

Pensieve header: 9413.82. No CF at the encapsulation of coefficients.

```
In[ ]:= Date []
```

```
Out[ ]:= {2020, 12, 24, 7, 42, 43.5247910}
```

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\BabyDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@"..\\Profile\\Profile.m"];
<< Objects.m
<< KT.m
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

## Engine

```
In[ ]:= CCF[ε_] := PP_CCF@ExpandDenominator@ExpandNumerator@Together[ε];
CF[ε_List] := CF /@ ε;
CF[ε_εSeries] := CF /@ ε;
CF[ε_] := PP_CCF@Module[
  {vs = Cases[ε, (y | x | η | ξ)_ , ∞] ∪ {y | x | η | ξ}},
  Total[CoefficientRules[Expand[ε], vs] /. (ps_ -> c_) -> CCF[c] (Times @@ vs^ps)]
];
(*CF[ε_] := PP_CCF@CCF[ε];*)
CF[ε_E] := CF /@ ε;
CF[E_sp___[εS___]] := CF /@ E_sp[εS];
```

```
In[ ]:= εSeries /: S1_εSeries ≡ S2_εSeries :=
  Length[S1] == Length[S2] ∧ Inner[CF[#1] == CF[#2] &, S1, S2, And];
εSeries[0] := εSeries @@ Table[0, $k + 1];
εSeries /: S1_εSeries + S2_εSeries :=
  εSeries @@ Table[S1[[k]] + S2[[k]], {k, Min[Length@S1, Length@S2]};
εSeries /: S1_εSeries * S2_εSeries := εSeries @@
  Table[Sum[S1[[j + 1]] * S2[[k - j + 1]], {j, 0, k}], {k, 0, Min[Length@S1, Length@S2] - 1};
εSeries /: c_ * S_εSeries := (c #) & /@ S;
εSeries /: ∂_vs___ S_εSeries := (s ↦ ∂_vs s) /@ S;
```

Variables and their duals:

```
In[ ]:= {y*, x*, η*, ξ*} = {η, ξ, y, x};
(vs_List)* := (v ↦ v*) /@ vs;
(u_i_)* := (u*)_i;
```

E operations:

```
In[ ]:=
E /: E[ω1_, Q1_, P1_] ≡ E[ω2_, Q2_, P2_] := CF[ω1 == ω2] ∧ CF[Q1 == Q2] ∧ (P1 ≡ P2);
E /: E[ω1_, Q1_, P1_] × E[ω2_, Q2_, P2_] := E[ω1 ω2, Q1 + Q2, P1 + P2];
Ed1→r1[E1s____] ≡ Ed2→r2[E2s____] ^:= (d1 == d2) ∧ (r1 == r2) ∧ (E[E1s] ≡ E[E2s]);
Ed1→r1[E1s____] Ed2→r2[E2s____] ^:= E(d1∪d2)→(r1∪r2) @@ (E[E1s] × E[E2s]);
Edr[Es____]$k := Edr @@ E[Es]$k;
```

```
In[ ]:=
Ed1→r1[E1s____] // Ed2→r2[E2s____] := Module[{is = r1 ∩ d2, lvs},
  lvs = Flatten@Table[{x$ei, y$ei}, {i, is}];
  E(d1∪Complement[d2,is])→(r2∪Complement[r1,is]) @@ (Ziplvs∪lvs*[lvs*.lvs, Times[
    E[E1s] /. Table[(v : x | y)i → v$ei, {i, is}],
    E[E2s] /. Table[(v : ξ | η)i → v$ei, {i, is}]
  ]])
]
```

```
In[ ]:=
Zipvs[F_, E_] := ⟨F, E⟩ // Zip1vs // Zip2vs // Zip3vs;
Zipvs[F_, E_] := ⟨F, E⟩ // Zip1vs // EZip23vs;
```

Getting rid of the quadratic.

**Lemma 1.** With convergences left to the reader,

$$\left\langle F : \mathcal{E} \otimes^{\frac{1}{2} \sum_{i,j \in B} G_{ij} z_i z_j} \right\rangle_B = \det(1 - GF)^{-1/2} \left\langle F(1 - GF)^{-1} : \mathcal{E} \right\rangle_B$$

```
In[ ]:=
Zip1{} = Identity;
Zip1vs@⟨F_, E[ω_, Q_, P_]⟩ := PPZip1@Module[{I, F, G, u, v},
  I = IdentityMatrix@Length@vs;
  F = Table[∂u,vF, {u, vs*}, {v, vs*}];
  G = Table[∂u,vQ, {u, vs}, {v, vs}];
  ⟨CF[vs*.F.Inverse[I - G.F].vs* / 2],
  E[CF@PowerExpand@Factor[ω Det[I - G.F]-1/2, CF[Q - vs.G.vs / 2], P]]
]
```

Getting rid of linear terms.

**Lemma 2.**  $\left\langle F : \mathcal{E} \otimes^{\sum_{i \in B} y_i z_i} \right\rangle_B = \otimes^{\frac{1}{2} \sum_{i,j \in B} F_{ij} y_i y_j} \left\langle F : \mathcal{E}|_{z_B \rightarrow z_B + F y_B} \right\rangle_B$ .

```
In[ ]:=
Zip2{} = Identity;
Zip2vs@⟨F_, E[ω_, Q_, P_]⟩ := PPZip2@Module[{F, Y, u, v},
  F = Table[∂u,vF, {u, vs*}, {v, vs*}];
  Y = Table[∂vQ, {v, vs}];
  CF / @ ⟨F, E[ω, Q - Y.vs + Y.F.Y / 2, P /. Thread[vs → vs + F.Y]]
]
```

Dealing with Feynman diagrams.

**Lemma 3.** With an extra variable  $\lambda$ ,  $Z_\lambda := \log[\lambda F : \mathbb{E}^P]_B$  satisfies and is determined by the following PDE / IVP:

$$Z_0 = P \quad \text{and} \quad \partial_\lambda Z_\lambda = \frac{1}{2} \sum_{i,j \in B} F_{ij} (\partial_{z_i} \partial_{z_j} Z_\lambda + (\partial_{z_i} Z_\lambda)(\partial_{z_j} Z_\lambda)).$$

Note that the power  $m$  of  $\lambda$  is at most  $k - 1 + \frac{2k+2}{2} = 2k$ . We write  $Z_\lambda = \sum Z[m] \lambda^m$ .

```
In[ ]:= Zip3_{vs} @ < \mathcal{F}_-, \mathbb{E}[\omega_-, Q_-, P_-] > := PPZip3 @ Module [ {Z, u, v, m, j},
  Z[0] = P;
  For [ m = 0, m < 2 $k, ++m,
    Z [ m + 1 ] = CF [ \frac{1}{2 (m + 1)}
      Sum [ \partial_{u^*, v^*} \mathcal{F} ( \partial_{u, v} Z [ m ] + Sum [ ( \partial_u Z [ j ] ) ( \partial_v Z [ m - j ] ), { j, 0, m } ] ), { u, vs }, { v, vs } ] ]
  ];
  \mathbb{E} [ \omega, Q, CF [ Sum [ Z [ m ], { m, 0, 2 $k } ] ] /. Table [ v \to 0, { v, vs } ] ] ] ]
```

```
In[ ]:= EZip23 {} = Identity;
EZip23_{vs} @ < \mathcal{F}_-, \mathbb{E}[\omega_-, Q_-, P_-] > := PPEZip23 @ Module [
  { nP, nF, nQ, j = 0, ps, c, t, rr = { (*release rules*) } },
  nP = Total [
    CoefficientRules [ #, vs ] /.
    ( ps_ \to c_ ) \Rightarrow ( AppendTo [ rr, t [ ++j ] \to c ]; t [ j ] ( Times @@ vs^{ps} ) )
  ] & /@ P;
  nQ = Total [ CoefficientRules [ Q, vs ] /.
    ( ps_ \to c_ ) \Rightarrow ( AppendTo [ rr, t [ ++j ] \to c ]; t [ j ] ( Times @@ vs^{ps} ) ) ];
  nF = Total [ CoefficientRules [ \mathcal{F}, vs^* ] /. ( ps_ \to c_ ) \Rightarrow
    ( AppendTo [ rr, t [ ++j ] \to c ]; t [ j ] ( Times @@ ( vs^* )^{ps} ) ) ];
  CF [ Expand [ < nF, \mathbb{E}[\omega, nQ, nP] > // Zip2_{vs} // Zip3_{vs} ] /. rr ]
]
```

## Profile

```
In[ ]:= BeginProfile [ ];
PopupWindow [ Button [ "Show Profile Monitor" ],
  Dynamic [ PrintProfile [ ], UpdateInterval \to 3, TrackedSymbols \to {} ] ]
```

```
Out[ ]:= Show Profile Monitor
```

$\$k = 1$

```
NewBit[K_] := Module[{Alex = Alexander[K][T]},
```

```
  T^3  $\frac{\text{Alex}^2}{T-1}$  Z[K][[3, 2]] // Factor]
```

```
$k = 1; NewBit /@ AllKnots[{3, 5}]
```

KnotTheory: Loading precomputed data in PD4Knots`.

```
Out[ ]:= {2 - T + T^2, (1 + T) (1 - 3 T + T^2),  $\frac{4 - 3 T + 5 T^2 - 3 T^3 + 3 T^4 - T^5 + T^6}{T^2}$ , 9 - 11 T + 7 T^2 - T^3}
```

(\*Two knots with equal Alexander, new bit does not agree\*)

```
Alexander[Knot[6, 1]] == Alexander[Knot[9, 46]]
```

```
$k = 1; Timing[NewBit[Knot[6, 1]] == NewBit[Knot[9, 46]]]
```

```
Out[ ]:= True
```

```
Out[ ]:= {26.1563, 5 - 11 T - T^2 + 3 T^3 == 7 - 21 T + 9 T^2 + T^3}
```

```
In[ ]:= PrintProfile[]
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 49.891
( 24) 0/ 0 above CF
( 237) 2.005/ 46.616 above EZip23
( 237) 1.454/ 3.275 above Zip1
Zip3: called 237 times, time in 25.465/26.961
( 237) 25.465/ 26.961 under EZip23
( 1422) 0.497/ 1.496 above CF
CCF: called 10873 times, time in 10.283/10.283
( 10873) 10.283/ 10.283 under CF
CF: called 4290 times, time in 9.949/20.232
( 948) 7.092/ 12.581 under EZip23
( 24) 0/ 0 under ProfileRoot
( 711) 0.980/ 1.821 under Zip1
( 1185) 1.380/ 4.334 under Zip2
( 1422) 0.497/ 1.496 under Zip3
( 10873) 10.283/ 10.283 above CCF
EZip23: called 237 times, time in 2.005/46.616
( 237) 2.005/ 46.616 under ProfileRoot
( 948) 7.092/ 12.581 above CF
( 237) 0.735/ 5.069 above Zip2
( 237) 25.465/ 26.961 above Zip3
Zip1: called 237 times, time in 1.454/3.275
( 237) 1.454/ 3.275 under ProfileRoot
( 711) 0.980/ 1.821 above CF
Zip2: called 237 times, time in 0.735/5.069
( 237) 0.735/ 5.069 under EZip23
( 1185) 1.380/ 4.334 above CF
```

```
$k = 1; equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};  
Length@Union[Z /@equiv]
```

**KnotTheory:** Loading precomputed data in KnotTheory/12N.dts.

**KnotTheory:** The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[ ]= 1

```
$k = 1; equiv =  
{Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};  
Length@Union[Z /@equiv]
```

**KnotTheory:** Loading precomputed data in KnotTheory/12A.dts.

Out[ ]= 1

In[ ]:= **PrintProfile[]**

```
Out[ ]= ProfileRoot is root. Profiled time: 255.299  
  ( 44)  0.016/ 0.031 above CF  
  ( 652) 12.609/ 245.480 above EZip23  
  ( 652)  4.003/  9.789 above Zip1  
CF: called 11780 times, time in 90.399/167.892  
  ( 2608) 82.347/ 146.030 under EZip23  
  ( 44)  0.016/ 0.031 under ProfileRoot  
  ( 1956) 2.880/  5.786 under Zip1  
  ( 3260) 3.651/ 11.608 under Zip2  
  ( 3912) 1.505/  4.439 under Zip3  
  ( 32286) 77.493/ 77.493 above CCF  
CCF: called 32286 times, time in 77.493/77.493  
  ( 32286) 77.493/ 77.493 under CF  
Zip3: called 652 times, time in 69.076/73.515  
  ( 652) 69.076/ 73.515 under EZip23  
  ( 3912) 1.505/  4.439 above CF  
EZip23: called 652 times, time in 12.609/245.479  
  ( 652) 12.609/ 245.480 under ProfileRoot  
  ( 2608) 82.347/ 146.030 above CF  
  ( 652)  1.719/ 13.327 above Zip2  
  ( 652) 69.076/ 73.515 above Zip3  
Zip1: called 652 times, time in 4.003/9.789  
  ( 652)  4.003/  9.789 under ProfileRoot  
  ( 1956) 2.880/  5.786 above CF  
Zip2: called 652 times, time in 1.719/13.327  
  ( 652)  1.719/ 13.327 under EZip23  
  ( 3260) 3.651/ 11.608 above CF
```

**\$k = 2**

```
$k = 2; equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};  
Length@Union[Z /@equiv]
```

Out[ ]= 2

```
In[ ]:= PrintProfile[ ]
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 2163.29
```

```
( 54) 0.062/ 0.141 above CF
( 813) 48.156/ 2150.540 above EZip23
( 813) 5.085/ 12.608 above Zip1
CF: called 16459 times, time in 1147.16/1909.66
( 3413) 1123.550/ 1787.150 under EZip23
( 54) 0.062/ 0.141 under ProfileRoot
( 2439) 3.725/ 7.523 under Zip1
( 4226) 7.463/ 30.970 under Zip2
( 6327) 12.365/ 83.876 under Zip3
( 52601) 762.499/ 762.499 above CCF
CCF: called 52601 times, time in 762.499/762.499
( 52601) 762.499/ 762.499 under CF
Zip3: called 813 times, time in 198.057/281.933
( 813) 198.057/ 281.933 under EZip23
( 6327) 12.365/ 83.876 above CF
EZip23: called 813 times, time in 48.156/2150.54
( 813) 48.156/ 2150.540 under ProfileRoot
( 3413) 1123.550/ 1787.150 above CF
( 813) 2.326/ 33.296 above Zip2
( 813) 198.057/ 281.933 above Zip3
Zip1: called 813 times, time in 5.085/12.608
( 813) 5.085/ 12.608 under ProfileRoot
( 2439) 3.725/ 7.523 above CF
Zip2: called 813 times, time in 2.326/33.296
( 813) 2.326/ 33.296 under EZip23
( 4226) 7.463/ 30.970 above CF
```

```
$k = 2; equiv =
```

```
{Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};  
Length@Union[Z /@ equiv]
```

```
Out[ ]:= 3
```

```
In[ ]:= PrintProfile []
```

```
Out[ ]:= ProfileRoot is root. Profiled time: 6959.65
( 69) 0.109/ 0.188 above CF
( 1071) 125.209/ 6942.996 above EZip23
( 1071) 6.539/ 16.464 above Zip1
CF: called 23956 times, time in 3897.16/6397.05
( 4703) 3844.241/ 6073.636 under EZip23
( 69) 0.109/ 0.188 under ProfileRoot
( 3213) 4.991/ 9.925 under Zip1
( 5774) 13.896/ 67.649 under Zip2
( 10197) 33.924/ 245.648 under Zip3
( 87339) 2499.885/ 2499.885 above CCF
CCF: called 87339 times, time in 2499.88/2499.88
( 87339) 2499.885/ 2499.885 under CF
Zip3: called 1071 times, time in 427.823/673.471
( 1071) 427.823/ 673.471 under EZip23
( 10197) 33.924/ 245.648 above CF
EZip23: called 1071 times, time in 125.209/6943.
( 1071) 125.209/ 6942.996 under ProfileRoot
( 4703) 3844.241/ 6073.636 above CF
( 1071) 3.031/ 70.680 above Zip2
( 1071) 427.823/ 673.471 above Zip3
Zip1: called 1071 times, time in 6.539/16.464
( 1071) 6.539/ 16.464 under ProfileRoot
( 3213) 4.991/ 9.925 above CF
Zip2: called 1071 times, time in 3.031/70.68
( 1071) 3.031/ 70.680 under EZip23
( 5774) 13.896/ 67.649 above CF
```

```
In[ ]:= Date []
```

```
Out[ ]:= {2020, 12, 24, 9, 53, 32.6680106}
```

```
$k = 2; equiv = {Knot[12, NonAlternating, 60],
Knot[12, NonAlternating, 61], Knot[12, NonAlternating, 219]};
Length@Union[Z /@ equiv]
```

```
Out[ ]:= 1
```

In[ ]:= **PrintProfile** []

```
Out[ ]:= ProfileRoot is root. Profiled time: 9413.82
( 84) 0.140/ 0.297 above CF
( 1329) 183.497/ 9393.325 above EZip23
( 1329) 7.965/ 20.198 above Zip1
CF: called 31453 times, time in 5125.33/8569.22
( 5993) 5045.434/ 8079.680 under EZip23
( 84) 0.140/ 0.297 under ProfileRoot
( 3987) 6.175/ 12.233 under Zip1
( 7322) 20.454/ 101.217 under Zip2
( 14067) 53.124/ 375.790 under Zip3
( 117202) 3443.890/ 3443.890 above CCF
CCF: called 117202 times, time in 3443.89/3443.89
( 117202) 3443.890/ 3443.890 under CF
Zip3: called 1329 times, time in 649.344/1025.13
( 1329) 649.344/ 1025.134 under EZip23
( 14067) 53.124/ 375.790 above CF
EZip23: called 1329 times, time in 183.497/9393.33
( 1329) 183.497/ 9393.325 under ProfileRoot
( 5993) 5045.434/ 8079.680 above CF
( 1329) 3.797/ 105.014 above Zip2
( 1329) 649.344/ 1025.134 above Zip3
Zip1: called 1329 times, time in 7.965/20.198
( 1329) 7.965/ 20.198 under ProfileRoot
( 3987) 6.175/ 12.233 above CF
Zip2: called 1329 times, time in 3.797/105.014
( 1329) 3.797/ 105.014 under EZip23
( 7322) 20.454/ 101.217 above CF
```

In[ ]:= **Date** []

```
Out[ ]:= {2020, 12, 24, 10, 37, 59.6086105}
```