

Pensieve header: Time ?. Zip3 computes most partial derivatives only once. (Bug, interrupted).

```
In[ ]:= Date []
```

```
Out[ ]:= {2020, 12, 25, 14, 22, 45.2981926}
```

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\BabyDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@"..\\Profile\\Profile.m"];
<< Objects.m
<< KT.m
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

Engine

```
In[ ]:= CCF[ε_] := PP_CCF@ExpandDenominator@ExpandNumerator@Together[ε];
(*CoefficientCanonical Form *)
CF[ε_List] := CF /@ ε;
CF[ε_εSeries] := CF /@ ε;
CF[ε_] := PP_CCF@Module[
  {vs = Cases[ε, (y | x | η | ξ)_ , ∞] ∪ {y | x | η | ξ}},
  Total[(CCF[#][2]] (Times @@ vs#[1]) & /@ CoefficientRules[Expand[ε], vs]]
];
(*CF[ε_] := PP_CCF@CCF[ε];*)
CF[ε_ε] := CF /@ ε;
CF[ε_sp___[εS___]] := CF /@ ε_sp[εS];
```

```
In[ ]:= eSeries /: S1_εSeries ≡ S2_εSeries :=
  Length[S1] == Length[S2] ∧ Inner[CF[#1] == CF[#2] &, S1, S2, And];
eSeries[0] := eSeries @@ Table[0, {k, 1};
eSeries /: S1_εSeries + S2_εSeries :=
  eSeries @@ Table[S1[[k]] + S2[[k]], {k, Min[Length@S1, Length@S2]};
eSeries /: S1_εSeries * S2_εSeries := eSeries @@
  Table[Sum[S1[[j+1]] * S2[[k-j+1]], {j, 0, k}], {k, 0, Min[Length@S1, Length@S2] - 1};
eSeries /: c_ * S_εSeries := (c #) & /@ S;
eSeries /: ∂_vs___ S_εSeries := (s ↦ ∂_vs s) /@ S;
```

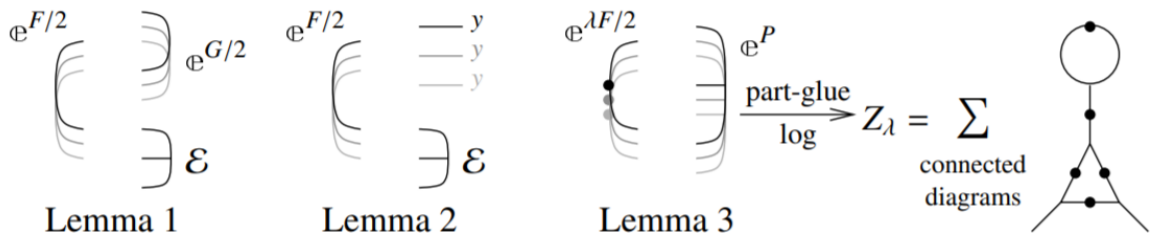
Variables and their duals:

```
In[ ]:= {y*, x*, η*, ξ*} = {η, ξ, y, x};
(vs_List)* := (v ↦ v*) /@ vs;
(u_i_)* := (u*)_i;
```

E operations:

```
In[*]:=
E /: E[ω1_, Q1_, P1_] ≡ E[ω2_, Q2_, P2_] := CF[ω1 == ω2] ∧ CF[Q1 == Q2] ∧ (P1 ≡ P2);
E /: E[ω1_, Q1_, P1_] × E[ω2_, Q2_, P2_] := E[ω1 ω2, Q1 + Q2, P1 + P2];
Ed1→r1[E1s___] ≡ Ed2→r2[E2s___] ^:= (d1 == d2) ∧ (r1 == r2) ∧ (E[E1s] ≡ E[E2s]);
Ed1→r1[E1s___] Ed2→r2[E2s___] ^:= E(d1∪d2)→(r1∪r2) @@ (E[E1s] × E[E2s]);
Edr[Es___]$k := Edr @@ E[Es]$k;
```

```
In[*]:=
Ed1→r1[E1s___] // Ed2→r2[E2s___] := Module[{is = r1 ∩ d2, lvs},
  lvs = Flatten@Table[{x$ei, y$ei}, {i, is}];
  E(d1∪Complement[d2,is])→(r2∪Complement[r1,is]) @@ (Ziplvs∪lvs*[lvs*.lvs, Times[
    E[E1s] /. Table[(v : x | y)i → v$ei, {i, is}],
    E[E2s] /. Table[(v : ξ | η)i → v$ei, {i, is}]
  ]])
]
```



```
In[*]:=
Zipvs[F_, E_] := ⟨F, E⟩ // Zip1vs // Zip2vs // Zip3vs;
Zipvs[F_, E_] := ⟨F, E⟩ // Zip1vs // EZip23vs;
```

Getting rid of the quadratic.

Lemma 1. With convergences left to the reader,

$$\left\langle F : \mathcal{E} \otimes \mathbb{E}^{\frac{1}{2} \sum_{i,j \in B} G_{ij} z_i z_j} \right\rangle_B = \det(1 - GF)^{-1/2} \left\langle F(1 - GF)^{-1} : \mathcal{E} \right\rangle_B$$

```
In[*]:=
Zip1_{ } = Identity;
Zip1vs @ ⟨F_, E[ω_, Q_, P_]⟩ := PPZip1 @ Module[{I, F, G, u, v},
  I = IdentityMatrix@Length@vs;
  F = Table[∂u,v F, {u, vs*}, {v, vs*}];
  G = Table[∂u,v Q, {u, vs}, {v, vs}];
  ⟨CF[vs*.F.Inverse[I - G.F].vs* / 2],
  E[CF@PowerExpand@Factor[ω Det[I - G.F]^{-1/2}, CF[Q - vs.G.vs / 2], P]]
]
```

Getting rid of linear terms.

Lemma 2. $\left\langle F : \mathcal{E} \otimes \mathbb{E}^{\sum_{i \in B} y_i z_i} \right\rangle_B = \mathbb{E}^{\frac{1}{2} \sum_{i,j \in B} F_{ij} y_i y_j} \left\langle F : \mathcal{E} \Big|_{z_B \rightarrow z_B + F y_B} \right\rangle_B$.

```

In[ ]:= Zip2_{ } = Identity;
Zip2_{vs_} @ < {F_, E[ω_, Q_, P_]} := PPZip2 @ Module[{F, Y, u, v},
  F = Table[∂_{u,v} F, {u, vs*}, {v, vs*}];
  Y = Table[∂_v Q, {v, vs}];
  CF /@ < {F, E[ω, Q - Y.v + Y.F.Y / 2, P /. Thread[v → vs + F.Y]]}
]

```

Dealing with Feynman diagrams.

Lemma 3. With an extra variable λ , $Z_\lambda := \log[\lambda F : \mathbb{C}^P]_B$ satisfies and is determined by the following PDE / IVP:

$$Z_0 = P \quad \text{and} \quad \partial_\lambda Z_\lambda = \frac{1}{2} \sum_{i,j \in B} F_{ij} (\partial_{z_i} \partial_{z_j} Z_\lambda + (\partial_{z_i} Z_\lambda)(\partial_{z_j} Z_\lambda)).$$

Note that the power m of λ is at most $k - 1 + \frac{2k+2}{2} = 2k$. We write $Z_\lambda = \sum Z[m] \lambda^m$.

```

In[ ]:= Zip3_{vs_} @ < {F_, E[ω_, Q_, P_]} := PPZip3 @ Module[{F, Z, u, v, m, j},
  F[u_, v_] := F[u, v] = ∂_{u*,v*} F;
  Z[j_, v_] := Z[j, v] = ∂_u Z[j];
  Z[0] = P;
  For[m = 0, m < 2 $k, ++m,
    Z[m + 1] = CF [
      1
      / (2 (m + 1))
      Sum[F[u, v] (∂_{u,v} Z[m] + Sum[Z[j, u] * Z[m - j, v], {j, 0, m}]), {u, vs}, {v, vs}]
    ];
  E[ω, Q, CF[Sum[Z[m], {m, 0, 2 $k}]] /. Table[v → 0, {v, vs}]]]
]

```

```

In[ ]:= EZip23_{vs_} @ < {F_, E[ω_, Q_, P_]} := PPEZip23 @ Module [
  {nP, nF, nQ, j = 0, ps, c, t, rr = {(*release rules*)}},
  nP = Total [
    CoefficientRules[#, vs] /.
    (ps_ → c_) ⇒ (AppendTo[rr, t[++j] → CF@c]; t[j] (Times @@ vs^{ps}))
  ] & /@ P;
  nQ = Total [CoefficientRules[Q, vs] /.
    (ps_ → c_) ⇒ (AppendTo[rr, t[++j] → CF@c]; t[j] (Times @@ vs^{ps}))];
  nF = Total [CoefficientRules[F, vs*] /. (ps_ → c_) ⇒
    (AppendTo[rr, t[++j] → CF@c]; t[j] (Times @@ (vs*)^{ps}))];
  CF [Expand[< nF, E[ω, nQ, nP] > // Zip2_{vs_} // Zip3_{vs_} /. rr]
]

```

Profile

```
In[ ]:= BeginProfile[];
        PopupWindow[Button["Show Profile Monitor"],
                    Dynamic[PrintProfile[], UpdateInterval -> 3, TrackedSymbols -> {}]]
```

Out[]:=

\$k = 1

```
In[ ]:= NewBit[K_] := Module[{Alex = Alexander[K][T]},
    T^3  $\frac{\text{Alex}^2}{T-1}$  Z[K][[3, 2]] // Factor]
```

```
In[ ]:= $k = 1; NewBit /@ AllKnots[{3, 5}]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[]:= $\left\{ 2 - T + T^2, (1 + T) (1 - 3 T + T^2), \frac{4 - 3 T + 5 T^2 - 3 T^3 + 3 T^4 - T^5 + T^6}{T^2}, 9 - 11 T + 7 T^2 - T^3 \right\}$

```
In[ ]:= (*Two knots with equal Alexander, new bit does not agree*)
        Alexander[Knot[6, 1]] == Alexander[Knot[9, 46]]
        $k = 1; Timing[NewBit[Knot[6, 1]] == NewBit[Knot[9, 46]]]
```

Out[]:= True

Out[]:= $\{19.1406, 5 - 11 T - T^2 + 3 T^3 == 7 - 21 T + 9 T^2 + T^3\}$

```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 37.36
  ( 24) 0.016/ 0.032 above CF
  ( 237) 1.830/ 35.150 above EZip23
  ( 237) 0.885/ 2.181 above Zip1
Zip3: called 237 times, time in 15.776/17.013
  ( 237) 15.780/ 17.010 under EZip23
  ( 1422) 0.504/ 1.237 above CF
CCF: called 15895 times, time in 9.369/9.369
  ( 15895) 9.369/ 9.369 under CF
CF: called 8837 times, time in 9.049/18.418
  ( 5495) 7.031/ 12.960 under EZip23
  ( 24) 0.016/ 0.032 under ProfileRoot
  ( 711) 0.656/ 1.296 under Zip1
  ( 1185) 0.842/ 2.891 under Zip2
  ( 1422) 0.504/ 1.237 under Zip3
  ( 15895) 9.369/ 9.369 above CCF
EZip23: called 237 times, time in 1.83/35.147
  ( 237) 1.830/ 35.150 under ProfileRoot
  ( 5495) 7.031/ 12.960 above CF
  ( 237) 0.451/ 3.342 above Zip2
  ( 237) 15.780/ 17.010 above Zip3
Zip1: called 237 times, time in 0.885/2.181
  ( 237) 0.885/ 2.181 under ProfileRoot
  ( 711) 0.656/ 1.296 above CF
Zip2: called 237 times, time in 0.451/3.342
  ( 237) 0.451/ 3.342 under EZip23
  ( 1185) 0.842/ 2.891 above CF
```

```
In[ ]:= $k = 1; equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};  
Length@Union[Z /@equiv]
```

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

```
Out[ ]:= 1
```

```
In[ ]:= $k = 1; equiv =  
{Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};  
Length@Union[Z /@equiv]
```

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

```
Out[ ]:= 1
```

```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 193.971
( 44) 0.032/ 0.048 above CF
( 652) 9.625/ 186.510 above EZip23
( 652) 2.791/ 7.418 above Zip1
CF: called 24383 times, time in 70.663/136.917
( 15211) 64.259/ 120.750 under EZip23
( 44) 0.032/ 0.048 under ProfileRoot
( 1956) 2.349/ 4.627 under Zip1
( 3260) 2.585/ 8.193 under Zip2
( 3912) 1.438/ 3.297 under Zip3
( 48358) 66.254/ 66.254 above CCF
CCF: called 48358 times, time in 66.254/66.254
( 48358) 66.254/ 66.254 under CF
Zip3: called 652 times, time in 43.268/46.565
( 652) 43.268/ 46.565 under EZip23
( 3912) 1.438/ 3.297 above CF
EZip23: called 652 times, time in 9.625/186.505
( 652) 9.625/ 186.510 under ProfileRoot
( 15211) 64.259/ 120.750 above CF
( 652) 1.370/ 9.563 above Zip2
( 652) 43.268/ 46.565 above Zip3
Zip1: called 652 times, time in 2.791/7.418
( 652) 2.791/ 7.418 under ProfileRoot
( 1956) 2.349/ 4.627 above CF
Zip2: called 652 times, time in 1.37/9.563
( 652) 1.370/ 9.563 under EZip23
( 3260) 2.585/ 8.193 above CF
```

\$k = 2

```
In[ ]:= $k = 2; equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};  
Length@Union[Z /@ equiv]
```

```
Out[ ]:= 2
```

```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 3111.
( 54) 0.329/ 0.673 above CF
( 813) 91.311/ 3101.300 above EZip23
( 813) 3.225/ 9.028 above Zip1
CF: called 35792 times, time in 1635.3/2880.95
( 22746) 1575.950/ 2474.770 under EZip23
( 54) 0.329/ 0.673 under ProfileRoot
( 2439) 2.881/ 5.803 under Zip1
( 4226) 6.644/ 30.633 under Zip2
( 6327) 49.493/ 369.073 under Zip3
( 95499) 1245.660/ 1245.660 above CCF
CCF: called 95499 times, time in 1245.66/1245.66
( 95499) 1245.660/ 1245.660 under CF
Zip3: called 813 times, time in 133.834/502.907
( 813) 133.834/ 502.907 under EZip23
( 6327) 49.493/ 369.073 above CF
EZip23: called 813 times, time in 91.311/3101.3
( 813) 91.311/ 3101.300 under ProfileRoot
( 22746) 1575.950/ 2474.770 above CF
( 813) 1.678/ 32.311 above Zip2
( 813) 133.834/ 502.907 above Zip3
Zip1: called 813 times, time in 3.225/9.028
( 813) 3.225/ 9.028 under ProfileRoot
( 2439) 2.881/ 5.803 above CF
Zip2: called 813 times, time in 1.678/32.311
( 813) 1.678/ 32.311 under EZip23
( 4226) 6.644/ 30.633 above CF
```

```
In[ ]:= $k = 2; equiv =
{Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};
Length@Union[Z /@ equiv]
```

```
Out[ ]:= 3
```

```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 11727.
( 69) 0.704/ 1.468 above CF
( 1071) 303.549/ 11713.470 above EZip23
( 1071) 4.378/ 12.110 above Zip1
CF: called 54383 times, time in 6829.48/11099.3
( 35130) 6664.608/ 9838.369 under EZip23
( 69) 0.704/ 1.468 under ProfileRoot
( 3213) 3.875/ 7.732 under Zip1
( 5774) 13.918/ 77.027 under Zip2
( 10197) 146.370/ 1174.702 under Zip3
( 177409) 4269.823/ 4269.823 above CCF
CCF: called 177409 times, time in 4269.82/4269.82
( 177409) 4269.823/ 4269.823 under CF
Zip3: called 1071 times, time in 317.426/1492.13
( 1071) 317.426/ 1492.128 under EZip23
( 10197) 146.370/ 1174.702 above CF
EZip23: called 1071 times, time in 303.549/11713.5
( 1071) 303.549/ 11713.470 under ProfileRoot
( 35130) 6664.608/ 9838.369 above CF
( 1071) 2.398/ 79.425 above Zip2
( 1071) 317.426/ 1492.128 above Zip3
Zip1: called 1071 times, time in 4.378/12.11
( 1071) 4.378/ 12.110 under ProfileRoot
( 3213) 3.875/ 7.732 above CF
Zip2: called 1071 times, time in 2.398/79.425
( 1071) 2.398/ 79.425 under EZip23
( 5774) 13.918/ 77.027 above CF
```

```
In[ ]:= Date []
```

```
Out[ ]:= {2020, 12, 25, 17, 49, 50.0036552}
```

```
In[ ]:= $k = 2; equiv = {Knot[12, NonAlternating, 60],  

    Knot[12, NonAlternating, 61], Knot[12, NonAlternating, 219]};  

Length@Union[Z /@ equiv]
```

```
Out[ ]:= $Aborted
```

```
In[ ]:= PrintProfile []
```

```
In[ ]:= Date []
```