Pensieve header: (“Contained” rational functions of T). No, vanilla. Failed.

In[∘]:= Date[]

Out[∘]= {2020, 12, 24, 12, 21, 15.7463997}

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\BabyDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@"../Profile/Profile.m"];
<< Objects.m
<< KT.m
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.
This version: April 2020. Original version: July 1994.
```

Engine

In[∘]:=
```
CCF[ℰ_] := PP_CCF@ExpandDenominator@ExpandNumerator@Together[ℰ];
(*CoefficientCanonical Form *)
CF[ℰ_List] := CF /@ ℰ;
CF[ℰ_𝜖Series] := CF /@ ℰ;
CF[ℰ_] := PP_CF@Module[
    {vs = Cases[ℰ, (y | x | η | ξ)_, ∞] ⋃ {y | x | η | ξ}},
    Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) :→ CCF[c] (Times @@ vs^ps)]
   ];
(*CF[ℰ_] := PP_CF@CCF[ℰ];*)
CF[ℰ_𝔼] := CF /@ ℰ;
CF[𝔼_sp___[ℰs___]] := CF /@ 𝔼_sp[ℰs];
PCF[ℰ_] := PP_PCF@Module[ (* Protected Canonical Form *)
    {vs = Cases[ℰ, (y | x | η | ξ)_, ∞] ⋃ {y | x | η | ξ}},
    Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) :→ CCF[c] (Times @@ vs^ps)]
   ];
ECF[ℰ_] := PP_ECF@Module[ (* Protected Canonical Form *)
    {vs = Cases[ℰ, (y | x | η | ξ)_, ∞] ⋃ {y | x | η | ξ}},
    Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) :→ CCF[c] (Times @@ vs^ps)]
   ];
```

In[ ]:=
```
ϵSeries /: S1_ϵSeries ≡ S2_ϵSeries :=
  Length[S1] == Length[S2] ∧ Inner[CF[#1] == CF[#2] &, S1, S2, And];
ϵSeries[0] := ϵSeries @@ Table[0, $k + 1];
ϵSeries /: S1_ϵSeries + S2_ϵSeries :=
  ϵSeries @@ Table[S1⟦k⟧ + S2⟦k⟧, {k, Min[Length@S1, Length@S2]}];
ϵSeries /: S1_ϵSeries * S2_ϵSeries := ϵSeries @@
   Table[Sum[S1⟦j + 1⟧ * S2⟦k - j + 1⟧, {j, 0, k}], {k, 0, Min[Length@S1, Length@S2] - 1}];
ϵSeries /: c_ * S_ϵSeries := (c #) & /@ S;
ϵSeries /: ∂_vs__ S_ϵSeries := (s ↦ ∂_vs s) /@ S;
```

Variables and their duals:

In[ ]:=
```
{y*, x*, η*, ξ*} = {η, ξ, y, x};
(vs_List)* := (v ↦ v*) /@ vs;
(u_i_)* := (u*)_i;
```

𝔼 operations:

In[ ]:=
```
𝔼 /: 𝔼[ω1_, Q1_, P1_] ≡ 𝔼[ω2_, Q2_, P2_] := CF[ω1 == ω2] ∧ CF[Q1 == Q2] ∧ (P1 ≡ P2);
𝔼 /: 𝔼[ω1_, Q1_, P1_] 𝔼[ω2_, Q2_, P2_] := 𝔼[ω1 ω2, Q1 + Q2, P1 + P2];
𝔼_{d1_→r1_}[ℰ1s___] ≡ 𝔼_{d2_→r2_}[ℰ2s___] ^:= (d1 == d2) ∧ (r1 == r2) ∧ (𝔼[ℰ1s] ≡ 𝔼[ℰ2s]);
𝔼_{d1_→r1_}[ℰ1s___] 𝔼_{d2_→r2_}[ℰ2s___] ^:= 𝔼_{(d1⋃d2)→(r1⋃r2)} @@ (𝔼[ℰ1s] 𝔼[ℰ2s]);
𝔼_{dr_}[ℰs___]_{$k_} := 𝔼_{dr} @@ 𝔼[ℰs]_{$k};
```

In[ ]:=
```
𝔼_{d1_→r1_}[ℰ1s___] // 𝔼_{d2_→r2_}[ℰ2s___] := Module[{is = r1 ⋂ d2, lvs},
  lvs = Flatten@Table[{x_{$@i}, y_{$@i}}, {i, is}];
  𝔼_{(d1⋃Complement[d2,is])→(r2⋃Complement[r1,is])} @@ (Zip_{lvs⋃lvs*}[lvs*.lvs, Times[
      𝔼[ℰ1s] /. Table[(v : x | y)_i → v_{$@i}, {i, is}],
      𝔼[ℰ2s] /. Table[(v : ξ | η)_i → v_{$@i}, {i, is}]
     ]])
 ]
```

In[ ]:=
```
Zip_vs_[ℱ_, ℰ_] := ⟨ℱ, ℰ⟩ // Zip1_vs // Zip2_vs // Zip3_vs;
Zip_vs_[ℱ_, ℰ_] := ⟨ℱ, ℰ⟩ // Zip1_vs // EZip23_vs;
```

Getting rid of the quadratic.

Lemma 1. With convergences left to the reader,

$$\left\langle F : \mathcal{E}\,\mathbb{e}^{\frac{1}{2}\sum_{i,j\in B} G_{ij} z_i z_j}\right\rangle_B = \det(1-GF)^{-1/2}\left\langle F(1-GF)^{-1} : \mathcal{E}\right\rangle_B$$

In[∘]:=
```
Zip1{} = Identity;
Zip1vs_@⟨𝓕_, 𝔼[ω_, Q_, P_]⟩ := PPZip1@Module[{𝓘, F, G, u, v},
   𝓘 = IdentityMatrix@Length@vs;
   F = Table[∂u,v 𝓕, {u, vs*}, {v, vs*}];
   G = Table[∂u,v Q, {u, vs}, {v, vs}];
   ⟨CF[vs*.F.Inverse[𝓘 - G.F].vs* / 2],
    𝔼[CF@PowerExpand@Factor[ω Det[𝓘 - G.F]^(-1/2)], CF[Q - vs.G.vs / 2], P]⟩
  ]
```

Getting rid of linear terms.

Lemma 2. $\left\langle F:\mathcal{E}\,e^{\sum_{i\in B}y_iz_i}\right\rangle_B=e^{\frac{1}{2}\sum_{i,j\in B}F_{ij}y_iy_j}\left\langle F:\ \mathcal{E}|_{z_B\to z_B+Fy_B}\right\rangle_B.$

In[∘]:=
```
Zip2{} = Identity;
Zip2vs_@⟨𝓕_, 𝔼[ω_, Q_, P_]⟩ := PPZip2@Module[{F, Y, u, v},
   F = Table[∂u,v 𝓕, {u, vs*}, {v, vs*}];
   Y = Table[∂v Q, {v, vs}];
   CF /@ ⟨𝓕, 𝔼[ω, Q - Y.vs + Y.F.Y / 2, P /. Thread[vs → vs + F.Y]]⟩
  ]
```

Dealing with Feynman diagrams.

Lemma 3. With an extra variable λ, $Z_\lambda:=\log[\lambda F:e^P]_B$ satisfies and is determined by the following PDE / IVP:

$$Z_0=P\quad\text{and}\quad\partial_\lambda Z_\lambda=\frac{1}{2}\sum_{i,j\in B}F_{ij}\left(\partial_{z_i}\partial_{z_j}Z_\lambda+(\partial_{z_i}Z_\lambda)(\partial_{z_j}Z_\lambda)\right).$$

Note that the power m of λ is at most $k-1+\frac{2k+2}{2}=2k$. We write $Z_\lambda=\sum Z[m]\,\lambda^m$.

In[∘]:=
```
Zip3vs_@⟨𝓕_, 𝔼[ω_, Q_, P_]⟩ := PPZip3@Module[{Z, u, v, m, j},
   Z[0] = P;
   For[m = 0, m < 2 $k, ++m,
    Z[m + 1] = CF[1/(2 (m + 1))
       Sum[∂u*,v* 𝓕 (∂u,v Z[m] + Sum[(∂u Z[j]) (∂v Z[m - j]), {j, 0, m}]), {u, vs}, {v, vs}]]
    ];
   𝔼[ω, Q, CF[Sum[Z[m], {m, 0, 2 $k}] /. Table[v → 0, {v, vs}]]]
  ]
```

In[∘]:=
```
EZip23{} = Identity;
EZip23vs_ @⟨ℱ_, 𝔼[ω_, Q_, P_]⟩ := PPEZip23@Module[
  {nP, nℱ, nQ, j = 0, ps, c, t, rr = {(*release rules*)}},
  nP = Total[
      CoefficientRules[#, vs] /.
       (ps_ → c_) :→ (AppendTo[rr, t[++j] → PCF@c]; t[j] (Times @@ vs^ps))
    ] & /@ P;
  nQ = Total[CoefficientRules[Q, vs] /.
     (ps_ → c_) :→ (AppendTo[rr, t[++j] → PCF@c]; t[j] (Times @@ vs^ps))];
  nℱ = Total[CoefficientRules[ℱ, vs*] /. (ps_ → c_) :→
      (AppendTo[rr, t[++j] → PCF@c]; t[j] (Times @@ (vs*)^ps))];
  ECF[Expand[⟨nℱ, 𝔼[ω, nQ, nP]⟩ // Zip2vs // Zip3vs] /. rr]
 ]
```

Profile

In[∘]:=
```
BeginProfile[];
PopupWindow[Button["Show Profile Monitor"],
 Dynamic[PrintProfile[], UpdateInterval → 3, TrackedSymbols → {}]]
```

Out[∘]= Show Profile Monitor

$k = 1

In[∘]:=
```
NewBit[K_] := Module[{Alex = Alexander[K][T]},
  T^3 Alex^2/(T - 1) Z[K][[3, 2]] // Factor]
```

In[∘]:=
```
$k = 1; NewBit /@ AllKnots[{3, 5}]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[∘]= $\left\{2-T+T^2,\ (1+T)\left(1-3T+T^2\right),\ \frac{4-3T+5T^2-3T^3+3T^4-T^5+T^6}{T^2},\ 9-11T+7T^2-T^3\right\}$

In[∘]:=
```
(*Two knots with equal Alexander, new bit does not agree*)
Alexander[Knot[6, 1]] == Alexander[Knot[9, 46]]
$k = 1; Timing[NewBit[Knot[6, 1]] == NewBit[Knot[9, 46]]]
```

Out[∘]= True

Out[∘]= $\left\{31.625,\ 5-11T-T^2+3T^3 == 7-21T+9T^2+T^3\right\}$

In[ ]:=
```
PrintProfile[]
```

Out[ ]=
```
ProfileRoot is root. Profiled time: 57.996
  (   24)   0.453/  0.842 above CF
  (  237)   6.596/ 53.376 above EZip23
  (  237)   1.562/  3.778 above Zip1
Zip3: called 237 times, time in 26.077/27.732
  (  237)  26.077/ 27.732 under EZip23
  ( 1422)   0.716/  1.655 above CF
CCF: called 13417 times, time in 12.036/12.036
  ( 8157)   5.246/  5.246 under CF
  (  237)   0.504/  0.504 under ECF
  ( 5023)   6.286/  6.286 under PCF
PCF: called 4548 times, time in 7.026/13.312
  ( 4548)   7.026/ 13.312 under EZip23
  ( 5023)   6.286/  6.286 above CCF
EZip23: called 237 times, time in 6.596/53.376
  (  237)   6.596/ 53.376 under ProfileRoot
  (  237)   0.405/  0.909 above ECF
  ( 4548)   7.026/ 13.312 above PCF
  (  237)   0.701/  4.827 above Zip2
  (  237)  26.077/ 27.732 above Zip3
CF: called 3342 times, time in 3.593/8.839
  (   24)   0.453/  0.842 under ProfileRoot
  (  711)   1.269/  2.216 under Zip1
  ( 1185)   1.155/  4.126 under Zip2
  ( 1422)   0.716/  1.655 under Zip3
  ( 8157)   5.246/  5.246 above CCF
Zip1: called 237 times, time in 1.562/3.778
  (  237)   1.562/  3.778 under ProfileRoot
  (  711)   1.269/  2.216 above CF
Zip2: called 237 times, time in 0.701/4.827
  (  237)   0.701/  4.827 under EZip23
  ( 1185)   1.155/  4.126 above CF
ECF: called 237 times, time in 0.405/0.909
  (  237)   0.405/  0.909 under EZip23
  (  237)   0.504/  0.504 above CCF
```

In[ ]:=
```
$k = 1; equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};
Length@Union[Z /@ equiv]
```

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[ ]= 1

In[∘]:=
```
$k = 1; equiv =
  {Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};
Length@Union[Z /@ equiv]
```

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

Out[∘]= 1

In[∘]:=
```
PrintProfile[]
```

Out[∘]=
```
ProfileRoot is root. Profiled time: 316.123
  (   44)   1.140/  2.249 above CF
  (  652)  74.043/ 298.970 above EZip23
  (  652)   4.192/ 14.902 above Zip1
CCF: called 39265 times, time in 86.339/86.339
  ( 22522)  17.269/ 17.269 under CF
  (  652)   2.642/  2.642 under ECF
  ( 16091)  66.428/ 66.428 under PCF
EZip23: called 652 times, time in 74.043/298.972
  (  652)  74.043/ 298.970 under ProfileRoot
  (  652)   1.805/  4.447 above ECF
  ( 12613)  68.993/ 135.420 above PCF
  (  652)   1.761/ 13.228 above Zip2
  (  652)  67.383/ 71.833 above Zip3
PCF: called 12613 times, time in 68.993/135.421
  ( 12613)  68.993/ 135.420 under EZip23
  ( 16091)  66.428/ 66.428 above CCF
Zip3: called 652 times, time in 67.383/71.833
  (  652)  67.383/ 71.833 under EZip23
  ( 3912)   1.831/  4.450 above CF
CF: called 9172 times, time in 11.607/28.876
  (   44)   1.140/  2.249 under ProfileRoot
  ( 1956)   5.157/ 10.710 under Zip1
  ( 3260)   3.479/ 11.467 under Zip2
  ( 3912)   1.831/  4.450 under Zip3
  ( 22522)  17.269/ 17.269 above CCF
Zip1: called 652 times, time in 4.192/14.902
  (  652)   4.192/ 14.902 under ProfileRoot
  ( 1956)   5.157/ 10.710 above CF
ECF: called 652 times, time in 1.805/4.447
  (  652)   1.805/  4.447 under EZip23
  (  652)   2.642/  2.642 above CCF
Zip2: called 652 times, time in 1.761/13.228
  (  652)   1.761/ 13.228 under EZip23
  ( 3260)   3.479/ 11.467 above CF
```

$k = 2

In[∘]:=
```
$k = 2; equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};
Length@Union[Z /@ equiv]
```

Out[∘]= $Aborted

In[∘]:=
```
PrintProfile[]
```

```
$k = 2; equiv =
 {Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};
Length@Union[Z /@ equiv]
```

Out[∘]=
```
3
```

In[∘]:=
```
PrintProfile[]
```

In[∘]:=
```
Date[]
```

Out[∘]=
```
{2020, 12, 24, 9, 53, 32.6680106}
```

```
$k = 2; equiv = {Knot[12, NonAlternating, 60],
  Knot[12, NonAlternating, 61], Knot[12, NonAlternating, 219]};
Length@Union[Z /@ equiv]
```

Out[∘]=
```
1
```

In[∘]:=
```
PrintProfile[]
```

In[∘]:=
```
Date[]
```

Out[∘]=
```
{2020, 12, 24, 10, 37, 59.6086105}
```