Define[lhs = rhs, ...] defines the lhs to be rhs, except that rhs is computed only once for each value of $k. Fancy Mathematica not for the faint of heart. Most readers should ignore.

(Alt) In[∘]:=
```
SetAttributes[Define, HoldAll];
Define[def_, defs__] := (Define[def]; Define[defs];);
Define[op_is__ = ℰ_] := Module[{SD, ii, jj, kk, isp, nis, nisp, sis}, Block[{i, j, k},
   ReleaseHold[Hold[
      SD[op_{nisp,$k_Integer}, Block[{i, j, k}, op_{isp,$k} = ℰ; op_{nis,$k}]];
      SD[op_isp, op_{{is},$k}]; SD[op_sis__, op_{sis}];
     ] /. {SD → SetDelayed,
      isp → {is} /. {i → i_, j → j_, k → k_},
      nis → {is} /. {i → ii, j → jj, k → kk},
      nisp → {is} /. {i → ii_, j → jj_, k → kk_}
     }] ]]
```

The Basic Tensors

(Alt) In[∘]:=
```
Define[m_{i,j→k} = 𝔼_{{i,j}→{k}}[1, -ξi ηj + (ηi + ηj) yk + (ξi + ξj) xk, ϵSeries[0]]]
```

(Alt) Out[∘]= $\text{Define}\left[\mathbb{E}_{\{i,j\}\to\{k\}}\left[1, y_k\,(\eta_i+\eta_j) - \eta_j\,\xi_i + x_k\,(\xi_i+\xi_j), \epsilon\text{Series}[0]\right]\right]$

(Alt) In[∘]:=
```
AllMonomials[{}, 0] = {1};
AllMonomials[{}, d_Integer] /; d > 0 := {};
AllMonomials[{v_, vs___}, d_Integer] :=
  Join @@ Table[v^(d-k) AllMonomials[{vs}, k], {k, 0, d}];
AllMonomials[vs_List, {d_}] := Join @@ Table[AllMonomials[vs, k], {k, 0, d}];
```

(Alt) In[∘]:=
```
Basis[js_List, m_] := Flatten@Outer[Times,
    AllMonomials[Table[y_j, {j, js}], m], AllMonomials[Table[x_j, {j, js}], m]];
Basis[js_List, {m_}] := Flatten@Table[Basis[js, k], {k, 0, m}]
```

(Alt) In[∘]:=
```
Basis[{i, j}, {2}]
```

(Alt) Out[∘]= $\{1, x_i\,y_i, x_j\,y_i, x_i\,y_j, x_j\,y_j, x_i^2\,y_i^2, x_i\,x_j\,y_i^2, x_j^2\,y_i^2, x_i^2\,y_i\,y_j, x_i\,x_j\,y_i\,y_j, x_j^2\,y_i\,y_j, x_i^2\,y_j^2, x_i\,x_j\,y_j^2, x_j^2\,y_j^2\}$

(Alt) In[∘]:=
```
GenericCombination[bas_, c_] := bas.Table[c_j, {j, Length@bas}];
GenericCombination[bas_, c_k_] := bas.Table[c_{k,j}, {j, Length@bas}];
```

(Alt) In[∘]:=
```
GenericCombination[Basis[{i, j}, {2}], c_1]
```

(Alt) Out[∘]= $c_{1,1} + x_i\,y_i\,c_{1,2} + x_j\,y_i\,c_{1,3} + x_i\,y_j\,c_{1,4} + x_j\,y_j\,c_{1,5} + x_i^2\,y_i^2\,c_{1,6} + x_i\,x_j\,y_i^2\,c_{1,7} + x_j^2\,y_i^2\,c_{1,8} + x_i^2\,y_i\,y_j\,c_{1,9} + x_i\,x_j\,y_i\,y_j\,c_{1,10} + x_j^2\,y_i\,y_j\,c_{1,11} + x_i^2\,y_j^2\,c_{1,12} + x_i\,x_j\,y_j^2\,c_{1,13} + x_j^2\,y_j^2\,c_{1,14}$

(Alt) In[ ]:=
```
R_{i_,j_} := 𝔼_{{}→{i,j}}[1, (-1 + T) x_j (y_i - y_j), ϵSeries @@
    Prepend[0]@Table[GenericCombination[Basis[{i, j}, {k + 1}], c_k], {k, $k}]];
R̄_{i_,j_} := 𝔼_{{}→{i,j}}[1, (-1 + 1/T) x_j (y_i - y_j), ϵSeries @@
    Prepend[0]@Table[GenericCombination[Basis[{i, j}, {k + 1}], d_k], {k, $k}]];
CC_{i_} := 𝔼_{{}→{i}}[√T, 0, ϵSeries @@ Prepend[0]@
    Table[GenericCombination[Basis[{i}, {k + 1}], e_k], {k, $k}]];
C̄C̄_{i_} := 𝔼_{{}→{i}}[1/√T, 0, ϵSeries @@ Prepend[0]@
    Table[GenericCombination[Basis[{i}, {k + 1}], f_k], {k, $k}]];
```

Setting the Coefficients

As solved in “Solving to k=3.nb”.

$$\{c_{1,1}, c_{1,2}, c_{1,3}, c_{1,4}, c_{1,5}, c_{1,6}, c_{1,7}, c_{1,8}, c_{1,9}, c_{1,10}, c_{1,11}, c_{1,12}, c_{1,13}, c_{1,14}, d_{1,1}, d_{1,2}, d_{1,3}, d_{1,4}, d_{1,5}, d_{1,6}, d_{1,7}, d_{1,8}, d_{1,9}, d_{1,10}, d_{1,11}, d_{1,12}, d_{1,13}, d_{1,14}, e_{1,1}, e_{1,2}, e_{1,3}, f_{1,1}, f_{1,2}, f_{1,3}\} = \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{2}(-1+T), 0, 1, \frac{1}{2}(1-3T), 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{-1+T}{T^2}, -\frac{1-T}{2T^3}, 0, -\frac{1}{T^2}, -\frac{-1-T}{2T^3}, 0, 0, 0, 0, -\frac{1}{T}, 0, 0, \frac{1}{T}, 0\right\};$$

(Alt) Out[ ]=

$$\left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{2}(-1+T), 0, 1, \frac{1}{2}(1-3T), 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{-1+T}{T^2}, -\frac{1-T}{2T^3}, 0, -\frac{1}{T^2}, -\frac{-1-T}{2T^3}, 0, 0, 0, 0, -\frac{1}{T}, 0, 0, \frac{1}{T}, 0\right\}$$

$$\{c_{2,1}, c_{2,2}, c_{2,3}, c_{2,4}, c_{2,5}, c_{2,6}, c_{2,7}, c_{2,8}, c_{2,9}, c_{2,10}, c_{2,11}, c_{2,12}, c_{2,13}, c_{2,14}, c_{2,15}, c_{2,16}, c_{2,17}, c_{2,18}, c_{2,19}, c_{2,20}, c_{2,21}, c_{2,22}, c_{2,23}, c_{2,24}, c_{2,25}, c_{2,26}, c_{2,27}, c_{2,28}, c_{2,29}, c_{2,30}, d_{2,1}, d_{2,2}, d_{2,3}, d_{2,4}, d_{2,5}, d_{2,6}, d_{2,7}, d_{2,8}, d_{2,9}, d_{2,10}, d_{2,11}, d_{2,12}, d_{2,13}, d_{2,14}, d_{2,15}, d_{2,16}, d_{2,17}, d_{2,18}, d_{2,19}, d_{2,20}, d_{2,21}, d_{2,22}, d_{2,23}, d_{2,24}, d_{2,25}, d_{2,26}, d_{2,27}, d_{2,28}, d_{2,29}, d_{2,30}, e_{2,1}, e_{2,2}, e_{2,3}, e_{2,4}, f_{2,1}, f_{2,2}, f_{2,3}, f_{2,4}\} =$$

$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, -\frac{-1+4T-3T^2}{6T}, 0, -\frac{1}{2T}, -\frac{1-3T}{2T}, -\frac{1-11T+16T^2}{6T}, 0, 0, -\frac{1}{2}, \frac{1}{6}(-1+7T), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1-T}{T^3}, -\frac{-1+T}{2T^4}, 0, \frac{1}{T^3}, -\frac{1}{2T^4}, 0, 0, 0, 0, -\frac{-1+T}{2T^3}, -\frac{3-4T+T^2}{2T^4}, -\frac{-3+4T-T^2}{6T^5}, 0, -\frac{1}{2T^3}, \frac{2}{T^4}, -\frac{4+T+T^2}{6T^5}, 0, 0, -\frac{1}{2T^4}, -\frac{-1+T}{6T^5}, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{T^2}, 0, 0\right\};$$

(Alt) Out[ ]=

$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, -\frac{-1+4T-3T^2}{6T}, 0, -\frac{1}{2T}, -\frac{1-3T}{2T}, -\frac{1-11T+16T^2}{6T}, 0, 0, -\frac{1}{2}, \frac{1}{6}(-1+7T), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1-T}{T^3}, -\frac{-1+T}{2T^4}, 0, \frac{1}{T^3}, -\frac{1}{2T^4}, 0, 0, 0, 0, -\frac{-1+T}{2T^3}, -\frac{3-4T+T^2}{2T^4}, -\frac{-3+4T-T^2}{6T^5}, 0, -\frac{1}{2T^3}, \frac{2}{T^4}, -\frac{4+T+T^2}{6T^5}, 0, 0, -\frac{1}{2T^4}, -\frac{-1+T}{6T^5}, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{T^2}, 0, 0\right\}$$

$$\{c_{3,1}, c_{3,2}, c_{3,3}, c_{3,4}, c_{3,5}, c_{3,6}, c_{3,7}, c_{3,8}, c_{3,9}, c_{3,10}, c_{3,11}, c_{3,12}, c_{3,13}, c_{3,14}, c_{3,15}, c_{3,16}, c_{3,17}, c_{3,18}, c_{3,19}, c_{3,20}, c_{3,21}, c_{3,22}, c_{3,23}, c_{3,24}, c_{3,25}, c_{3,26}, c_{3,27}, c_{3,28}, c_{3,29}, c_{3,30}, c_{3,31}, c_{3,32}, c_{3,33}, c_{3,34}, c_{3,35}, c_{3,36}, c_{3,37}, c_{3,38}, c_{3,39}, c_{3,40}, c_{3,41}, c_{3,42}, c_{3,43}, c_{3,44}, c_{3,45}, c_{3,46}, c_{3,47}, c_{3,48}, c_{3,49}, c_{3,50}, c_{3,51}, c_{3,52}, c_{3,53}, c_{3,54}, c_{3,55}, d_{3,1}, d_{3,2}, d_{3,3}, d_{3,4}, d_{3,5}, d_{3,6}, d_{3,7}, d_{3,8}, d_{3,9}, d_{3,10}, d_{3,11}, d_{3,12}, d_{3,13}, d_{3,14}, d_{3,15}, d_{3,16}, d_{3,17}, d_{3,18}, d_{3,19}, d_{3,20}, d_{3,21}, d_{3,22}, d_{3,23}, d_{3,24}, d_{3,25}, d_{3,26}, d_{3,27}, d_{3,28}, d_{3,29}, d_{3,30}, d_{3,31}, d_{3,32}, d_{3,33}, d_{3,34}, d_{3,35}, d_{3,36}, d_{3,37}, d_{3,38}, d_{3,39}, d_{3,40}, d_{3,41}, d_{3,42}, d_{3,43}, d_{3,44}, d_{3,45}, d_{3,46}, d_{3,47}, d_{3,48}, d_{3,49}, d_{3,50}, d_{3,51}, d_{3,52}, d_{3,53}, d_{3,54}, d_{3,55}, e_{3,1}, e_{3,2}, e_{3,3}, e_{3,4}, e_{3,5}, f_{3,1}, f_{3,2}, f_{3,3}, f_{3,4}, f_{3,5}\} =$$

$$\Big\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1-T}{6T}, 0, 0, \frac{1}{2T}, -\frac{-2+5T}{2T},$$
$$0, 0, 0, \frac{5}{6}, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1-12T+27T^2-16T^3}{24T^2}, 0, \frac{1}{6T^2}, -\frac{-1+3T}{4T^2},$$
$$-\frac{-1+11T-16T^2}{6T^2}, -\frac{-1+31T-131T^2+125T^3}{24T^2}, 0, 0, \frac{1}{T}, -\frac{-5+23T}{6T}, -\frac{-5+69T-142T^2}{24T},$$
$$0, 0, 0, \frac{1}{6}, \frac{1}{24}(1-15T), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{-1+T}{T^4}, -\frac{1-T}{2T^5}, 0, -\frac{1}{T^4},$$
$$\frac{1}{2T^5}, 0, 0, 0, 0, -\frac{1-T}{T^4}, -\frac{-7+9T-2T^2}{2T^5}, -\frac{7-9T+2T^2}{6T^6}, 0, \frac{1}{T^4}, -\frac{9-T}{2T^5}, \frac{3}{2T^6},$$
$$0, 0, \frac{1}{T^5}, -\frac{1}{3T^6}, 0, 0, 0, 0, 0, -\frac{-1+T}{6T^4}, -\frac{2-3T+T^2}{T^5}, -\frac{-16+27T-12T^2+T^3}{6T^6},$$
$$-\frac{16-27T+12T^2-T^3}{24T^7}, 0, -\frac{1}{6T^4}, -\frac{-3+T}{T^5}, \frac{3(-3+T)}{2T^6}, -\frac{-27+5T-T^2-T^3}{24T^7}, 0, 0, -\frac{1}{T^5},$$
$$\frac{2}{T^6}, -\frac{12-T-5T^2}{24T^7}, 0, 0, 0, -\frac{1}{6T^6}, -\frac{-1-T}{24T^7}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{T^3}, 0, 0, 0\Big\};$$

(Alt) Out[∘]=

$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1-T}{6T}, 0, 0, \frac{1}{2T}, -\frac{-2+5T}{2T}, 0, 0, 0, \frac{5}{6}, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1-12T+27T^2-16T^3}{24T^2}, 0, \frac{1}{6T^2}, -\frac{-1+3T}{4T^2}, -\frac{-1+11T-16T^2}{6T^2}, -\frac{-1+31T-131T^2+125T^3}{24T^2}, 0, 0, \frac{1}{T}, -\frac{-5+23T}{6T}, -\frac{-5+69T-142T^2}{24T}, 0, 0, 0, \frac{1}{6}, \frac{1}{24}(1-15T), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{-1+T}{T^4}, -\frac{1-T}{2T^5}, 0, -\frac{1}{T^4}, \frac{1}{2T^5}, 0, 0, 0, 0, -\frac{1-T}{T^4}, -\frac{-7+9T-2T^2}{2T^5}, -\frac{7-9T+2T^2}{6T^6}, 0, \frac{1}{T^4}, -\frac{9-T}{2T^5}, \frac{3}{2T^6}, 0, 0, \frac{1}{T^5}, -\frac{1}{3T^6}, 0, 0, 0, 0, 0, -\frac{-1+T}{6T^4}, -\frac{2-3T+T^2}{T^5}, -\frac{-16+27T-12T^2+T^3}{6T^6}, -\frac{16-27T+12T^2-T^3}{24T^7}, 0, -\frac{1}{6T^4}, -\frac{-3+T}{T^5}, \frac{3(-3+T)}{2T^6}, -\frac{-27+5T-T^2-T^3}{24T^7}, 0, 0, -\frac{1}{T^5}, \frac{2}{T^6}, -\frac{12-T-5T^2}{24T^7}, 0, 0, 0, -\frac{1}{6T^6}, -\frac{-1-T}{24T^7}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{T^3}, 0, 0, 0\right\}$$

(Alt) In[∘]:= $\mathbf{f}_{3,2}$

(Alt) Out[∘]= $\frac{1}{T^3}$