[ ]:= $k = 2; equiv = {Knot[12, NonAlternating, 60],  

    Knot[12, NonAlternating, 61], Knot[12, NonAlternating, 219]};  

Length@Union[Z /@ equiv]
```

```
Out[ ]:= 1
```



```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 6036.26
( 84) 0.107/ 0.250 above CF
( 1329) 128.137/ 6021.620 above EZip23
( 1329) 5.501/ 14.390 above Zip1
CF: called 31453 times, time in 3366.78/5701.19
( 5993) 3314.122/ 5364.841 under EZip23
( 84) 0.107/ 0.250 under ProfileRoot
( 3987) 4.208/ 8.889 under Zip1
( 7322) 13.164/ 68.822 under Zip2
( 14067) 35.181/ 258.384 under Zip3
( 117202) 2334.404/ 2334.404 above CCF
CCF: called 117202 times, time in 2334.4/2334.4
( 117202) 2334.404/ 2334.404 under CF
Zip3: called 1329 times, time in 198.733/457.117
( 1329) 198.733/ 457.117 under EZip23
( 14067) 35.181/ 258.384 above CF
EZip23: called 1329 times, time in 128.137/6021.62
( 1329) 128.137/ 6021.620 under ProfileRoot
( 5993) 3314.122/ 5364.841 above CF
( 1329) 2.703/ 71.525 above Zip2
( 1329) 198.733/ 457.117 above Zip3
Zip1: called 1329 times, time in 5.501/14.39
( 1329) 5.501/ 14.390 under ProfileRoot
( 3987) 4.208/ 8.889 above CF
Zip2: called 1329 times, time in 2.703/71.525
( 1329) 2.703/ 71.525 under EZip23
( 7322) 13.164/ 68.822 above CF
```

```
In[ ]:= Date []
```

```
Out[ ]:= {2020, 12, 26, 11, 31, 49.3241941}
```