

251211  **Paper.** *Car traffic on Knots and the Seifert Pairing.*

251210 Challenge ChatGPT on $\{\text{genus, fibered, ribbon}\} \times \{\Delta, \theta\}$.

230811 **Thm** (cf. Lickorish pp. 50). Module presentations $R_i \xrightarrow{\alpha_i} G_i \rightarrow M$, $i = 1, 2$ are equivalent iff $\exists \beta_i, \gamma_i, \eta_i$ as here s.t. $\beta_1 \alpha_1 = \alpha_2 \gamma_1$, $\beta_2 \alpha_2 = \alpha_1 \gamma_2$, $\beta_2 \beta_1 + \alpha_1 \eta_1 = I$, and $\beta_1 \beta_2 + \alpha_2 \eta_2 = I$. Then $\begin{pmatrix} I & 0 \\ \beta_1 & \alpha_2 \end{pmatrix} = \begin{pmatrix} \alpha_1 & \beta_2 \\ 0 & I \end{pmatrix} \begin{pmatrix} \eta_1 & -\gamma_2 \\ \beta_1 & \alpha_2 \end{pmatrix}$ and $\begin{pmatrix} \alpha_1 & \beta_2 \\ 0 & I \end{pmatrix} = \begin{pmatrix} I & 0 \\ \beta_1 & \alpha_2 \end{pmatrix} \begin{pmatrix} \alpha_1 & \beta_2 \\ -\gamma_1 & \eta_2 \end{pmatrix}$ so elementary ideals make sense and (if $\exists \alpha_i^{-1}$) $\alpha_1^{-1} = \gamma_2 \alpha_2^{-1} \beta_1 + \eta_1$ and $\alpha_2^{-1} = \gamma_1 \alpha_1^{-1} \beta_2 + \eta_2$. Implement for null vertices / chain graphs! Implement for all Alexander presentations!

251208b In Projects/Theta2, **Paper.** “Gaussian Integration, Tangles, Seifert Surfaces, and Θ ”: \S as functorially as possible; the Wirtinger Θ for tangles and \S ; a revised Seifert algorithm; Θ for tangled thick graphs; the genus bound. Preliminary: Implement.

251208a **Claim.** For some fixed V , given a Seifert surface Σ of genus g and complexity n , in time $\sim n^3$ (?) we can compute \mathcal{L} on $V \otimes H_1(\Sigma)$ such that $\theta \sim \int_{V \otimes H_1} \oplus^{\mathcal{L}}$ is computable in time $\sim g^5$ (?).

251202 Tube equivalence for thick graph Seifert surfaces is local!

251127 Sconza, Wildi: “A Knot-based Key Exchange protocol”.

251124 The Turaev cobracket is naively well-defined on closed curves, but only non-naively using emergent knots, requiring a detour through long knots. Does this indicate that we should be looking at a restricted or extended class of emergent knots?

251117 BBS: SantosK-251117: Dehn is a $V \rightarrow F$ Alexander presentation. There’s also a (co-Dehn?) $F \rightarrow V$ presentation.

251118b Is there a compatible expansion for emergent knotted graphs in a PDS? Is it enough to recover the Turaev cobracket?

251118a **Knot Families.** Torus knots; pretzel knots; 2-bridge knots; twisted torus knots [arXiv:2411.13003](https://arxiv.org/abs/2411.13003); knots with diagonal grid diagrams [arXiv:2412.13796](https://arxiv.org/abs/2412.13796); Montesinos knots; double-torus knots (Hirasawa, Murasugi); torti-rational knots; the Conway and Kinoshita-Terasaka families [arXiv:2409.03557](https://arxiv.org/abs/2409.03557).

251108 **Q.** Does every finite type invariant have a Seifert formula? With τ for thickening, $\tau_* : \coprod \mathcal{A}(\text{graphs}) \rightarrow \coprod \mathcal{A}^\vee(\bigcirc^*)$.

251031 Perhaps the key to the homomorphicity of the GoTu co-action is to add “vertices”?

251021 **Proj.** SeifertBox.nb, with container TG[seq, BX[uc, ui, uj, lc, li, lj]..., BT[?]] with TG: Thick Graph, BX: Band Xing, uc: upper colour, etc., BT: Band Twist. With conversions to/from PD, with Draw, DeTwist, DeXing, reductions, and Δ .

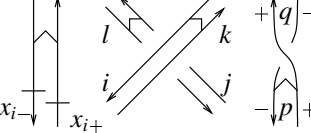
250731 With Santos: • A classicality condition for tangles from the arrow polynomial. • Categorify the arrow polynomial for tangles. • Does Miller’s homological arrow polynomial generalizes to other R -matrix invariants?

251014 **Q.** Ribbon concordances and Θ ?

251009 **Talk.** “Braids, a Representation, a Cocycle, an Integral, an Invariant”.

250414b Boninger: The torus is flat so rotation numbers make sense so YB+R invariants of knots make sense for knots on a torus.

Q. Does the affinity with the Kontsevich integral persist?

251006  Band flows: $n_i := x_{i+} - x_{i-}$ “net flow” and $a_i := x_{i+} + x_{i-}$ “absolute flow”. Satisfy: $n_k = n_i$, $n_l = n_j$, $a_k = a_i$, $a_l = a_j + (T^{-1} - 1)n_i$, $x_{i-} = x_{i+} - n_i$, $n_q = n_p$, $a_q = T a_p + (T - 1)n_p$.

250925 **Fact.** Every n -crossing knot is the \pm -twist thickening of a bipartite planar graph with $\sim n$ edges. A Reidemeister theory?

251003b **Proj.** “Categorify” the Goldman-Turaev operations.

251003a Circles are evil! Can we say “skein relations” without saying “circles”?

250929a What’s next? • A homological interpretation of \tilde{g}_{ab} ? • Prove that ρ_1 is palindromic? • Prove that $\rho_1 = -\theta(T, 1)$? • Work out “Seifert formulas from integration”? • “Traffic on the edge” of a Seifert surface, to a genus bound? • Finally read Kricker/Ohtsuki? • Fix the issue(s) and write up signatures for tangles? • A paper on an $F \times F$ Alexander for tangles? • Finish the DoPeGDO paper? • Some Fiedler work? • Some GoTu/KV work?

250929b **Q.** Is there a tangle version of the higher Alexander polynomials, to drop the complexity from $\exp(Cn)$ to $\exp(C\sqrt{n})$?

250928 **Q.** Does every finite type invariant have a Seifert formula?

250923 **Wanted.** GPV formulas for algorithm-style Seifert surfaces.

250814 **Q.** Is there an arrow version of the Kauffman polynomial?

250812 Naef, Willwacher [arXiv:2508.08081](https://arxiv.org/abs/2508.08081): KV/GRT computations to weight 29.

250516 **Q.** Is there a tangle proof that all Seifert surfaces of a knot are tube-equivalent? What’s the “right” tangle tube-equivalence?

250711 Is there a presentation matrix meaning to $TU - U^T$, where U is the $F \times F$ semi-Seifert pairing?

250709 Brittenham, Hermiller: [arXiv:2506.24088](https://arxiv.org/abs/2506.24088), *Unknotting number is not additive under connected sum*.

250704 With \mathcal{A}^{ugl} as in BBS:Dancso-170815–BBS:Dancso-170817, with \mathcal{A}^{sgl} similarly and as in BBS:Dancso-250704 and with $sDer := \text{im}(\mathcal{P}\mathcal{A}^{ugl} \rightarrow \mathcal{P}\mathcal{A}^{wgl})$, the commutator is the graded Goldman bracket and div is the graded Turaev cobracket. How are \mathcal{A}^{em} and \mathcal{A}^{wgl} related?

250702 **Q.** tDer, sDer, etc., are they the gr of anything? Do they have a tensorial interpretation?

250624b **Q.** Is there a relationship between SolKV and 1-loop associators? Between 1-loop associators and emergent associators?

250624a **Q.** Does \mathcal{A}_p^w act on $\mathcal{A}_{p,s}^{em}$? (Probably not).

250603 **Q.** Is there an ideal triangulation formula for Δ ? And θ ?

250529 **Talk.** “Piggyback Matchbox Burau Representations”. Needs a few further root system examples.

250515 **Proj.** Dirty-implement normal surfaces.

231013b **Def.** A Measured Partial Quadratic (MPQ) on a v.s. A is a triple $Q = (D, v, q)$ where q is a non-degenerate quadratic on a subspace $D \subset A$ and v is a volume element on D . Given such Q and a $\varphi \in A^*$, $\int_D dv \exp(-q/2 + \varphi)$ makes sense.

Conj. Given a linear $L: A \rightarrow B$ and an MPQ Q on A there is a unique MPQ $L_*Q = (D_*, v_*, q_*)$ on B such that for every $\varphi \in B^*$ we have $\int_{D_*} dv_* \exp(-q_*/2 + \varphi) = \int_D dv \exp(-q/2 + L^*\varphi)$.

250414a Proj. Find formulas for the weight systems of ρ_1 and θ .

250309b Q. Is there a direct $\{\text{expansions for emergent KO}\} \rightarrow \{\text{expansions for wKO}\}$?

250309a Chal. Understand non-orientable surfaces and the Goeritz Alexander formula.

250227 Q. Is there a (semi)-Seifert formula for the MVA?

250131 Proj. Find a Seifert-slides invariant face formula for θ .

241218 Proj. Use BF to solve KV.

241214 Talk. “Goldman-Turaev by ambience and constraint”.

241112 Q. What means Alekseev-Torossian’s ν : $\text{grt}_1 \rightarrow \text{frv}_2$ by $\varphi \mapsto (\varphi(y, x), \varphi(x, y))$? A Lie algebra map by their Prop. 4.9.

241113 Proj. Find an ss vertex that solves the pss R4 (and unitarity and cap) in \mathcal{A}^{wgh} and in \mathcal{A}^{wem} .

221028 Needed. A homological interpretation of $g_{\alpha\beta}(K)$. Is it the $?$ -invariant of a 3-component tangle $K \cup L_{\alpha\beta}$?

241105 Q. Do all homotopy invariants satisfy the H_1 relation? Is this an equivalence?

160330 UGP: An “Insolubility of the Quintic” web site.

241104a UGP: Properly draw knots, even large.

241029 Proj. Figure out Chern-Simons with \mathfrak{g}_+^ϵ .

241022 Q. Is there a $-\bigcirc-$ invariant for a pair of homology classes on a knotted surface?

240918 Proj. Study “emergent KV equations”. What’s the ss filtration? By tails on strands? By arrows on strands? By chords on strands?

240904 Is there a determinant formula for the Conway polynomial?

240721 Needed: A groupoid view of π_1 , Δ , ρ_1 , and θ .

240715 \mathcal{K}^{wem} : w-knotted objects with a marked “emergent” subgraph, meaning we mod out by having two semi-virtuals / arrows whose over / tail side is emergent. **Proj.** Use pole dancing ideas to find Goldman-Turaev within \mathcal{K}^{wem} and construct a homomorphic expansion for \mathcal{K}^{wem} (hence for Goldman-Turaev) using a homomorphic expansion for \mathcal{K}^w (hence using Kashiwara-Vergne).

240628 Q. Is $\text{grt} \simeq \text{frv}$ a version of “braids act nearly transitively on simple curves”?

240624 Proj. Study “timelined tangles”. Are there homomorphic expansions?

240523 What if Conway preceded Alexander?

240516 AP: Projects: HigherRank: TheCast has the Feynman ring for sl_3 , Heisenberg version.

240510 Started **AP: People: Kuno: CheatSheet**.

240502 The Drinfel’d Double: Given bialgebras A, B , an invertible non-degenerate $R \in A \otimes B$ defines double-meta-bialgebra morphism $\{POUB_{X,Y}\} \rightarrow \{A^{\otimes X} \otimes B^{\otimes Y}\}$. **Thm?** Up to ??, there is a unique bialgebra structure on $D := A \otimes B$ such that the diagram on the right commutes.

$$\begin{array}{ccc} \{POUB_{X,Y}\} & \xrightarrow{Z:=X \sqcup Y} & \{PB_Z\} \\ \downarrow & & \downarrow \\ \{A^{\otimes X} \otimes B^{\otimes Y}\} & \xrightarrow{Z:=X \sqcup Y} & \{D^{\otimes Z}\} \end{array}$$

240322 AP: Projects: HigherRank: Rank-2 Gassner:

R2G_{1,j}[_] $[\varepsilon_-] := \text{Expand}[\varepsilon / \{$

$\mathbf{e}_j \mapsto \mathbf{T}_i \mathbf{e}_j + (1 - \mathbf{T}_i) \mathbf{e}_i, \mathbf{f}_j \mapsto \mathbf{S}_i \mathbf{f}_j + (1 - \mathbf{S}_i) \mathbf{f}_i,$

$\mathbf{g}_j \mapsto \mathbf{T}_i \mathbf{S}_i \mathbf{g}_j + (1 - \mathbf{T}_i \mathbf{S}_i) \mathbf{g}_i$

$+ (\mathbf{S}_i + \mathbf{T}_i - 2 \mathbf{S}_i \mathbf{T}_i) \mathbf{e}_i \mathbf{f}_i + \mathbf{S}_i (\mathbf{T}_i - 1) \mathbf{e}_i \mathbf{f}_j + \mathbf{T}_i (\mathbf{S}_i - 1) \mathbf{e}_j \mathbf{f}_i \}$

The last line is a “cross-rep glow”. **Prob.** Find all cross-rep glows between products of Gassners at different T parameters. There too: a more general rank 2 Burau.

240415 What are burves, plane curves mod braid moves?

240404 MAT 198 Cryptology dreams: Enigma, compression, DES, RSA, Bible codes, Bitcoin, homomorphic computation.

240401 Projects: APAI: APAILinks2.nb: Why does ρ_1 makes sense for links? Does it always vanish when the MVA does? An $MV\rho_1$?


240328 Proj. Say “Rasmussen’s s ” w/o saying “spectral sequence”.

240305a Boninger’s conjecture: The coefficients of $\rho_1(K)$ in z are uniform-sign for positive K . Tested in **Rho_d-Positivity.nb** in **AP: T-: Oaxaca-2210**. True for Δ by Cromwell’s “Homogeneous Links”.

240319 Hartley: *The Conway Potential Function for Links*. Benheddi, Cimasoni: *Link Floer Homology Categorifies the Conway Function*.

240311 2024-03: Implementation of Kauffman States for tangles. Is there a pull-push formalism for PA operations?

231114 Ozsváth, Szabò: [arXiv:1603.06559](#), *Kauffman states, bordered algebras, and a bigraded knot invariant*, [arXiv:2212.11885](#), *The pong algebra*, [arXiv:2311.07503](#), *Planar graphs deformations of bordered knot algebras*. Zibrowius: [arXiv:1601.04915](#), *Kauffman states and Heegaard diagrams for tangles*. Roberts: [arXiv:math/0607244](#), **AGT 2009**, *Heegaard-Floer homology and string links*.

240305b : Kauffman states \leftrightarrow tree-dual-tree pairs in the checkerboard graphs \leftrightarrow one-long-path smoothings.

240304 Is the a local / size respecting description of knots \hookrightarrow bnots? In some completion? Being liberal about R 1s? Allowing virtuals?

240302 JFF. Compute the sign of a length 10^6 permutation, 10 of whose entries are hidden.

240301 Q. A local description of the Wirtinger-Alexander det-signs?

240221b In Kauffman’s *Formal Knot Theory*, A state model for the *normalized* Alexander polynomial. Implementation: **VxF_Alexander.nb** in **AP: Pe-: Martchenkov**. \rightarrow **p4:230609**.

240221a “The Alexander module dogma is wrong”.

170317 Wikipedia: q -derivative: $D_{q,x}f(x) = \frac{f(qx)-f(x)}{qx-x}$; has $D_{q,x}\mathbf{e}_q^x = \mathbf{e}_q^x$ (and $\mathbf{e}_q^0 = 1$); seek it and \mathbf{e}_q^x and $xy = qyx$ in nature. Finds: $[a, x] = x \Rightarrow \mathbf{e}^{ta}x = \mathbf{e}^t x \mathbf{e}^{ta}$. Also, in tensor powers with $X_k := \mathbf{e}^{t(a_1+\dots+a_{k-1})}x_k$, have $X_k X_l = \mathbf{e}^t X_l X_k$ for $k < l$.


240213 Q. Do braids act on multi-Fox profiles of free words?

190320 Proj. Analyze **2016-06/Turbo-Gassner** (also **Talks/Toronto-1912/GvIExamples**). Is it homological? Find it in Artin’s representation. Following **Ito@M19**, relations with Garside lengths?

240209b Jim Davis: A Witt-valued quadratic pushforward story?

240209a Calaque, Roca i Lucio [arXiv:2402.05539](#) *Associators from an operadic point of view*, a survey of associators.

240130 Garoufalidis, Kashaev [arXiv:2311.11528](#) *Multivariable knot polynomials from braided Hopf algebras with automorphisms*. Also has a Drinfel’d double alternative.

240122 : With Z a domain and Q its field of fractions, given $\partial: R \rightarrow G$ invertible over Q and $(\cdot, \cdot): R \otimes G \rightarrow Z$ with $(r_1, \partial r_2) = (r_2, \partial r_1)$ get a symmetric $\langle \cdot, \cdot \rangle: (\text{coker}_Z \partial)^{\otimes 2} \rightarrow Q/Z$ by $\langle \bar{g}_1, \bar{g}_2 \rangle := (\partial_Q^{-1} g_1, g_2)/Z$. **Q.** When do two presentations yield equivalent $\langle \cdot, \cdot \rangle$ ’s?

240118 Proj. Understand “tangle nullities” (cf. knot nullities).

240115 Borodzik, A. Conway, Politarczyk [arXiv:2111.10632](#), Section 1.1: TL signatures from matrices presenting Blanchfield. Kearton (1978): A edge-centric matrix presenting Blanchfield.

240103 Formal perturbed Gaussian integration, sans the determinant prefactor, is invariant under *all* linear coordinate changes.

240102b Projects: [APAI: PerturbedGaussianIntegration.nb](#): With K a knot diagram, w its writhe, πK its rotation by 180° , \mathcal{R}_1 the integrand for ρ_1 , $\mathcal{P} = \left\{ \begin{matrix} x_{2n+1} \rightarrow p_1, \\ p_{2n+1} \rightarrow x_{2n+1} \end{matrix} \right\} \cup \bigcup_{c:(s,i,j) \in K} \left\{ \begin{matrix} x_i \rightarrow T^{v_i}(p_1 - p_{i+1}), p_i \rightarrow T^{-v_i} x_i, p_j \rightarrow T^{-v_i - s} x_j, \\ x_j \rightarrow T^{v_i}((1 - T^s)p_{i+1} + T^s p_1 - p_{j+1}) \end{matrix} \right\}$ with v the

Alexander numbering, get $\{\overline{\mathcal{R}_1(\pi K)} = T^{-w} \mathcal{R}_1(K) / \mathcal{P}$.

240102a 2023-12: BridgesAndTunnels.nb: Palindromicity using bridges and tunnels.

231214b Q. Are there “dual presentations” signature formulas?

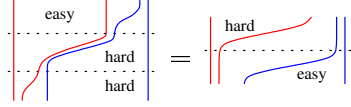
231214a Q. Do virtual knots have a meaningful Dehn fundamental group? Is it related to the two Wirtinger ones? Are there others?

231212 In the knot complement cyclic cover context, $0 \rightarrow H_1(\tilde{X}) \rightarrow H_1(\tilde{X}, \tilde{p}) \xrightarrow[\partial]{\partial} \ker(i_*) \rightarrow 0$ is split exact, where $\mathbb{Z}[T^{\pm 1}] = H_0(\tilde{p}) \xrightarrow{i_*} H_0(\tilde{X}) = \mathbb{Z}$ so $H_1(\tilde{X}) \simeq \ker \partial \simeq H_1(\tilde{X}, \tilde{p}) / \text{im } s$.

230406 The AKKN operational envelope: Cuts, then doubles (and reversals), then letter substitutions, then infusion of constants, then merges.

231205 López Neumann, van der Veen: [arXiv:2312.02070](#), “Genus bounds from unrolled quantum groups at roots of unity”.

230404 Q. Is the pentagon in emergent 2-poles 2-strands equivalent to the standard FL -pentagon? (The spaces grow slower!) Can Φ_{pps} be found degree by degree? Is there a GT group? Isomorphic to a known variant?



220720 In framed Σ^2 , mod homotopy, curves are the same as “sailing curves”, immersions whose tangent avoids direct front winds.

231105a Implicit in Chrisman, Todd [arXiv:2307.09387](#): $\mathcal{X}K(D) = D \cup \omega$, where $\omega \in H^1(\underline{D})$ is the self-intersection 1-form of \underline{D} , the underlying graph of D . Is there a rotational version? \rightarrow [p5:141113b](#)

231103 Tautology: Alexander numbering \Leftrightarrow homologically trivial.

210818 Abbasi’s $uK \hookrightarrow vK$, “opposite inner-most pairs of non-local R2s can be removed”: An “even set of xings” has even incidence with every face. Non-empty ones transport through local R-moves, get created when a non-local R2 is performed, and unions of even sets can be projected preserving R-moves. Now use Abbasi’s zigzag trick to reduce the xing-number profile between non-local moves. **Qs.** Links? Rotational virtuals? Braid-like R-moves? Does $uK \hookrightarrow vK$ preserve 1-cycles? Can these ideas be used to prove Satoh’s conjecture?

231014 Papers/kts: Why the dots?

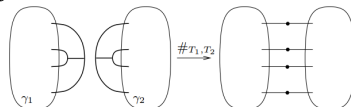
• $\rightarrow v^{1/2}$, with $v = Z(\bigcirc)$.

231013a Turaev’s [arXiv:math/0310218](#)

“Virtual Strings” has “based matrices” and sliceness criteria.

231009 Best w -practices:

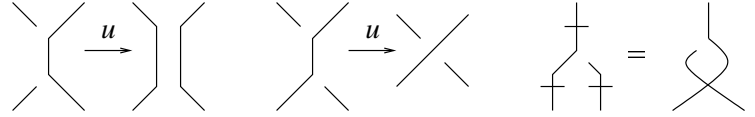
1. Trivalent tangles are end-labeled, to make a circuit algebra.
2. Tubes are bare (no colours and/or orientations).
3. Crossings have no signs; filtration is by comparison with virtual crossings.



4. Vertices are oriented and have marked legs: stem (s), upper (u), and lower (l). They are classical: they satisfy both R4s.

5. Wenjugating interchanges the two vertex types, and adds a virtual $l \leftrightarrow u$ crossing.

6. Only stems can be unzipped. Unzipping unwened edges connects u to u above a connection of l to l . Unzipping through a wen is defined by wenjugating it out.



231010 In [arXiv:1612.05641](#), a factorization of half-Dehn:



231008 Is there a finitely presented algebraic structure made of un-oriented pure tangles?

231002b Projects: [FullDoPeGDO: cR.nb](#): in sl_{2+}^0 , $(x_1, x_2) \cdot R_{12} = R_{12} \cdot (x_1 + (1 - B_1)x_2, B_1x_2)$, and $(y_1, y_2) \cdot R_{12} = R_{12} \cdot (y_1, \frac{b_2(1-B_1^{-1})}{b_1}y_1 + B_1^{-1}y_2)$.

231002a Proj. “Seven formulas for ρ_1 ”.

221228 Missing. A fully defined theory of pushing forward Gaussians (better with determinants and signatures). **Q.** Does the signature pushforward work also for \det' ?

230915 Frohlich’s trace: Let $A = \mathcal{U}(sl_{2+}^0)$ and $\phi: \mathbb{Q}[b, z, a] \rightarrow A_A$ by $b^k z^n a^m \mapsto b^k y^n a^m x^n / n!$ (surjective as $0 = [a, b^k y^n a^m x^l] = (l - n)b^k y^n a^m x^l$ in A_A). In A with $f = f(a)$ and $\nabla f := f(a) - f(a - 1)$, $[x, f] = -\nabla f \cdot x$ so in A_A , $0 = [x, y^{n+1} f x^n] = (n + 1)b y^n f x^n - y^{n+1} \nabla f x^{n+1}$ so $\phi(b z^n f) = \phi(z^{n+1} \nabla f)$. Ergo $\mathcal{G}(\text{tr}): \text{tr}(\otimes^{\beta b} \otimes^{\eta y} \otimes^{\alpha a} \otimes^{\xi x}) = \phi(\otimes^{\beta b} \otimes^{\eta y} \otimes^{\xi z} \otimes^{\alpha a}) = \phi(\otimes^{\beta z} \otimes^{\eta \xi z} \otimes^{\alpha a}) = \phi(\otimes^{\alpha a + (\eta \xi + \beta(1 - e^{-\alpha}))z})$. \rightarrow [p7:200906](#), \rightarrow [p7:180909a](#).

230911 $\mathcal{A}(*_*)$ is a contraction algebra.

230904 If cobrackets come from the asymmetry of coproducts, wherefore the Turaev cobracket?

230823 With $\Lambda := \mathbb{Z}[T^{\pm 1}]$ and $\Lambda_0 := \mathbb{Z}[T + T^{-1}]$, is it that for every Λ -module A there is a Λ_0 -module B with $A \oplus \bar{A} \equiv \Lambda \otimes_{\Lambda_0} B$?

230822 Figure out the Kashaev-formula weight system (for Δ^2 ?).

230817 Is there a Goeritz formula for Δ ?

230228b In [VanDerVeen_Journal](#): Given a diagram D for a long K , the phase ϕ along a curve $\gamma \subset D^c$ multiplies by T^s whenever γ passes over D with sign s . **Conj.** $\text{lk}_K(\alpha, \beta) = (T - 1) \langle \text{flow: generated by } \alpha, \text{ measured by } \beta \rangle + \langle \text{phased } \alpha \text{ over } \beta \text{ count} \rangle$. α generates $\pm \phi$ -flow when it runs over D . β measures $\pm \phi^{-1}$ -flow when it runs under D .

230804b Fox derivatives: in $H_1(\tilde{D}_n, \tilde{p})$, $[\gamma] = \sum (\partial_i \gamma) [x_i]$.

230804a Given a long K , is $K = (-K) \bmod \Delta \Delta$?

140422 Yajima’s “On Simply Knotted Spheres in \mathbb{R}^4 ”: all are ribbon. *Pf.* Enough: every simply-knotted balloon forest is equivalent to a ribbon-certificate. Take an inner-most double line on a balloon, slide out string ends and string transverses, and compress to a new string. When no double lines are left, float balloons to un-nest them, and iron wens to make string ends external. See [BBS: KAL-140520](#). I don’t fully understand the case of tubes.

230725 In w , what about $g \ltimes (g^* \oplus g^*)$? And flying graphs?

230702 BBS with Suzuki on Heisenberg doubles. Also her [arXiv:1612.08262](#), Kashaev's [arXiv:q-alg/9503005](#), Baseilhac's [arXiv:math/0202272](#) and [arXiv:1101.3440](#), and Baseilhac-Benedetti's [arXiv:1101.1851](#).

230425b López-Neumann@[K-OS], twisted Drinfel'd doubles: Let H be a f.d. \mathbb{N} -graded Hopf algebra and let $\theta(h) = t^{|h|}h$, $h \in H$. Virelizier (2000): for each $n \in \mathbb{Z}$ let $D_n = H^* \otimes H$ with multiplication as on the right. This is NOT a Hopf algebra (if $n \neq 0$) but there is a “co-product” $\Delta_{n,m}: D_{n+m} \rightarrow D_n \otimes D_m$ and antipode $S_n: D_n \rightarrow D_{-n}$ satisfying graded versions of Hopf axioms.

230425a López-Neumann@[K-OS]: $X \in \text{Rep}(DH)$ iff $X \in \text{Rep}(H)$ together with a collection of *half-braidings* $\{\sigma_{Y,X}: Y \otimes X \rightarrow X \otimes Y\}_{Y \in \text{Rep}(H)}$. Thus, $\text{Rep}(DH)$ is the *Drinfel'd Centre* of $(\text{Rep}(H), \otimes)$ (centre as for monoids, but it's a monoidal category!). It is a braided monoidal category.

230711 What's the canopoly envelope of Khovanov's Frobenius algebra as generated by $\{\succ, \odot_{\pm}, \otimes_{\pm}\}$?

230612b For matrices A, B , is $\{\lambda: \exists v Av = \lambda Bv \neq 0\}$ always finite? (No; take $A, B: \mathbb{R}_{x,y}^2 \rightarrow \mathbb{R}$ with $A = x + y$, $B = x$. Then with $v = (1, \lambda - 1)$, $Av = \lambda Bv = \lambda \neq 0$.)

230609 AlexanderUsingDehn.nb in [AP: P-: Martchenkov](#). [→p2:240221b](#).

230607 Abstract version of $\det \begin{pmatrix} A & B \\ C & U \end{pmatrix} = \det(A) \det(U - CA^{-1}B)$?

230606 Alexander: $tA - A^T$. Tristram-Levine: $(1 - \omega)A + (1 - \bar{\omega})A^T = (\bar{\omega} - 1)(\omega A - A^T)$.

230525 Are there other nearly-linear invariants of tangles, beyond linking numbers and signatures? [→p6:210114a](#)

230419 Sydney, Mar-Apr 2023. In [AP: A-: Alekseev_Kawa-: Annotated AKKN1](#). In [BBS: Hogan: Multiplication, exp, and \$\odot\$ in \$\mathcal{A}^{1p2s}\$ \(1 pole 2 strands\); implementation in \[AP: —: Hogan: Comp1ss.nb\]\(#\). In \[BBS: Dancso: “weakly parenthesized” tangles in a PDS, searching for div in strand doubling \\$\mathcal{A}^{2p1s} \rightarrow \mathcal{A}^{2p2s}\\$ \\(more likely KV2 is in pole doubling \\$\mathcal{A}^{1p1s} \rightarrow \mathcal{A}^{2p1s}\\$, in degree 1\\), the pentagon in \\$\mathcal{A}^{2p2s}\\$ \\(\\[→p3:230404\\]\\(#\\)\\).\]\(#\)](#)

230417 Proj. Extend knot colouring to tangles in Zombian language.

200611a Q. What conditions on $(\mathbb{A}, \mathbb{B}, \langle \cdot \rangle)$ are enough to make the Drinfel'd double associative? Examples beyond Hopf algebras? Restrict attention to braids?

230321b Proj. Develop a language to describe $\mathcal{A}^{/(k+1)\text{-co}}$ in terms of FA/FL . What are the atomic spaces and operations?

230321a López-Neumann and van der Veen, [arXiv:2211.15010](#), [\[K-OS\]](#), [email](#): A “twisted double” that may replace the need to deform the co-product.

230317b Q. In [PDS](#), is there an expansion for v -tangles?

230317a Proj. In [PDS](#), study emergent v -tangles and/or v_1 -tangles.

230309 In [PDS](#), is $\mathcal{A}^{*/+1}$ isomorphic to “*FL-dirty* \mathcal{A} ”? Is there a homomorphic ss -degree expansion $\mathcal{A}^* \rightarrow \mathcal{A}^{*/+1}$? Does the sequence $0 \rightarrow \langle \text{wheels} \rangle \otimes FA \rightarrow \mathcal{A}_H^{/2} \rightarrow FA \rightarrow 0$ split?

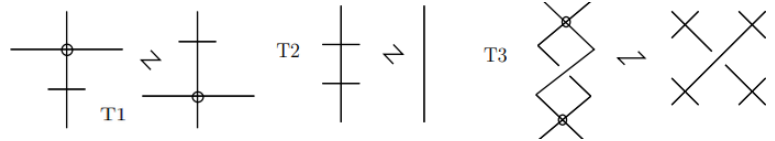
230315 In [2023-03: AlexanderLogConcavity.nb](#): Testing log-concavity for alternating knots, inspired by [arXiv:2303.04733](#) by Hafner, Mészáros, Vidinas.

230313 Hiroe, Negami, [arXiv:2303.05770](#) “Long-Moody Construction of Braid Representations and Katz Middle Convolution”.

220712b Construct homomorphic expansions for tangles in PDS_n ss -degree by ss -degree starting from $FG_n \rightarrow FA_n$.

230227 Equivariant linking numbers: • Kearton (~ 1978): Blanchfield formulas. • Kojima and Yamasaki (1979): First definition. • Kricker's and Garoufalidis' [arXiv:math/0105028](#): Definition from the Kontsevich Integral. • Garoufalidis' and Teichner's [arXiv:math/0206023](#): Abstract definition (also, indication that ρ_1 sees genus). • Ohtsuki 2007 “Invariants ... Surgery Presentations”: Some computations, probably no PD formula. • Lescop's [arXiv:1001.4474](#), [arXiv:1008.5026](#): Abstract definitions (also appearing: $(\log \Delta)'$). • Friedl's and Powell's [arXiv:1512.04603](#): Blanchfield formulas from Seifert surfaces.

230228a Negi, Prabhakar, Kamada [arXiv:2302.13244](#), twists in v -knots:



230217 Groningen, Jan-Feb 2023. In [Projects: MetaCalculi: UnitaryGamma.nb](#): A pairing with a Lagrangian property and an $\omega/\bar{\omega} = \det$ property for Γ -calculus (single variable). In [Talks: MoscowByWeb-2104: Hodge.nb](#): Hodge infrastructure for \mathcal{A} -calculus; exponentials are preserved, but unfinished unitarity property. Lashing fails at [Projects: APAI: Lashings.nb](#).

221118 Do. Unify Goldman-Turaev (GT) with equivariant intersection numbers. Understand the action of the braid group on GT (and its expansions). Related to the unitarity of Gassner?

230213b Do. Digest $S(V)/\Lambda(V)$ -exponentiation as “unfurling”.

230213a Re. $\omega\epsilon\beta/\text{mo21}$, is there a fast zero-test for linear combinations of Fermionic Gaussians?

230103 Q. Are there “*R-matrix*” long knot invariants in which a distinction is made between $R_{i < j}$ and $R_{i > j}$? “Scheduled tangles”?

230209 Chal. Find topological interpretations for QA Alexander formulas. I.e., what do specific matrix entries mean?

230203b Q. Might it be that determinant formulas for Alexander should depend on a presentation matrix along with a certificate that it presents a torsion module?

230203a Do. Analyze Alexander determinants for tangles similarly to the signature analysis.

230201b In Γ , is there an infinitesimal version? Are there $\text{im}(\alpha)$ “classicality” conditions?

230201a In Γ , for classical tangles $\bar{A} = \mathcal{L}_0 A$ with analytic \mathcal{L}_0 . Is $(\bar{A}, \bar{\omega}) = \mathcal{L}(A, \omega)$ with analytic \mathcal{L} ?

230118 Proj. Write `ValidatePD` and `PlanarQ` for `KnotTheory`.

230131 In Γ , truly understand the “flatness property” of ω .

221213 The signed number of ascending crossings is invariant under $R2/R3$.

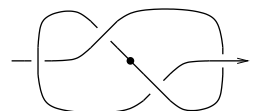
221212 Q. Is Alexander a counting of representations?

221207 Which Gauss diagram formulas vanish on classical knots/tangles? Tanglify “the linking matrix of a classical link is symmetric”.

221128 Q. Is there a 3D understanding of the balancing of $\Delta(K)$?

221114 Burton [tabulated](#) knots to 19 crossings.

221025b Boyle@Oaxaca: Develop a finite type theory for SNACKs, Strongly Negative AmphiChiral Knots (long knots in-



variant under 180° rotation with vertical flip, as 4_1). Probable examples in [arXiv:2206.03598](#). **Q.** If SNACKs are equivalent as knots, are they equivalent within SNACKs? Are they the same as knots in \mathbb{RP}^3 ? (No and no).

2210a5a Learn how to compute invariants of knots presented by surgeries. E.g., Boyle's@Oaxaca knot, [arXiv:2207.12593](#).

221007 With V the Burau representation of wB_n , there is a non-trivial splitting of $\mathbb{Q}wB_n \times \mathbb{H}(V \otimes V^*)[[\epsilon]]_{\text{doc}} \rightarrow \mathbb{Q}wB_n$. So what?

220928 **Q.** Given a classical K , \exists ? automorphism $\phi: \pi(K) = \pi \hookrightarrow$ s.t. $\bar{\phi}: \pi/[\pi, \pi] \cong \mathbb{Z} \hookrightarrow$ is (-1) ?

220920 In [Projects: RibbonKnots: Rho4Ribbons.nb](#): ($p = 1 - 3T + T^2$) $\mid \Delta(11_{n66})$ yet $p \nmid \rho_1(11_{n66})$. Is there a Fox-Milnor K and a p s.t. $p^2 \mid \Delta(K)$ yet $p \nmid \rho_1(K)$? \rightarrow [p5:220816](#)

220712a A quadratic form Q on V induces Q' on $V/\langle u \rangle$ via $Q'(x, y) = Q(u, u)Q(x, y) - Q(x, u)Q(u, y)$. Best if $Q(u, u) \neq 0$. Leads to a characterization of signatures?

220821 **Proj.** Find Duflo in Goldman-Turaev.

220816 **Q.** When the Alexander polynomial factors, is there a reason? (It factors more than Jones). Is every ribbon knot the sum of a w-knot with its mirror, modulo Alexander skein moves?

220807 **Proj.** Develop a "universal" Goldman-Turaev theory: With G a group and F a free group, with $|G| := G/\text{Ad } G$ and $|GG| := G \times G/\text{Ad } G$ (note the maps $|GG| \rightarrow |G| \times |G|$ and $|GG| \rightarrow |G|$) there are $b: \mathbb{Q}[F] \wedge \mathbb{Q}[F] \rightarrow \mathbb{Q}[FF]$ and $\delta: \mathbb{Q}[F] \rightarrow \mathbb{Q}[FF]/\text{alt}$. (1) Really? (2) What Lie-bialgebra-like properties hold? (3) Expansions? (4) KV?

150224b Fadell-Neuwirth: For $0 < r < n$, $m \geq 0$, and $M = \Sigma_{g \geq 1}^2 \mid D^2$, $1 \rightarrow PB_{n-r}(M \setminus \underline{m+r}) \rightarrow PB_n(M \setminus \underline{m}) \rightarrow PB_r(M \setminus \underline{m}) \rightarrow 1$ is exact. **Q.** An infinitesimal version?

220615b **Q.** Which maps $\varphi: \mathbb{Q}F_n \rightarrow \mathbb{Q}F_n$ preserve the filtration(s)? Just homomorphisms combined with translations? Which decrease the filtration? Riffled splittings with homomorphisms with intersection countings?

220615a Does the recovery formula of [HUJI-1912](#),

$P^{(1)} = T\omega\dot{\omega} - p_1(T-1)^2/T + 2T\omega\dot{\omega}a + 2T\omega\dot{\omega}xy/(1-T)$, hold for virtuals?

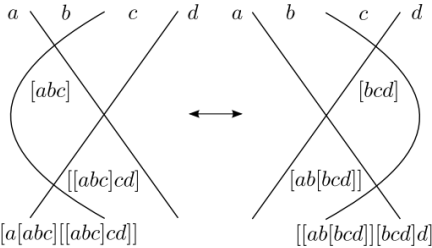
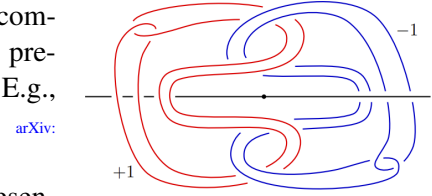
220530 Is the \diamond -algebra generating function poly-computable?

220426 Lamm lists ribbons with up to 12 crossings.

220421 **Proj.** Use AI to rediscover Fox-Milnor.

220413 Niebrzydowski, Pilitowska, Zamojska-Dzienio [arXiv:1805.07817](#) "Knot-theoretic Ternary Groups". Also, Chavez, Nelson [arXiv:2204.05851](#) " Δ -Tribrackets and Link Homotopy".

140312 Minsky: Fary-Milnor: $\gamma: S^1 \rightarrow \mathbb{R}^3$, κ its total curvature. Then $\kappa < 4\pi \Rightarrow \gamma$ unknotted. *Pf.* Find a projection direction $p \in S^2$ in which γ has ≤ 3 criticals, hence 2 criticals, hence γ is 1-bridge. As $\dot{\gamma}$ travels length κ on S^2 , $(\dot{\gamma})^\perp$ spans



area $< 4\kappa$, hence some p is covered < 4 times. \square *Further proofs.* Petrunin, Stadler [arXiv:2203.15137](#). *Generalization.* $2\pi(\text{bridge number}) = (\text{infimal } \kappa)$.

220321 **Q.** What are the ADO (Akutsu-Deguchi-Ohtsuki) invariants?

220302 Giroux, Goodman, [arXiv:math/0509555](#): fibered links all come from plumbing.

201225b **Proj.** Develop poly-dimensional braid representations via "Burau/Gassner homology" (cf. [2020-07: HeisenbergPerturbations.nb](#)).

220128 Miller: [All the Ways I Know to Define the Alexander Polynomial](#).

220102 **Dream.** (1) An extension theorem for Heisenberg modules. (2) All quantum invariants arise in this way.

211225 **Q.** A quantum mechanical interpretation of coloured Jones?

211222b Owens' [arXiv:2112.10706](#) has a list of sliceness criteria.

211222a Owens and Swenton's "An Algorithm to Find Ribbon Disks for Alternating Knots", [arXiv:2102.11778](#) and [here](#), has programs, data, and stats. \rightarrow [p6:210517](#)

211221 **Q.** Mirror and glue a slice disk $(\mathbb{R}_+^2, S = \mathbb{R}) \subset (\mathbb{R}_+^4, \mathbb{R}^3)$ to get a w-knot $K = \mathbb{R}^2 \subset \mathbb{R}^4 = \mathbb{R}_+^4 \cup \mathbb{R}_-^4$. Is $A(S) = |A(K)|^2$?

211206 Signatures: I know neither the domain (tangles as what algebraic structure?) nor the range (as algebro-geometric object, without evaluation in \mathbb{R}/\mathbb{C}). Are signatures compatible with strand doubling? What's the multi-variable version?

211114 Is there a universal way to express the quotient $\text{MVA} \rightarrow \text{Alexander}$?

211014 Aizawa, Harada, Kawaguchi, Otsuki, [arXiv:math/0612781](#): All Link Invariants for Two Dimensional Solutions of YBE.

210930 In Ruppik's [talk](#): Any epimorphism $\pi_1(\Sigma_g) \rightarrow FG_g$ is realized by a handlebody.

210923 Roland: $\mathbb{C}^{\hbar x}$ makes sense in $(\mathbb{Z}/p)[\hbar]/(\hbar^p)$.

210918 Compact \Leftrightarrow Every open cover that's closed under pair unions contains the full set.

210916 **Do.** Implement and double the Taft Hopf algebra (e.g. sec. 4 of Montgomery, Schneider, "Skew derivations of finite-dimensional algebras and actions of the double of the Taft Hopf algebra"). Compute invariants. Compare with [arXiv:2103.01081](#) by Feng, Hu, and Li, [arXiv:1805.10340](#) by Cline, and [Cline's thesis](#).

210908 Farley's [arXiv:2109.02815](#): The Planar Pure Braid Group, $\Gamma_n := \pi_1(\mathbb{R}^n \setminus \{\text{triple intersections}\})$.

210905b The thickening $\{\text{signed bipartite planar graphs}\} \rightarrow \mathcal{K}$ is surjective. Is there a local theory? Finite-type? Virtual?

210905a Checkerboard / Alexander knots: Reidemeister theory? Algebraic structure? Finite-type theory? Relations with almost classical? Signatures? Other invariants?

141113b Boden: A v-knot is "Almost Classical (AC)" if it is homologically trivial on a surface. Equivalent to "image in $\mathcal{K}(\bigcirc_v \uparrow_w)$ splits"? Is there a FT theory for AC knots? Is there " H^1 of the carrying surface", an invariant of v-knots? \rightarrow [p3:231105a](#)

210806 Y-C. Tang: $u\text{-width}(K) = 6 > 4 = v\text{-width}(K)$ for $K \in \{9_{18}, 9_{27}, 10_{24}, 10_{25}, 10_{42}, 10_{44}\}$.

210802 Write Duflo in **DoPeGDO** language.

170316 **Wikipedia: Wigner-Weyl transform**, [arXiv:hep-th/0011137](#) (& within): Gaussians compose hyperbolically, $\exp(-a(q^2 + p^2)) \star \exp(-b(q^2 + p^2)) = \frac{1}{1 + \hbar^2 ab} \exp\left(-\frac{a+b}{1 + \hbar^2 ab}(q^2 + p^2)\right)$.

210414 Signatures: The “standard” à la Tristram–Levine, KnotTheory’, Goeritz (e.g. Ozsváth–Stipsicz–Szabó), Kashaev’s [arXiv:1801.04632](#), A. Conway’s [arXiv:1903.04477](#). Also Cimasoni–Conway [arXiv:1507.07818](#) and Merz’ [arXiv:2104.02993](#) on additivity defects of signatures of braid closures.

210106 The “Iterates Completion” \mathcal{M} of a **Vect**-enriched category \mathcal{C} with an ideal \mathcal{I} : adjoin “iterates” $a^* := (1 - a)^{-1}$ for endomorphic $a \in \mathcal{I}$. Commutes with additive completion, with positive coefficients: $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^* = \begin{pmatrix} (a + bd^*c)^* & a^*b(d + ca^*b)^* \\ d^*c(a + bd^*c)^* & (d + ca^*b)^* \end{pmatrix}!$ If \mathcal{C} is monoidal, contains $\det(1 - A)^{-1}$ for $A \in \mathcal{I}$? Relations beyond $a^*b(d + ca^*b)^* = (a + bd^*c)^*bd^*$ with $a, (b|c), d \in \mathcal{I}$ at

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} b & c \\ c & d \end{pmatrix} ?$ Also/just $(a + b)^* = b^*(ab^*)^* = a^*(ba^*)^*?$

Canonical forms for morphisms? In poly-time? Carries Alexander? **2021-01: Det3x3.nb**, \rightarrow p7:**181222b**, “weighted automata”, “rational series”.

151216 **Proj.** Braidors & weak associators: $B = \Phi^{012} R_u^{12} \Phi^{-021}$,
 $B^{012} B^{02,1,3} B^{023} = B^{01,2,3} B^{013} B^{03,1,2}.$

Extensibility / uniqueness in \mathcal{A}^u , sder, Γ ? A KV/WKO3 variant? Are they enough to quantize Lie bialgebras? To solve YB?

210525 **TIL** (Kuperberg on FB, **2021-05: FubiniCounterexample.nb**) $\int_{[0,\infty]^2} \frac{(x-y)dx dy}{(1+(x-y)^2)^2}$, a nice Fubini counterexample.

210517 Manolescu@BIRS: Sherry Gong has a program to find ribbons for (ribbon) knots. \rightarrow p5:**211222a**

210512 **Q.** What is docility for maps between polynomial rings?

210506 $\pi_1(SL_2(\mathbb{R})) = \mathbb{Z}.$

210429 On the Blanchfield pairing: [mo:7411](#).

210408 Lobb’s [talk](#), “Four-Sided Pegs Fitting Round Holes Fit All Smooth Holes”, inspires plotting Möb = $\text{Conf}_2(S^1 \subset \mathbb{C}) \rightarrow \mathbb{C}^2$ via $(z, w) \mapsto ((z+w)/2, (z-w)^2)$ to get a circle-boundary Möbius band in \mathbb{R}^3 .

210329 **Q.** What values take singular Gaussian Fermionic integrals?

210318c **Chal.** Interpret Leclerc’s “On Identities Satisfied by Minors of a Matrix” in terms of Fermionic Gaussians.

210318b Grinberg’s book, “Notes on the Combinatorial Fundamentals of Algebra”.

210316 **Q.** Why are Fermionic Gaussian compositions so similar to Bosonic ones?

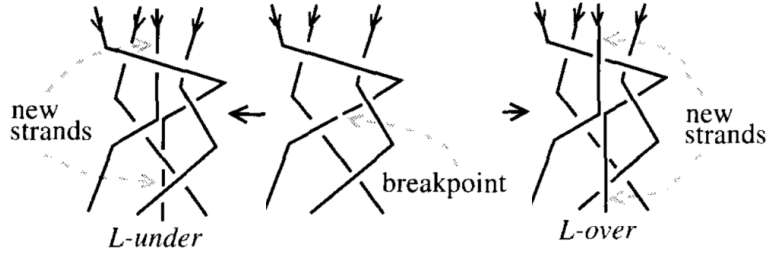
180821b I don’t understand the MVA (and yet it’s there). E.g., does the Halacheva meta-trace have a tensorial representation \rightarrow p7:**180909a**? Why is it independent of the component left open?

210211a $\Lambda^*(X^* \cup Y)$ with $\deg X^* = -1$, $\deg Y = 1$ is a contraction algebra with $c_{x,y} := \partial_{\eta} \partial_x$: $\Lambda^*(X^* \cup Y) \rightarrow \Lambda^*((X-x)^* \cup (Y-y))$ hence $\mathcal{A}(X) := \Lambda^*(X^* \cup X)_0$ is a traced meta-monoid with $m_z^{xy} := c_{xy}/(\zeta \rightarrow \xi, y \rightarrow z)$ and $\text{tr}_x(A) := c_{xx}$. Halacheva (\sim): Contains Γ (w/ fixed colours) via $\Upsilon_X: (\omega, M) \mapsto \omega \oplus \sum \xi M_{\xi y}$. Predict \mathcal{A} from Γ ? Interpret \mathcal{A} in *ybox*? Related to super-algebras? Raise \mathcal{A} to meta-Hopf? Understand $\text{im}(\Upsilon)$?

210217 **Do.** In **DoPeGDO** for *ybox*, do a first?

201217a **Proj.** A concise proof of Alexander / Markov (including an $n^{3/2}$ complexity bound and an imple-

mentation). **Q.** Is there a framed Markov theorem? Lambropoulou, Rourke “Markov’s Theorem in 3-Manifolds”: A “Markov theorem” using only L -moves:



Also: Sundheim, Traczyk, Morton, Birman, Birman-Brendle.

210211c **Do.** Super-**DoPeGDO**.

210211b **Do.** \mathcal{A}^w and super-Lie-algebras.

190314b Wherefore $(\sum_{n \geq 0} \text{tr } \Lambda^n A) \exp\left(\sum_{n > 0} \frac{(-1)^n}{n} \text{tr } \Lambda^n\right) = 1?$ **2021-02/rdet.nb**: use for quick det computations in rings in which non-zero integers are invertible. \rightarrow p6:**210130**

200703a **Q.** An Archibald calculus that includes strand doubling? A common generalization of Archibald- and Γ -calculus? \rightarrow p7:**200804**, \rightarrow p6:**210211a**

210204 Conway’s [talk](#), “Knotted Surfaces with Infinite Cyclic Knot Group”: a topological classification of surfaces Σ with $\pi_1(\Sigma^c) = \mathbb{Z}$ in a simply-connected 4-manifold.

210130 Itai: Computing det over a ring R in poly-time: compute $\det(1 - \epsilon(1 - A))$ in $R[[\epsilon]]$ by Gaussian elimination and formal inversions. But it’s a polynomial in ϵ ! Now set $\epsilon = 1$. \rightarrow p6:**190314b**

210114a Feller’s [talk](#): the fractional Dehn twist coefficient, the unique homogeneous quasimorphism $\omega: B_n \rightarrow \frac{1}{n}\mathbb{Z}$ with defect $\sup_{g,h \in G} |f(gh) - f(g) - f(h)| = 1$ s.t. $\omega(\Delta^2) = 1$, $\omega(B_{n-1}) = 0$. Invariant under conjugation, probably under strand doubling. Also Malyutin, “Twist Number of (Closed) Braids”.

210128b **Do.** Use \oplus to compute $\mathcal{A}^w(\bigcirc_n)$. \rightarrow p7:**200906**

210128a Hom’s [talk](#), “Infinite Order Rationally Slice Knots”: Rationally slice := bounds a smooth disk in a $\mathbb{Q}HS^4$. **Thm.** Using Heegaard-Floer, there is a $\mathbb{Z}^\infty \oplus (\mathbb{Z}/2)^\infty$ in rationally slice knots modulo concordance. Levine 69’: algebraic concordance group := {Seifert forms}/(metabolic, vanishes on half-dimensional subspace) $\equiv \mathbb{Z}^\infty \oplus (\mathbb{Z}/2)^\infty \oplus (\mathbb{Z}/4)^\infty$.

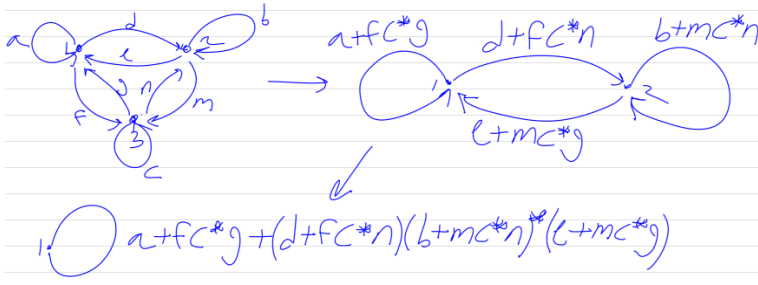
210124b Manturov’s philosophy: “If something is wrong, it’s because it’s not drawable on the plane. This must be because of a homological obstruction, which leads to a parity, leading to projections, brackets, coverings.”

210124a Manturov (zoom): his “Parity and Projection from Virtual Knots to Classical Knots” has a combinatorial proof of Kuperberg’s theorem.

210121 Khovanov’s [talk](#), “Bilinear Pairings and Topological Theories”: “near TQFTs” regressed from arbitrary invariants; especially in 2D.

210114b **TIL.** “Digit ratio”.

210107 The denominators miracle:



$$= a + ((1-c)de + (1-b)fg + dm g + fne) / ((1-b)(1-c) - mn)$$

$$= a + (db^*e + fc^*g + db^*mc^*g + fc^*nb^*e)(mb^*nc^*)$$

200917 Do. Zipping in the 1PI context. Convert diagram from “external assembly” to “merging of completed”.

201225a Zipping twice:

$$\alpha = \frac{r^2}{1 - (a + \frac{b^2}{1-c})} = \frac{r^2(1-c)}{(1-a)(1-b)-b^2}, \quad \beta = \frac{r^2(1-c)}{(1-a)(1-b)-b^2}, \quad \gamma = \frac{tsb}{1-c} \frac{1}{1 - (a + \frac{b^2}{1-c})}$$

$$\frac{tsb}{(1-a)(1-b)-b^2}, \quad \gamma = \frac{tsb}{(1-a)(1-b)-b^2} + \frac{s^2}{1-c} = \frac{s^2b^2 + s^2((1-a)(1-c)-b^2)}{(1-c)((1-a)(1-c)-b^2)} = \frac{s^2(1-a)(1-c)}{(1-c)((1-a)(1-c)-b^2)} = \frac{s^2(1-a)}{(1-a)(1-c)-b^2}.$$

201230 People: VanDerVeen: Singu-larGamma.nb: S_{ab} (on right) is a general singular point in Γ -calculus, with $S_{ab}|_{e=1} = R_{ab}$ and $S_{ab}|_{e=t^{-1}} = R_{ba}^{-1}$.

201217b Q. If $\beta_{1,2} \in B_n$ then $\beta_1\sigma_n\beta_2\sigma_n^{-1}$ and $\beta_1\sigma_n^{-1}\beta_2\sigma_n$ have the same closure. How are they related by Markov moves?

Sol’n from Birman-Brendle [arXiv:math/0409205](https://arxiv.org/abs/math/0409205), Fig. 13:

“Markov” the red strands on the right.

201214b Polyak: {planar diagrams}/(R1r, R2b, R3b) describes knots via braids, but by counting counterclockwise cycles in the oriented smoothing, are more than knots.

201214a Q. What are {planar curves}/(R1l, R1r, R2b, R3b)?

201209 Bolan: $\sqrt{2}^{\log_2 9} \in \mathbb{Q}$.

201117 Q. Are there GPV formulas for tangles and links?

201112 Piccirollo’s talk, “A Users Guide to Straightforward Exotica”: has a calculus for handle decompositions of 4-manifolds.

201105 Dynnikov’s talk, “An Algorithm for Comparing Legendrian Knots”: Legendrian knot: in $\ker(xdy + dz)$. Have Reidemeister theory. Related to grid diagrams. Topological meaning?

201023 TIL. Merge “only” in “she told him that she loved him”.

181222b For $A := \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & b\bar{d} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a - b\bar{d}c & 0 \\ 0 & d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \bar{d}c & 1 \end{pmatrix} \in M_{(p+q) \times (p+q)}$, $|A| = |d||a - b\bar{d}c|$, and $A^{-1} = \begin{pmatrix} (a - b\bar{d}c)^{-1} & -(a - b\bar{d}c)^{-1}b\bar{d} \\ -\bar{d}c(a - b\bar{d}c)^{-1} & \bar{d} + \bar{d}c(a - b\bar{d}c)^{-1}b\bar{d} \end{pmatrix}$, with $\bar{d} := d^{-1}$.
2019-03, →p9:190312, →p6:210106, Bareiss’ “Sylvester’s Identity...”.

181222a If $A \in M_{n \times n}$ and $\omega = |A|$, then each entry of A^{-1} has denominator ω , so expect $|A^{-1}| \propto \omega^{-n}$. Yet $|A^{-1}| = \omega^{-1}$.

201011 The Steinberg relations between elementary matrices: $e_{ij}(\lambda)e_{ij}(\mu) = e_{ij}(\lambda + \mu)$, $[e_{ij}(\lambda), e_{jk}(\mu)] = e_{ik}(\lambda\mu)$ for $i \neq k$, and $[e_{ij}(\lambda), e_{kl}(\mu)] = 1$ for $i \neq l$ and $j \neq k$.

201006 Q. Let $I_{v \downarrow w} := \ker(PB \rightarrow PwB)$ and let $\mathcal{A}_{v \downarrow w} := \prod I_{v \downarrow w}^m / I_{v \downarrow w}^{m+1}$. What is $\mathcal{A}_{v \downarrow w}$? Are there “expansions”? Are there “Burau/Gassner expansions”?

200925 Riddle (Matthew Bolan) $\exists?$ cont. $f: \mathbb{R} \rightarrow \mathbb{R}$ s.t. $f \circ f = \cos$?

200216 Direct proof that $uB_n \hookrightarrow vB_n$ (cf. Gaudreau, [arXiv:2008.09631](https://arxiv.org/abs/2008.09631)): For $w = \prod_{\alpha=1}^l \sigma_{i_\alpha j_\alpha}^{s_\alpha}$ define “depths” $(d_{k,\alpha})_{1 \leq k \leq n, 0 \leq \alpha \leq l}$ inductively by $d_{k0} = k$ and for $\alpha > 0$, $d_{k\alpha} = d_{k,\alpha-1} + s_\alpha(\delta_{ki_\alpha} - \delta_{kj_\alpha})$. Drop from w every $\sigma_{i_\alpha j_\alpha}^{s_\alpha}$ for which $s_\alpha \neq d_{j_\alpha, \alpha-1} - d_{i_\alpha, \alpha-1}$ (well defined on vB_n !). Iterate. The result is a retraction $vB_n \rightarrow uB_n$.

200906 With $g = sl_{2+}^0 = \langle y, b, a, x \rangle / ([a, x] = x, [a, y] = -y, [x, y] = b, [b, -] = 0)$, $\mathcal{U}(g)_g$ is freely generated by $\{b^k a^n\}_{k \leq n}$, also $\{y^k a^n x^k\}_{k, n \geq 0}$, for $[a, y^l a^m x^n] = (n-l)y^l a^m x^n$ forces $l = n$ and $xa = (a-1)x$ implies $xf(a) = f(a-1)x$ so $[x, f(a)] = (f(a-1) - f(a))x$ so with $a^{(k)} := \binom{a+k}{k} = (a+1)(a+2) \cdots (a+k)/k!$ and $a^{(-1)} := 0$, $[x, a^{(k)}] = -a^{(k-1)}x$ so $[x, y^n a^{(k)} x^{n-1}] = nby^{n-1}a^{(k)}x^{n-1} - y^n a^{(k-1)}x^n$. Thus in $\mathcal{U}(g)_g$, $by^n x^n = 0$ and $y^n a^{(k)} x^n = n!b^n a^{(k+n)}$. So $\{b^k a^n\}$ generates and $b^{n+1}a^n = 0$. The same relations also follow from $[y^{n-1}a^{(k)}x^n, y] = -y^n a^{(k-1)}x^n + nby^{n-1}a^{(k)}x^{n-1}$, and these are all the relations in $\mathcal{U}(g)_g$. →p3:230915

180909a The meta-trace (simpler before quantization?): At $k = 0$, $\text{tr}_m = \mathbb{E} \left(\beta_m b_m, \frac{\mathcal{A}_m(1-B_m)}{\mathcal{A}_m-1} \xi_m \eta_m \right)$. Then ruled by $C_{abc} \xrightarrow{d^2} C_{12} \xrightarrow{d^0} C_0$ via $d^1 = \mu_0^{12} - \mu_0^{21}$ and $d^2 = (\sigma_1^a \mu_2^{bc} + \text{cyc. perm.})$, where $\mu_k^{ij} = (\alpha_k \rightarrow \alpha_i + \alpha_j, \xi_k \rightarrow \mathcal{A}_j \xi_i + \xi_j, \eta_k \rightarrow \eta_i + \mathcal{A}_i \eta_j, \beta_k \rightarrow \beta_i + \beta_j - \xi_i \eta_j)$. More generally, $\text{tr}_m = \mathbb{E} \left(r\alpha_m \beta_m / \hbar + s\beta_m b_m + t\alpha_m a_m, \frac{\mathcal{A}_m^{-r}(\mathcal{A}_m^r - B_m^s)}{\mathcal{A}_m-1} \xi_m \eta_m \right)$. Pensieve: **2018-08: Trace.nb, →p6:180821b, →p3:230915.**

200827 Baby DoPeGDO in 2020-07: HeisenbergPerturbations.nb and **People: VanDerVeen: TimidHeisenbergRGeneralForm@.nb.**

190304 Exponential zipping: With $[f]_\lambda := e^{\lambda \partial_z \partial_\zeta} f = \langle f |_{z \rightarrow z+\zeta', \zeta \rightarrow \zeta+\zeta'} \rangle_{\lambda; \zeta' \leftrightarrow \zeta'}$ (so $\langle f \rangle_\lambda = [f]_\lambda|_{\zeta=\zeta'=0}$) and $P_\lambda := \log[e^f]_\lambda$, have $P_0 = f$ and $\partial_\lambda P_\lambda = (\partial_z \partial_\zeta P_\lambda) + (\partial_z P_\lambda)(\partial_\zeta P_\lambda)$. **Proj.** Extend DoPeGDO to EDDO (Exponentiated Docile Differential Operators). **2019-03:** requires the “wake equation” $\partial_\lambda W^j = W^i \partial_{\zeta_i} W^j$.

200811 Q. In CU , if Q is quadratic and P is docile, are $\log \mathbb{O}(e^Q)$ quadratic and $\log \mathbb{O}(e^P)$ docile?

160312 Proj. Truly understand $Kh(K) = 1 \Rightarrow K = 1$. If $Kh(D) = 1$, constructively reduce D to 1. Kronheimer, Mrowka [arXiv:1005.4346](https://arxiv.org/abs/1005.4346), “Khovanov homology is an unknot-detector”.

200807 Q. Is there a unique factorization $K = \kappa \# K'$, with K, K' virtual knots, and κ maximal classical?

200806 Q. Is there a link invariant $UMVA$ such that $UMVA(L)$ dominates all the MVAs of satellites of L ? →p6:200703a

200804 Costantino-Le [arXiv:1907.11400](https://arxiv.org/abs/1907.11400): $O_{q^2}(SL(2))$ in skein theory.

20721 Q. Is there a unique factorization $T = \beta' T'$, with $T, T' \in \mathcal{T}_n$, and $\beta' \in \mathcal{B}_{2n}$ “maximal”? What is $\mathcal{T}_n / \mathcal{B}_{2n}$?

200703b Kassel-Turaev: Σ a punctured disk with basepoint d , $\varphi: \pi_1(\Sigma) \rightarrow G$ a surjective homomorphism, $(\tilde{\Sigma}, \tilde{d})$ the cover of (Σ, d) corresponding to $\ker \varphi$. Then $\tilde{H} := H_1(\tilde{\Sigma}, \tilde{d})$ is a left module over $\mathbb{Z}G$ of free rank $\text{rk } H_1(\Sigma)$.

200625 Beliakova@KOS: “Partial trace property”.

200618 Meusburger@KOS: Mapping class groups presentations by

$$\begin{array}{ccccc}
\mathcal{T} & \xrightarrow[\text{filtered}]{Z^v} & \mathcal{A}^v & \xrightarrow{\text{graded}} & \mathcal{A}^v \\
\downarrow \tilde{\tau} & \searrow & \downarrow & \searrow & \downarrow \\
\mathcal{A} & \xrightarrow{Z_{URT}} & \mathcal{A} & \xrightarrow{Z_{\mathfrak{g}}} & \mathcal{A} \\
\downarrow & & \downarrow & & \downarrow \\
\mathcal{U}_h(\mathfrak{g}) & \xrightarrow{Z_{\mathfrak{g}}} & \mathcal{U}(\mathfrak{g}) & \xrightarrow{\hbar} & \mathcal{U}(\mathfrak{g})
\end{array}$$

thereof? ν -Claspers? A closure of \odot ? \rightarrow [p9:190105a](#).

[190603](#) Kofman in [Da Nang](#): The Vol-Det Conjecture: For a hyperbolic alternating link K , $\text{Vol}(K) \leq 2\pi \log \det(K)$.

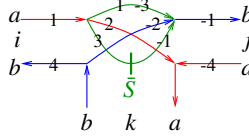
[190530](#) Porti in [Da Nang](#): For a hyperbolic $K \subset S^3$ and $|\zeta| = 1$, $\lim_{N \rightarrow \infty} N^{-2} \log |\Delta_k^{\rho_N}(\zeta)| = \text{vol}(S^3 \setminus K)/4\pi$, with ρ_N the N -dim representation of $SL_2(\mathbb{C})$.

[190523](#) [BBS:Itai-190523](#)+: L^p inequalities: $\|u\|_q \leq C_{p,d} \|\nabla u\|_p$ in \mathbb{R}^d with $\frac{1}{q} = \frac{1}{p} - \frac{1}{d}$; $\|u\|_p \leq \|u\|_{p_0}^{1-\theta} \|u\|_{p_1}^\theta$.

[190113](#) The non-linear Schrödinger eqn: $\square \partial_t \psi = -\frac{1}{2} \Delta \psi - |\psi|^{p-1} \psi$.
[BBS:Itai-190523](#): conserves $\int |\psi|^2$ and $\int \left(\frac{1}{2} |\nabla \psi|^2 - \frac{2}{p+1} |\psi|^{p+1} \right)$.

[180619](#) Does the Drinfel'd double generate all “suppressed-cycle diagrams”?

[190509](#) Snyder, Tingley @[arXiv:0810.0084](#): T with $R = T_1^{-1} T_2^{-1} \Delta_{12} T$. In [DoPeGDO](#)?



[190501b](#) Merkulov @[arXiv:1904.13097](#): “Grothendieck-Teichmüller Group, Operads and Graph Complexes: a Survey”.

[190501a](#) Livingston @[arXiv:1504.03368](#): “Doubly Slice Knots with Low Crossing Number”. An AKT description? \rightarrow [p10:181218b](#)

[190425b](#) [DoPeGDO₂](#): Quadratics are of weight precisely 2, interactions of weight at most 2, with $\text{wt}(x, y, \xi, \eta, a, b, \alpha, \beta, \epsilon) = (1, 1, 1, 1, 2, 0, 0, 2, -2)$.

[190425a](#) Darné @[arXiv:1904.10677](#): w-braids up to homotopy.

[190417](#) **Riddle**. If a box of sides (b_i) is contained in a box of sides (a_i) , then $\sum b_i \leq \sum a_i$. Khesin’s and Itai’s Sol’ns in %.

[190414](#) With Ens. • Establish a cluster \Leftrightarrow DK dictionary. • Do syzygy operators always eliminate the image of some “error operator”, like $\tilde{d}^1 \tilde{d}^2 = 0$ in [papers/GT1](#)?

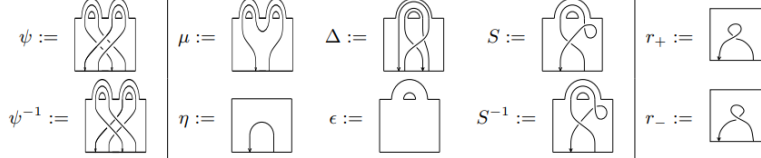
[190412b](#) **Chal**. Mix braidors and solvable approximation.

[190412a](#) **Q**. Is there an interesting “braidor-liberator” algebra?

[190407](#) DCA := Directed Circuit Algebra = Symmetric Strict Spherical Category with singly-generated set of objects.

[190409](#) Khovanskii @[arXiv:1904.03341](#): “One Dimensional Topological Galois Theory”.

[190404](#) **Do**. Implement Habiro-Massuyeau [arXiv:1702.00830](#), section 9.1:



[190324](#) In rec-tangles: rectangle = handle = hand = sleeve = protected zone = input of op. Is there a 3D description?

[190325](#) **TIL**. Mathematica’s [Notation](#) package.

[190322](#) Fiedler @[arXiv:1902.06091](#): “A refinement of the first Vassiliev invariant can distinguish the orientation of knots”.

[171205](#) (Approx.) On $H^{*cop} \otimes H$ with $R = Id = \rho \otimes r$ (summed), $\int \phi \otimes x := \langle \phi \bar{\rho} \mid xr \rangle$ is an integral. $\frac{1}{2}$ **Pf**. $x_1 \int \phi \otimes x_2 = x_1 \langle \phi \bar{\rho} \mid x_2 r \rangle = x_1 r^a r^b \langle \phi \bar{\rho} \bar{\rho}^a \rho^b \mid x_2 r \rangle \sim x_1 r_1 r^b \langle \phi \bar{\rho} \rho^b \mid x_2 r_2 \rangle \sim (xr)_1 r^b \langle \phi \bar{\rho} \mid (xr)_3 \rangle \langle \rho^b \mid (xr)_2 \rangle \sim (xr)_1 (\bar{x}r)_2 \langle \phi \bar{\rho} \mid (xr)_3 \rangle = \langle \phi \bar{\rho} \mid xr \rangle = \int \phi \otimes x$. **Verify!** Attempt in [Projects/SL2Portfolio2/DoubleIntegration.nb](#).

[190314a](#) **Do**. A graphical calculus for [DoPeGDO](#) and [DoPeGDO₂](#).

[190321](#) After Rushworth: Is Jones on $\mathbb{Q}HS$ a polynomial? A skein relation? A Kauffman bracket? Categorifies? \rightarrow [p17:140116](#)

[190312](#) **Def**. $\omega \| A$ means $\forall k \omega^{k-1} |\Lambda^k(A)|$. **Ex**. With $\omega = |A|$, $\omega \| (\omega B + \text{adj}(A))$ (tested [2018-12](#)). **Q**. An effective $\omega \| A$ certify strong enough to certify the example? \rightarrow [p7:181222b](#)

[190310b](#) Tree generation: $\partial_\lambda T_\lambda = (\partial_z T_\lambda)(\partial_\zeta T_\lambda)$. Legendre transform: $(Lf)(\eta) := \text{crit}_y(\eta y - F(y)) = \eta y_0 - F(y_0)$, with $dF_{y_0} = \eta$.

[190310a](#) KV in Γ in [Projects/MetaCalculi](#):

$$\begin{array}{c} \begin{array}{cc} \begin{array}{c} \left(\frac{T_1-1}{t_1} \right)^{\frac{1}{4}} \left(\frac{T_2-1}{t_2} \right)^{\frac{1}{4}} \\ \left(\frac{T_1 T_2 - 1}{t_1 t_2} \right)^{\frac{1}{4}} \end{array} & \begin{array}{c} S_1 \\ S_2 \\ \Gamma \end{array} & \begin{array}{c} S_2 \\ S_1 \\ \sqrt{T_1} \end{array} \\ \begin{array}{c} t_1 + \sqrt{\frac{t_1 t_2 (t_1+t_2) T_1 (T_2-1)}{(T_1-1) (T_1 T_2-1)}} \\ t_2 - \sqrt{\frac{t_1 t_2 (t_1+t_2) T_1 (T_2-1)}{(T_1-1) (T_1 T_2-1)}} \\ 1 \end{array} & \begin{array}{c} t_1+t_2 \\ t_1+t_2 \\ 1 \end{array} & \begin{array}{c} t_1-T_2 \sqrt{\frac{t_1 t_2 (t_1+t_2) (T_1-1) T_1}{(T_2-1) (T_1 T_2-1)}} \\ t_2+T_2 \sqrt{\frac{t_1 t_2 (t_1+t_2) (T_1-1) T_1}{(T_2-1) (T_1 T_2-1)}} \\ \sqrt{T_1} \end{array} \end{array}$$

[190225](#) “Set-theoretically-induced delusions of greatness”.

[131203](#) Ševera quantization, [arXiv:1401.6164](#): Given a BMC \mathcal{D} (with Manin (∂, g, g^*) , set $\mathcal{D} := \mathcal{U}(\partial)\text{-Mod}^\Phi$), a co-braided co-algebra $(M, \Delta: M \rightarrow M^2, \epsilon: M \rightarrow 1_{\mathcal{D}})$ in it ($M := \mathcal{U}(g) = \mathcal{U}(\partial)/\mathcal{U}(\partial)g^*$), a second BMC C (**Vect**), a functor $F: \mathcal{D} \rightarrow C$ ($F(X) := X/gX$) and a comonoidal structure c (natural $c_{X,Y}: F(XY) \rightarrow F(X)F(Y)$ and $c_1: F(1_{\mathcal{D}}) \rightarrow 1_C$ respecting braiding and associativity) so that

$$F(XMY) \xrightarrow{F(1\Delta_1)} F(XMMY) \xrightarrow{c_{XM,MY}} F(XM)F(MY)$$

and $F(M) \xrightarrow{F(\epsilon)} F(1_{\mathcal{D}}) \xrightarrow{c_1} 1_C$

are isomorphisms (the clear $c_{X,Y}: XY/g(XY) \rightarrow (X/gX)(Y/gY)$), construct a Hopf algebra structure on $H := F(M^2)$:

$$\begin{array}{l} \Delta_H: F(M^2) \xrightarrow{F(\Delta\Delta)} F(M^4) \xrightarrow{F(1R_1)} F(M^4) \xrightarrow{c_{M,M}} F(M^2)^2, \\ m_H: F(M^2)^2 \xleftarrow[\sim]{c_{M^2,M^2} \circ F(1\Delta_1)} F(M^3) \xrightarrow{F(1\epsilon_1)} F(M^2), \\ S_H: F(M^2) \xrightarrow{F(R)} F(M^2). \end{array}$$

Set also $G: X \mapsto F(MX)$ ($G: X \mapsto \frac{\mathcal{U}(g)X}{g(\mathcal{U}(g)X)}$), the “twist”.

– Is H the symmetry algebra of something? – In the non-quasi case, can we reconstruct $\mathcal{U}(g)$ from the category of ∂ -modules?
– In the abstract context, what is the relation between H and M ?
– How does this restrict to AT/AET in the commutative case?
– H pairs with the quantization of g^* ? Ševera in [LD15/II](#): No.

[190221](#) **Do**. Understand “two tube surgery”:

$$\begin{array}{c} \begin{array}{ccc} + & & + \\ \swarrow & & \swarrow \\ X & \text{---} & Y \end{array} \leftrightarrow \begin{array}{ccc} + & & + \\ \swarrow & & \swarrow \\ X & \text{---} & Y \end{array} \end{array}$$

[190205](#) **Q**. When/why does the universal Verma module see all invariants?

[190124](#) **TIL**. “Groupoid algebras” are “weak Hopf algebras”.

[181106a](#) Guo@Hefei: Algebras: • Rota-Baxter: Associative with unary P with $P(f) * P(g) = P(f * P(g)) + P(P(f) * g)$. Think $P(f) = \int f$, $f * g = \int f'g'$. • Dendriform; pre-Lie; averaging; diassociative.

[190105b](#) **Q**. Why is there a Cartan involution for classical $sl_{2,\epsilon}$ (multiplicative co-multiplicative θ preserving $r + r^{21}$)? Is there a quantum analog?

[190105a](#) **Q**. If A_h is the quantization of a Lie bialgebra \mathfrak{a} , is there always a multiplicative expansion $A_h \rightarrow \mathcal{U}(\mathfrak{a})[[\hbar]]$? \rightarrow [p8:171001](#).

[190104](#) **Wanted**. Examples around “ $\partial_x(x^{-1})|_{x=0}$ is undefined, yet $\partial_x^{\xi} \partial_x(x^{-1})|_{x=0} = \xi^{-1}$ ”.

190102 Etingof-Schiffmann 2.2.2: $\langle a, x \rangle / ([a, x] = x)$ is also a Lie bialgebra with $\delta(a, x) = (a \wedge x, 0)$.

180629 The Zipping Thm (verification [2018-12](#)). If P has a finite ζ -degree and \tilde{q} is the inverse matrix of $1 - q$: $(\delta_j^i - q_j^i)\tilde{q}_k^j = \delta_k^i$, then

$$\left\langle P(z_i, \zeta^j) e^{c + \eta^i z_i + y_j \zeta^j + q_j^i z_i \zeta^j} \right\rangle = |\tilde{q}| \left\langle P(z_i, \zeta^j) e^{c + \eta^i z_i} \middle| z_i \rightarrow \tilde{q}_i^k (z_k + y_k) \right\rangle$$

$$= |\tilde{q}| e^{c + \eta^i \tilde{q}_i^k y_k} \left\langle P(\tilde{q}_i^k (z_k + y_k), \zeta^j + \eta^j \tilde{q}_i^j) \right\rangle.$$

Do. • Sort $\langle \cdot \rangle_{\text{int}} \leftrightarrow \langle \cdot \rangle \leftrightarrow \oint$. • Sort denominators.

181218b Freedman: an AKT description of slice knots? \rightarrow [p9:190501a](#)

181218a **Q** (following Boden via Gaudreau). Is the crossing number of a virtual link equal to that of its irreducible representative?

181202 **TIL.** Vibration modes of mugs (Tadashi Tokieda).

181201 **Do.** A better narrative for φ .

181123 **Proj.** Clasp number k knots: AKT-definable? Alexander properties? \rightarrow [p14:161009](#).

181119 **Do.** EK in terms of pegged tangles:

181116 Chang, Cui [@arXiv:1710.09524](#) relate Kuperberg/Turaev-Viro-Barrett-Westbury with Hennings-Kauffman-Radford/Witten-Reshetikhin-Turaev.

181104 $\text{AC} \Rightarrow \text{Zorn}$: Assume by contradiction that in $(X, <)$ every chain C has a (chosen) *strict* bound $M(C)$, and let $\mathcal{W} := \{W \subset X : W \text{ well ordered}, \forall x \in W M(\{w \in W : w < x\}) = x\}$. Then $\bigcup \mathcal{W}$ is a maximal element of \mathcal{W} (effort here), contrary to the existence of $M(\bigcup \mathcal{W})$. (The key: transfinite constructions have a “maximal extent” M ; here leading to a contradiction. AC is not needed for M , yet it has a busy beaver feel.)

150609d Fibrations $p: E \rightarrow B$ on right. Any $X \xrightarrow{\phi} Y$ is $X \xrightarrow{i_0} E_\phi \xrightarrow{p_1} Y$ with i_0 a homotopy equivalence and p_1 a fibration. Here $E_\phi \sim \{(x \in X, \gamma : [0, 1] \rightarrow Y) : \gamma(0) = \phi(x)\}$; $i_0(x) = (x, \bar{x})$, $p_1(x, \gamma) = \gamma(1)$, and the “homotopy fiber” is $p_1^{-1}(y) = \{(x, \gamma) : \gamma(0) = \phi(x), \gamma(1) = y\}$. E.g., for $\text{Emb}(\mathbb{R} \hookrightarrow \mathbb{R}^3) \hookrightarrow \text{Imm}(\mathbb{R} \rightarrow \mathbb{R}^3)$, the homotopy fiber is framed knots.

Cofibrations $f: A \rightarrow X$ on right (e.g. cones, $A \rightarrow CA$). Any $X \xrightarrow{\phi} Y$ is $X \xrightarrow{i_0} M_\phi \xrightarrow{p_1} Y$ with i_0 a cofibration and p_1 a homotopy equivalence.

181106b **Drummond-Cole** knows dimensions of the BiAlg PROP.

181028 Wherefore these relations, with $n \in 2\mathbb{Z}$? $\left| \begin{smallmatrix} n+1 \\ n \end{smallmatrix} \right| = \left| \begin{smallmatrix} n \\ n+1 \end{smallmatrix} \right| = \left| \begin{smallmatrix} n \\ n \end{smallmatrix} \right| = \left| \begin{smallmatrix} n-1 \\ n \end{smallmatrix} \right| = \left| \begin{smallmatrix} n \\ n-1 \end{smallmatrix} \right|$

181017a “Topological Expansionism”: “Quantum Topology” is a mix of topology, algebra, representation theory, and quantum field theory. I will explain how to expand the territory of topology within that mix at the expense of representation theory and algebra.

181022 **Do.** Understand the swirl:

181019 Bruguières, Virelizier [@arXiv:math/0505119](#): “Hopf diagrams and quantum invariants”.

181017b Hass, Thompson, Tsvietkova [@arXiv:](#)

1809.10996 “alternating links have at most polynomially many Seifert surfaces of fixed genus”: In AKT language?

181014 **Do.** Show that KV is a full triangularity equation, as Duflo is triangularity in co-invariants.

181013 Khanin: $\sum 1/n^2 = \pi^2/6$ by comparing coefficients of x^2 in $\frac{\sin x}{x} = \prod (1 - x^2/\pi^2 n^2)$, itself true by comparing roots and constant terms.

181009 de Mesmay, Rieck, Sedgwick, Tancer [@arXiv:1810.03502](#): realizations of some logic circuits in KO \Rightarrow many KT problems are NP-hard.

160611 Majid’s Primer §8.1: the quantum double $\mathcal{DA} := A^{*,op} \otimes A$ with $(\phi a)(\psi b) := \langle S a_1, \psi_1 \rangle \langle a_3, \psi_3 \rangle \langle \psi_2 \phi \rangle (a_2 b)$ (“op” for multiplication). What problem does it solve? Two layers of wrong: 1. R isn’t in the result so it shouldn’t be in the motivation. 2. The result is degree-non-decreasing, the formula should be the same. \rightarrow [p11:180725](#). **Q.** It’s an image construction. A kernel one?

181001 **Riddle** (Dylan). Warden to 100 prisoners: I’ve chosen a permutation π of your names and tomorrow I will place each of you in an isolated room with 100 boxes storing π . You will each get to open 50 boxes and each must open their “own” box. Maximize the probability of success. Sol’n [2018-10](#).

180424 **BBS: VanDerVeen-170622**, verification [Doubling.pdf](#), in $R_{12}^{-1} = S_2(R_{12})$ conventions. The Drinfel’d’s cuap: $u := R_{12}/S_1/m^{21}$, $v^2 := S(u)u$, $C := uv^{-1}$. Properties: $S^2(z) = u^{-1}zu$ (pf?), $uS(u) = S(u)u$ is central. Issues: Invariance property of C ? R is in the total-rotation-0 subspace, and all operations preserve it. How can they generate C ? Perhaps it’s the distinction between the pre- and post-doubling S ? \rightarrow [p11:180724](#).

180910 “Locally Euclidean Knotted Objects”, leKO. **Thm.** $\text{leKO} \Leftrightarrow \mathbb{R}\text{-rotation vKO}$; and $\mathbb{Z}\text{-rotation vKO} \Leftrightarrow \text{leKO}$ with all measurable rotation numbers in \mathbb{Z} .

180905 **Riddle** (Itai). Fairly select 1 in 1,001 in 2 tosses of some fair p -dice, $p < 1,000$ prime. Sol’n in %.

180909b Green, Nichols, Taft: “Left Hopf Algebras”, $S * I = \eta \epsilon$, $I * S \neq \eta \epsilon$. Also Lauve, Taft [@arXiv:0908.3718](#): “[Left $SL_q(n)$]”. \rightarrow [p8:200205](#).

180904b **Riddle** (Tsimmerman). How many not-necessarily-fair coins to fairly select 1 of n in finitely many tosses? Sol’n in %.

180830 **Riddle** (Khovanova) Among 14 coins, 7 weigh a each, and 7 weigh $b < a$ each. You’re told which is which. Confirm this with three uses of a balance scale. (“leverage”)

180904c What’s “tangle planarity” in evaluation diagrams?

180904a Dylan’s sutured 3-manifolds: $\partial M = R_+ \cup_\sigma R_-$. Balanced: $\chi(R_+) = \chi(R_-)$ (per component?). Taut: if $\Sigma \subset M$ with $\partial \Sigma = \sigma$ then $\chi(\Sigma) \leq \chi(R_\pm)$ (wherefore?). Equivalence: add or remove a $D_1 \times D_2$ with suture $\{0\} \times D_2$ along $S^0 \times D_2$ or along $D_1 \times S^1$. Marking: by multiple disjoint arrows in R_+ and in R_- , with ends on σ . Gluing with 90° rotation! Contains m, Δ, S if $S^2 = 1$. **Q.** Faithfulness and completeness? Separation algorithm? $S^2 \neq 1$? **Proj.** Write “3-manifolds and the Drinfel’d double construction”.

180821a The open Hopf ϕ maps dual co-invariants to invariants.

180827 Riddle (Ido). On an n -vertex directed graph with letter-marked vertices, every length 2^n word can be formed with walks. Prove that the same is true for all words. Sol'n **2018-08**.

180818 Kirk's unitarity: X is the complement of a pure tangle in $D^2 \times [0, 1]$, $X_i := X \cap (D^2 \times \{i\})$ for $i = 0, 1$. Choose a generic $U(1)$ representation $\alpha: H_1(X; \mathbb{Z}) = \mathbb{Z}^n \rightarrow U(1)$ with $\alpha(m_k) = t_k$. Then $\alpha|_{X_0} = \alpha|_{X_1}$. Since $t_k \neq 1$, the cohomologies (with coefficients in α) of meridians and tubes in ∂X vanish.

By Mayer-Vietoris the restriction map

$$H^1(\partial X; \alpha) \rightarrow H^1(X_0; \alpha) \oplus H^1(X_1; \alpha) \quad (*)$$

is an isomorphism. By the long exact sequence $H^1(X_i, \partial X_i; \alpha) \cong H^1(X_i; \alpha)$. Thus the non-degenerate skew-Hermitian (ndsH) cup product $H^1(\partial X; \alpha)^2 \rightarrow H^2(\partial X; \mathbb{C})$ decomposes using $(*)$ as $\text{diag}(A, -A)$ with A the ndsH inner product $H^1(X_0, \partial X_0; \alpha)^2 \rightarrow \mathbb{C}$, as the X_i 's are disjoint with opposite orientations in ∂X .

Claim. $x^T A y = g(x)^T A g(y)$ for g the Gassner representation.

Pf. 1. The image $H^1(X) \rightarrow H^1(\partial X)$ is a Lagrangian L (Poincaré duality). 2. The two composites $H^1(X) \rightarrow H^1(\partial X) \rightarrow H^1(X_i)$ on the summands in $(*)$ are isomorphisms (Le Dimets, also [KLW]). Using 1 and 2, $x \in H^1(X_0)$ has a unique lift to $(x, g(x)) \in L$. If $x, y \in H^1(X_0)$, as L is Lagrangian,

$$0 = (x, g(x)) \cup (y, g(y)) = x \cup y - g(x) \cup g(y).$$

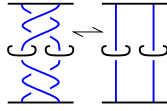
(the cross terms vanish because of the 0s off diagonal.) but $x \cup y = x^T A y$ and $g(x) \cup g(y) = g(x)^T (-A) g(y)$, hence $x^T A y = g(x)^T A g(y)$ \square

180815 Blair, Sack @arXiv:1801.00230: the tangle category is Karoubi complete: $f^2 = f \Rightarrow f = gh$ with $hg = 1$. **Q.** v,w?

180809 Meta-monoids aren't equivalent to monoid objects in a monoidal category: $m_k^{ij}[S]: M_{S \sqcup \{i,j\}} \rightarrow M_{S \sqcup \{k\}}$ isn't induced from $m_k^{ij}[\emptyset]$. Yet if M is a monoid object in a symmetric strict monoidal $(C, \otimes, \mathbb{1})$, then $M_S := \text{mor}(\mathbb{1}, M^{\otimes S})$ is a meta-monoid.

180812 Riddle (Ido). In an n -cards game of war, can the sides jointly ensure finiteness from any initial position? **Q.** Effective? Poly-time?

170928 Riddle (Chterental, May 2014). Get left to right moving only blue.



180811 Tangloids, Medusas. **Q.** Expansions? Intermediate to u and v , so implied by neither; yet implies $\mathcal{U}(g)$.

150610 Andrews-Curtis Conjecture: balanced group presentations differ by Nielsen trans.: $(g_i) \rightarrow (g_{\sigma i}), g_i \rightarrow g_i^{-1}, g_i \rightarrow g_i g_j$. Myasnikov $\times 2$, Shpilrain @arXiv:math/0302080: potential counterexamples.

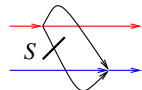
180806 Schwinger-Dyson is translation invariance of the path integral measure, written using post-integration handles:

$$0 = \int \mathcal{D}\phi \partial_\phi e^{\phi \cdot Q \phi / 2 + V(\phi) + J \cdot \phi} = (Q(\delta_J) + (\partial_\phi V)(\delta_J) + J) Z(J).$$

180803 Q. What axioms befall meta-Hopf-actions?

180722 $\mu: \{\text{DK-Associators}\} \rightarrow \text{SolKV}$ is injective as $\mu/(- \rightarrow \{\text{sder -Associators}\})$ is the identity modulo wheels (also Furusho, Schneps, Enriquez@LD16). Surjectivity? \rightarrow p11:180721a.

180725 Wanted. A limited-foresight narrative for (want a stitching- and cabling-compatible invariant of u -tangles) \implies (look at am ,



bm, R, P , rotational virtual knots, and the meta-Drinfel'd double procedure). \rightarrow p10:160611.

180724 (w/ Dylan). $(l+)$ -kink $= (r+)$ -kink in RVT implies $s = \mathbb{1}$ in \mathcal{A}^{rvt} . \rightarrow p12:170520, \rightarrow p10:180424.

180721a Dream (w/ Zsuzsi). $d^3 \gamma \pi d^2 f = 0$ when $d^n = \sum_{k=0}^{n+1} (-)^k d_k^n$ the co-Hochschild differential, $\pi: \text{sder}_n \rightarrow FL_{n-1}$ the projection on x_n -degree 1, $\gamma: FL_{n-1} \rightarrow \text{sder}_n$ the Lie morphism with $x_i \mapsto t_{in}$, and with $f \in \text{sder}_2$. Leads to an algebraic construction of DK associators? \rightarrow p11:180722.

180721b $FL(V \oplus W) \cong FL(FA(V) \otimes W) \oplus FL(V)$ like $FA(V \oplus W) \cong FA(FA(V) \otimes W) \oplus FA(V)$.

180721c Conj (w/ Zsuzsi). $\text{sder}_n \cong \bigoplus_{k=1}^n FL_k^{\text{palindromic}}$.

180716 Proj. Direct proofs of $u\mathcal{T} \hookrightarrow v\mathcal{T}$ and $u\mathcal{T} \hookrightarrow w\mathcal{T}$.

180708 With $\mathcal{Z}_\lambda = \{(Z: B \rightarrow A = \text{gr } B): \text{gr } Z = \lambda^{\deg}\}$, have $Z_0 \in \mathcal{Z}_0$ by $b \mapsto [b]_I \oplus 0 \dots$. Hence $\text{GRT} \rightarrow \mathcal{Z}_0$. When injective? Surjective? Bijective for $B = PaB$! For $\epsilon^2 = 0$, $\mathcal{Z}_\epsilon \neq \emptyset$ iff there is $\beta_\kappa \in B_\kappa$ for every kind κ , such that for every $\text{op } \rho$, $\rho(\beta \dots \beta) - \beta \in I^2$ (set $Z_1(b) = [b]_I \oplus \epsilon[b - \beta]_{I^2} \oplus 0 \dots$). In that case, $\text{GRT} \rightarrow \mathcal{Z}_\epsilon$. Injective? Surjective? Always $\mathcal{Z}_\epsilon \rightarrow \mathcal{Z}_0$; a bijection for PaB .

180615 Q. For the BMC crowd, are tangles a Hopf algebra variant?

180611 Q. Does $t = \epsilon a - \gamma b$ have a topological meaning?

180528b Not every v -knot has a Seifert surface. Are there "Seifert v -tangles"? Is this related to unitarity and to Fox-Milnor?

180515 Q. Wherefore the zip algebra, $\langle z_n \rangle_{n \geq 0}$ with $z_m z_n = \sum_{k=0}^{\min(m,n)} k! \binom{m}{k} \binom{n}{k} z_{m+n-k}$? Representation: $z_n \mapsto \hat{x}^n \partial_x^n$. Isomorphic to $\mathbb{Z}[y]$ via $z_n \mapsto y(y-1) \dots (y-n+1)$.

180508 Etingof: Mod ϵ^2 , co-Jacobi is not needed for δ .

180507 Proj. Unravel the topology behind Enriquez-Furusho @arXiv:1605.02838 *A Stabilizer Interpretation of Double Shuffle Lie Algebras*; also at SCGP.

180423 Riddle (Masbaum). Can you partition a rectangle exactly one of whose sides is irrational into finitely many squares?

180413 Itai: $\{\text{partitions of } n \text{ into odd numbers}\} \leftrightarrow \{\text{partitions of } n \text{ into distinct numbers}\}$.

180411 Riddle (Tsimmerman). The area of a projection of a unit cube on a plane P is the length of its projection on P^\perp .

180402 BBS: VanDerVeen-180402: $y \rightsquigarrow x^{-1}$ via $y = (\omega + (T - T^{-1})a)x^{-1}$.

180320 $M_{2 \times 2}(\mathbb{C})/(\text{conj})$ isn't Hausdorff.

180318 In CU/QU, t_i 's are "semi-scalars" — scalars for one tensor factor, Lie elements for m, Δ, S . Reduction by $\langle t_i \rangle$ kills interest. $\langle t_i = t_j \rangle$ is interesting for m but fully destructive for Δ/S .

180317 Burton @arXiv:1712.05776: HOMFLY-PT is $\mathcal{O}(\sqrt{n \log n})$.

140405 In **Projects: Mathematica: Localization.pdf**: `a=1; c:=b; Command[{a=a, b=2}, x:=a; y=c]; ?x; ?y`. Block: local values, `x:=a, y=2`. Module: local symbols, `x:=a$1, y=b` with `a$1=1` and `unset b`. With: internal replacements, `x:=1, y=b`.

170427 Faddeev @arXiv:math/9912078 (10), then Quesne @arXiv:math-ph/0305003: $\log \phi_q^x = \sum_{k \geq 1} \frac{(1-q)^k x^k}{k(1-q^k)}$. Readable proof in Zagier's "The Dilogarithm Function", pp. 28-30.

180206 "Diagrammatic" \implies depends on g continuously; meaningful "group-like"; an intrinsic upper bound on what can be done.

180208 The Benkart-Witherspoon representation of 2017-06/BW.nb is commented out here.

180205 Task. Re-examine the relation between 2-parameter and 1-parameter quantum groups. Wherefore moding out by t ? Can it be recovered? Where is ϵ hiding in the 1-parameter picture?

180126 LuaTeX marries Lua and TeX.

180118 Outreach talk idea “Humans’ Art and God’s Art”: pattern recognition in Schwartz’s factorization, then in an ArrayPlot of primes, then a word on crypto, then K250 poster.

170520 ∇ is graded! Related to spinners (sol’s of $[a_{12}, s_1 + s_2] = 0$ and $\delta//[\cdot, \cdot] = [-, s]$). s is inseparable from the tadpole $\bigcirc = (a_{12} - a_{21})//m_-^{12}$ by products, co-products, primitivity, and degree. \rightarrow p11:180724.

180113 Dylan: “elevator pitch”.

180106b Refined BCH: What’s $\log \oplus^x \rightarrow \oplus^y \rightarrow \oplus^z$ in $\mathcal{A}^{v,ac}(x^*y^*z)$? Is it of z -degree 1? What if replacing $\oplus^? \rightarrow \oplus^z$ by another \exp ?

180106a In $\mathcal{A}^{v,ac}$, can bring all c vertices to before all b vertices.

180104b Q. Let B_L^U be the RAAG generated by $(R_x^a)_{a \in U, x \in L}$ modulo $(R_x^a, R_y^b) = 1$ whenever locality, $a \neq b$ and $x \neq y$. Let $A_L^U = \text{gr } B_L^U$, the RAAA generated by $(r_x^a)_{a \in U, x \in L}$ modulo $[r_x^a, r_y^b] = 0$ whenever locality. Let $Z: B_L^U \rightarrow A_L^U$ by $R_x^a \mapsto r_x^a$. Are there $\{\Delta_{a,yz}^{bc}, \Delta_{yz}^x\}$ on A_L^U compatible with the natural ones on B_L^U ?

180104a Costello, Witten, Yamazaki arXiv:1709.09993: “Gauge Theory and Integrability, I”.

171228b Nosaka arXiv:1712.02060: “the Orr invariant of degree k is equivalent to the tree $[]$ Kontsevich invariant of degree $< 2k$ ”.

171228a González-Meneses, Silvero arXiv:1712.01552: Polynomial braid combing.

170829b Przytycki arXiv:1707.07733: With HOMFLYPT $P(\bigcirc) = 1$, $aP(\text{?}) + a^{-1}P(\text{?}) = zP(\text{?})$, expand $P = \sum_i P_{2i}(a)z^{2i}$. Then P_{2i} is of complexity $O(n^{2+3i})$, likely $O(n^{2+2i})$. **Q.** Does P_{2i} factor through type $2 + 2i$ invariants? Ito arXiv:1710.09969: (1) Related to “low genus invariants” in the $\mathcal{A} \rightarrow \mathcal{M}$ sense. (2) Even the cables of P_0 are mutation invariant.

171214 Wanted. In $\mathcal{A}^{\sim v}$, a tail-strand/head-strand pairing P , a co-product P -dual to m and a P -compatible antipode (R would then be the P -inverse of P ?). Breaks: (1) A spinner / a homotopy w -strand. (2) The Cartan-criterion relation.

171213 Q. What’s transmutation? (In Majid and in Habiro’s “Bottom Tangles”).

171212 Q. An \mathcal{A}^v analog of the \mathcal{A}^u notion “ $gl(N)$ genus 0”?

170923 Q. Is \hat{h} injective on $\mathcal{U}_h(\mathfrak{g})$? What’s $\text{gr } \mathcal{U}_h(\mathfrak{g})$? Expansion? Is it inductive? Can I trust a non-universal inductivity proof?

171202 If $(C(S), \Delta_{bc}^a)$ is a meta coalgebra (less is enough; “symmetric set comodule?”) and $C^n := C(\{0\} \cup n)$ then $d: C^n \rightarrow C^{n+1}$ by $dE := \sum_{k=1}^{n+1} (-)^k E // \sigma_{k+1, \dots, n+1} // \Delta_{0k}^0$ has $d^2 = 0$. **Q.** Find H^n when $\Delta_{ix}^f(t, S)$ is $D_1 = f(t, S)$, $D_2 = f(t + x, S)$ or $D_3 = D_2 - D_1$ on $FA(*, S)$. For D_3 , a spectral seq. with $D_{1,2}$?

170625 $\mathcal{U}_{h;\gamma\epsilon}$ conventions in Projects: PPSA: CS-PPSA.pdf.

171117 Kotorii arXiv:1705.10490: n -equivalence on $v\mathcal{K}(\uparrow) \Leftrightarrow$ equivalence modulo $\text{LCS}(AB)$.

171116 The proofreader’s clasper transpose.

171109 Cheng, Jackson, Stanley arXiv:1601.01377: With $q = e^{\hbar/2}$, $(n)_q = (q^n - q^{-n})/(q - q^{-1})$ under $[h, x] = 2x$, $[h, y] = -2y$, $[x, y] = (h)_q$, have $\frac{x^a}{(a)_q!} \frac{y^b}{(b)_q!} = \sum_{i \geq 0} \binom{h+b-a}{i} \frac{y^{b-i}}{q^{(b-i)_q!}} \frac{x^{a-i}}{(a-i)_q!}$ (and more). Also in People: VanDerVeen: Generalxy.nb.

171108b Def. $\mathcal{K} = \mathcal{K}^{gcs-rvt}$: ground (some components are on the ground) ceiling (some are ceiling) surgery (some g/c components are surgery-slippy) rotational virtual tangles. \mathcal{K} is $\textcircled{0}$.

Conj. (1) $\mathcal{K}^{gcs-rvt}$ has a combinatorial description. (2) \mathcal{K} has a surgery-compatible expansion; v -Hopf surgeries split. (3) \mathcal{K} has a stitching-compatible “universal dequantizer” expansion.

171108a Q. Do volumes / homologies extend to rotational v -knots?

171018 Taylor: If $f(a + x) = \sum_{k=0}^n \frac{f^{(k)}(a)x^k}{k!} + R_{n,a}(x)$ then $\exists \xi_{1,2} \in (0, x)$ s.t. $R_{n,a}(x) = \int_0^x \frac{f^{(n+1)}(a+\xi)}{n!} (x - \xi)^n d\xi = \frac{f^{(n+1)}(a+\xi_1)}{n!} x(x - \xi_1)^n = \frac{f^{(n+1)}(a+\xi_2)}{(n+1)!} x^{n+1}$.

171107 Manin ’89 “multiparametric quantum deformation”. Garcia, Gavarini arXiv:1708.05760 “multiparameter quantum groups” (MpQG).

140213 $\$:$ F , mathtools: $:=$, mathabx: \hookrightarrow Ψ , babel: $\lambda\lambda$, txfonts: \mathbb{I} . Hupfer knows jumplines. pdfcomment.

171102 Khovanskii: an algebraic formula is expressible in radicals iff its monodromy group is solvable. \Rightarrow : Arnol’d argument. \Leftarrow middle lemma: a finite Abelian group A acts on a ring R that contains all roots of 1. Then every element of R is a linear combination of roots of elements in R^A .

171015 Ito’s arXiv:1411.5418 “Topological formula of the loop expansion of the colored Jones polynomial” has (multi-)forks.

171010 An expansion $\mathcal{U}_{h;\gamma\epsilon} \rightarrow \text{gr } \mathcal{U}_{h;\gamma\epsilon}$?

171009b Q. A name for $\oplus^v(\xi x + \eta y + \delta xy - t\xi\eta)$?

171009a Q. If $\phi: (V = \mathbb{R}_{\xi^i}^n) \rightarrow (W = \mathbb{R}_{\eta^j}^m)$ and $W = \langle y_j \rangle$ with $\eta^j(y_k) = \delta_k^j$, what do you call $\Phi(\xi^i, y_j) := \sum y_j \phi^*(\eta^j)$?

141226b Proj. FT invariants of fixed-linking-numbers (uvw)-KO. A Goussarov view? FT relative to CO/CU (Commute Overcrossings/Undercrossings)? Is there a good presentation of tangles with fixed linking numbers?

170919 Talk idea: “ $\textcircled{0}$ -Tangles & the Quantum Groups Conspiracy”.

170128 Cartan’s criterion: $\mathfrak{g} \subset \text{End}(V)$ is solvable iff $\forall x \in \mathfrak{g}, y \in [\mathfrak{g}, \mathfrak{g}], \text{tr}_V(xy) = 0$. Induces a quotient of \mathcal{A}^v ; what is it?

170917 Gautam: an explicit $\mathcal{U}(sl_n)[\hbar] \cong \mathcal{U}_\hbar(sl_n)$.

170914 Q. Is there a canonical isomorphism between quantizations of sl_2 with varying r ?

170913b Do. Center poly-poly at Lie algebra contractions.

170908 Livingston arXiv:1709.00732: $\sigma: S^1 \rightarrow \mathbb{Z}$ is a knot signature function iff all discontinuities are at roots of an Alexander polynomial and $[\dots]$. **Q.** How fits with w and with Γ -calculus?

170519 Bonahon’s “miraculous cancellations” arXiv:1708.07617 link Ito with PPSA?

170813 Given a Hopf H , is there a “pair one” $\text{op } H^* \otimes H \rightarrow H^* \otimes H$?

170309 In 2017-03/geps.nb: $w, u, b, c = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ -\epsilon & 0 \end{pmatrix},$

$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \frac{1}{2} \begin{pmatrix} -1 - \epsilon^{-1} & 0 \\ 0 & 1 - \epsilon^{-1} \end{pmatrix}$ obey $g^\epsilon: [w, c] = w, [c, u] = u, [u, w] = b - 2\epsilon c$. Then $r = (b_1 - \epsilon c_1)c_2 + u_1 w_2 = \frac{1}{4\epsilon} \begin{pmatrix} 1 - \epsilon & 0 \\ 0 & 1 + \epsilon \end{pmatrix} \otimes \begin{pmatrix} 1 + \epsilon & 0 \\ 0 & 1 - \epsilon \end{pmatrix} - \epsilon \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$.

170807 Naef: For $D \in \text{der}(FL)$ (any!), $\text{div}(D) := \sum_x \text{tr } \partial_x D(x)$ satisfies $\text{div}[D_1, D_2] = D_1 \text{div}(D_2) - D_2 \text{div}(D_1)$.

170804 Given $f(x), g(\xi)$, have $f(\partial_\xi)g(\xi)|_{\xi=0} = g(\partial_x)f(x)|_{x=0}$.

170802 Does every infinitesimal deformation of a solvable Lie algebra globalize? Does some H^2 parameterise knot invariants?

170708 In w, inner $q\Delta$'s automatically lead to sder- hence tree-level u- associators.

170707 Which doubling makes the diagram commute? $\mathcal{A}^w(\uparrow) \xrightarrow{?} \mathcal{A}^w(\uparrow_2)$
 $\downarrow \tau \qquad \qquad \downarrow \tau$

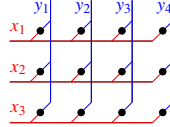
170703 What characterizes “PBW” maps $\mathcal{U}(sl_2^0) \xrightarrow{q\Delta} \mathcal{U}(sl_2^0)^{\otimes 2}$
 $\mathcal{S}(\mathfrak{g}) \rightarrow \mathcal{U}(\mathfrak{g})$?

170702 In $\mathcal{U}_{\hbar;\gamma\beta}$, $\prod_i e^{\eta_i y} e^{\alpha_i a} e^{\xi_i x} = e^{\eta y} e^{\alpha a} e^{\xi x} e^{\sigma} (1 + \sum_{k \geq 1} \Lambda_k \beta^k)$, with $\alpha = \sum \alpha_i$, $\eta = \sum_i \eta_i e^{-\gamma \sum_{j < i} \alpha_j}$, $\xi = \sum_i \xi_i e^{-\gamma \sum_{j > i} \alpha_j}$, $\sigma = \frac{1-T}{\hbar} \sum_{i < j} \xi_i \eta_j e^{-\gamma \sum_{l: i < l < j} \alpha_l}$ and Λ_k is ...

170412b Rote (2001), [BBS:Dancso-170529](#): An n^4 -time division-free algorithm for det.

170602 Word-pairing in Hopf algebras:

$$\left\langle \prod_{i \in \underline{n}} x_i, \prod_{j \in \underline{m}} y_j \right\rangle = \prod_{i \in \underline{n}, j \in \underline{m}} \langle x_i^{(j)}, y_j^{(i)} \rangle.$$



170529 Is “ $(\mathfrak{g}, [\cdot, \cdot], \delta)$ non-negatively graded with Abelian degree 0” same as “ $D\mathfrak{g}$ is a sum of Kac-Moody and inhomogeneous factors”? A sense by which these are precisely “the quantizeables”?

170323b Is there a “Heisenberg-Drinfel’d Double Construction”?

170224 \mathfrak{g} solvable $\Leftrightarrow [\mathfrak{g}, \mathfrak{g}]$ nilpotent $\stackrel{?}{\Leftrightarrow} \mathfrak{g} \cong \mathfrak{a} \ltimes \mathfrak{n}$ with Abelian \mathfrak{a} and nilpotent \mathfrak{n} . Also, \mathfrak{g} solvable \Leftrightarrow there is a finite decreasing filtration $(\mathfrak{g}_k)_{k \geq 0}$, $\mathfrak{g}_0 = \mathfrak{g}$, with $\mathfrak{g}_0/\mathfrak{g}_1$ Abelian and $[\mathfrak{g}_k, \mathfrak{g}_l] \subset \mathfrak{g}_{k+l}$. Then $\mathcal{U}(\mathfrak{g})$ also has a multiplicative decreasing filtration.

131009 Let $\Gamma_{1,2,3}$ be thickened surfaces. Is there an expansion for the structure $\mathcal{K}(\Gamma_1 \hookrightarrow \Gamma_2) \times \mathcal{K}(\Gamma_2 \hookrightarrow \Gamma_3) \xrightarrow{\cong} \mathcal{K}(\Gamma_1 \hookrightarrow \Gamma_3)$? (Handlebodies in Habiro-Massuyeau [arXiv:1702.00830](#)).

170518 Given unital algebras B, C and $R \in B \otimes C$, when is there a swap $s: C \otimes B \rightarrow B \otimes C$ so that $R^{12}R^{13}R^{23} = R^{23}R^{13}R^{12}$ would hold in $B \bowtie_s C$?

170512 Le: “Every $\mathcal{U}_q(\mathfrak{g})$ embeds in some quantum torus”. A smidge version? Does “every \mathfrak{g} embeds in a Heisenberg” quantize to Le’s?

170508 Majid’s Primer §4: $A \mapsto M(A)$, algebras to bialgebras.

131023 Markl: “like a bottle under a waterfall”. Psychology buzzword: “cognitive overload”.

170413a Shortcut / characterize / replace / generalize $\mathcal{U}(\mathfrak{g})^* \cong \mathcal{S}(\mathfrak{g})^* \cong \mathcal{S}(\mathfrak{g}^*) \cong \mathcal{U}(\mathfrak{g}^*)$. Are “doubling the Cartan” a/o “ Δ -conjugations” more fundamental than “pairing”?

131130b **Proj.** “Alexander Recovery”. Conway relation; relations as in Archibald; factorization as in Levine [arXiv:q-alg/9711007](#), Tsukamoto-Yasuhara [arXiv:math/0405481](#); cabling; Fox-Milnor (is there for links?); genus property; crossing-number property; split-link property; u-range; w-range; unitarity; concordance; mutations; behaviour under mirror/strand reversal; Torres conditions; Hartley’s property; cheirality properties as in [arXiv:1608.04453](#); Alexander/Thurston norms as in [McMullen:alex.pdf](#).

170413b In [2009-01/KAL-090128...pdf](#) and [BBS:KAL-090128](#):

170411 [Vogtmann@MSRI](#): action of trivial trees on the $\text{gr}(\pi_1(\Sigma_g))$ by derivations via contractions; relation between $H^*(\text{Out}(F_n))$ and some H^* of trees modulo AS & IHX.

$$\mathcal{A}^v(\uparrow) \xrightarrow{T_g} \mathcal{U}(\mathfrak{g})$$

$$\downarrow \Delta \qquad \qquad \uparrow$$

$$\mathcal{A}^v(\uparrow \uparrow^{ab}) \xrightarrow{T_{g^+}} \mathcal{U}(\mathfrak{g}^+)$$

$$\text{Gaussian pairing: } \left\langle \exp\left(\frac{x \subset}{2}\right) \mid \exp\left(\frac{\supset y}{2} + \sum_i \bullet \rightarrow i\right) \right\rangle = \exp\left(\frac{1}{2} \log\left(\frac{1}{1-xy}\right) \circ + \sum_{i,j} \frac{x i \bullet \rightarrow j}{1-xy}\right).$$

131014 Problems with the projectivization paradigm: No room for negative degrees and for degree-decreasing ops. No built-in \hbar .

170325b **Q.** Is there a “Vogel group” acting on $\mathcal{A}^w(S)$? In general, is there topology behind the Vogel action?

170325a Generalized Weyl: If $f \in \mathcal{S}(V)$ and $\psi \in \mathcal{S}(V^*)$ then in $\mathcal{U}(HV)$, $f\psi = \psi_1 f_1 \langle \psi_2, f_2 \rangle$, where $\Delta f = \sum f_1 \otimes f_2$ and $\Delta \psi = \sum \psi_1 \otimes \psi_2$. Is there a version with $\mathcal{U}(\mathfrak{g})$ replacing $\mathcal{S}(V)$?

170325c **Q.** Is there a \mathcal{K}^u interpretation of the Vogel action on \mathcal{A}^u ?

170318 **Q.** Are there easy θ -invariant braidors for \mathfrak{g}_0 ?

170323a **Proj.** Study $\mathcal{K}^w/\mathcal{A}^w$ with “solvable heads”.

170322 With $f_t = e^t - 1$, $f_{x+y} = f_x + e^x f_y = e^y f_x + f_y$.

170320a **Q.** Is there a good algebraic structure of “groups with a fixed Abelianization”?

170310 Tentative ID: I’m a selfish rationalist atheist permissive individual-rights free-market socialist global citizen. Note to self: read defs! (Liberal? Democrat?)

170308 Mine “ $\mathcal{S}(\mathfrak{g}): \mathcal{S}(\mathfrak{g}^*)$ pairing” \Leftrightarrow “commutation in $\mathcal{U}(H\mathfrak{g})$ ”.

170302 **Conj.** Every rotational classical YB structure can be quantized. Related to spectral parameters? (Chari-Pressley 15.2.B).

170306 For sl_2^+ $= \langle e, f, h, c \rangle / ([h, e] = 2e, [h, f] = -2f, [e, f] = h, [c, \cdot] = 0)$, $r_{ij} := e_i f_j + h_i h_j / 4 + \alpha(h_i c_j - c_i h_j)$ solves CYBE.

170301 **Prob.** Given ad and a solvable structure on \mathfrak{g} , fully implement the group-likes in $\hat{\mathcal{U}}(\mathfrak{g})$.

170223d VdV on gmail/161122: A $\mathfrak{g}_0 / gl(1|1)$ relationship.

170223c **Q.** What’s the internal kernel in \mathcal{A}^v for 2-loop \mathcal{Z}^u ? What’s the nearest-dual Lie bialgebra?

170223b **Do.** Compute $Z_{I\mathfrak{g}}$ for general \mathfrak{g} w/o back reference to \mathcal{A}^w .

170223a By nilpotent approximation, all semi-simple weight systems come from nilpotent Lie algebras. Do the latter make more?

170222 Is there a Chern-Simons theory for degenerate Casimirs?

141107 Claim. \mathfrak{g} a Lie algebra, $d \in \mathfrak{g}$ fixed, c a “new” central element, $\mathfrak{g}_1 := \mathfrak{g} \oplus \langle c \rangle$, $\delta: \mathfrak{g}_1 \rightarrow \mathfrak{g}_1 \otimes \mathfrak{g}_1$ by $c \mapsto 0$ and $\mathfrak{g} \ni x \mapsto [d, x] \otimes c + c \otimes [d, x]$, then \mathfrak{g}_1 is a Lie bialgebra. Extends to a non-cocommutative seed? Eckhard: may be related to Medina-Revoy “double extensions”, a structure theorem for metrized Lie algebras. \rightarrow [p8:141114a](#).

170221 Teichner on [mo:7052](#): K is slice iff $K \# R$ is ribbon for some ribbon R .

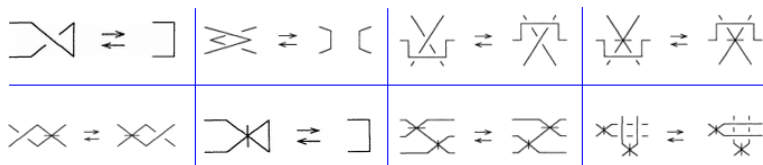
170126b **Q.** What does Ado give for \mathfrak{g}_1 ? For $I\mathfrak{g}$ (\rightarrow [p8:190709](#))?

161027a Describe \mathcal{B}^{rv} and $\alpha: \mathcal{A}^u \rightarrow \mathcal{A}^{rv} := \mathcal{A}^v / \langle [a_{ij}, s_i + s_j] \rangle$.

170126a From Roland’s Poly.pdf: Under $[F, E] = 1 - t - (1 + t)\epsilon L$, $[F, L] = F$, $[L, E] = E$, $t = e^c$, $s = 1 - t$ and $\nu = (1 - s\delta)^{-1}$, have $\odot(FE|e^{\alpha E + \beta F + \delta EF + \epsilon(s-2)P(E,F)}) = \odot(ELF|(1 + \epsilon(s-2)(P(\partial_\alpha, \partial_\beta) + \partial_s((\partial_\alpha + \partial_\beta)/2 + \partial_\delta + L) + s\partial_s^2/4))\nu e^{\nu(\alpha\beta s + \alpha E + \beta F + \delta EF)})$

131103 The Yoshikawa moves (usefulness limited by the before/after unknottedness condition; Chterental: the Swenton proof of completeness may be broken, new ones by Kearton-Kurlin and him-

self exist):



170108a `Table[As[n,6], As[k,0,n], Binomial[n,k]]` in [2017-01/As.nb](#).

170107b What do coverings of the annulus say about annular braids?

170107a Is there strand-doubling for handle-strands ($|_h$'s)? In general, what do maps between surfaces of (possibly different) genera say about $\mathcal{A}(|_h^g \uparrow^n)$?

161122 What's θ , in \mathcal{A}^w language?

161101 Stein's paradox: with $\theta \in \mathbb{R}^{n \geq 3}$, given a single measurement x_i of n independent normal Gaussians with mean θ_i and variance 1, the estimator $\hat{\theta} = x$ for θ is dominated by another (across all θ), if aiming to minimize $E[\|\theta - \hat{\theta}\|^2]$.

161027b What's the abstract relation between Roland's g_0 and mine?

161009 Kondo (1979): Any Alexander polynomial is attained with unknotting number 1.

160911 **T/F?** The only solutions to Morrison's equation for $p \in PB_4$ are 1 and σ_2^{-2} .

160801 Is there a topological meaning to primitive-exponential hybrids like $\Phi^{-1}t_{14}\Phi$?

160721 **Q.** Are braidors related to quandle cohomology?

160616 Representing \mathcal{A}^w on functions on a 2D Lie algebra, what is the functional representation of the braid group we get?

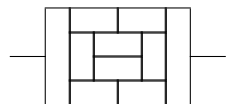
160613 Find Gassner and dual-Gassner in the topology of PwB_n .

160609 $\text{rad } \mathfrak{g} := (\text{maximal solvable ideal})$. Levi: $\text{rad } \mathfrak{g}$ has a complementary Lie algebra.

160519 **Proj.** Extendibles extend to extendibles, the group case.

160508 Mrowka's dodecahedron:

160505 Chterental: Brunnian 2-component links have manifestly Brunnian diagrams.



160503b **Prob.** Find a methodology for promoting invariants of braid-like virtuals to full invariants of classicals.

160503a **Q.** What's ω , as seen from linear control theory?

160414b **Q.** In $\mathbb{Q}G$, is $\varprojlim_{m \geq n} I^n / I^m = \left(\varprojlim_m I / I^m \right)^n$?

160414a **Q.** Suppose $A \otimes B \twoheadrightarrow C$ and all are filtered compatibly. Does $\widehat{A \otimes B} \twoheadrightarrow \widehat{C}$? Does $\widehat{A} \otimes \widehat{B} \twoheadrightarrow \widehat{C}$?

160410 **Q.** Can the filtrations of $\mathbb{Q}G^{(n)}$ and of $\widehat{\mathbb{Q}G}$ be defined from their (Hopf-)algebraic structures?

160408 **Q.** Integrate Lie algebra 2-cocycles to Lie group 2-cocycles.

160403b **Prob.** Characterize $\{\exp F(\log X, \log Y)\}$ in $\widehat{\mathbb{Q}FG_2}$ language.

160315b Is $FG_{a,b,c,d} / \{a = c^b, b = d^a, c = e^f, d = f^e\}$ free? Expansion faithful?

160315a $\pi_2(n\text{-ring complement}) = \mathbb{Z}(FG_n \times \underline{n})$?

160311 For wB , why does the π_1 action determine the π_2 action? For wT , it doesn't.

160308 Naor's $\sqrt{2} \notin \mathbb{Q}$: Else $0 < (\sqrt{2} - 1)^n \rightarrow 0$ but with $\sqrt{2} = p/q$, $(\sqrt{2} - 1)^n = a_n \sqrt{2} + b_n = (pa_n + qb_n)/q \geq 1/q$.

160304 **Riddle** (saw Ω by Gracia-Saz after M. Bernstein). 100 prisoners strategize, then are sealed in rooms with the same countable sequence of "boxes with reals" in each. Can each open all but one of their boxes and guess the remaining one so that at most one prisoner would be wrong? Hint: 0-1 boxes, finitely many 1s.

160113 A meta-monoid M is *factored* if it has a \square compatible with all operations. What structure form the primitives P of M ? Does P determine M ?

151214 **BBS:** [Schneps-151209](#), the spherical \diamond :

$$\varphi(t_{12}, t_{23})\varphi(t_{34}, t_{45})\varphi(t_{51}, t_{12})\varphi(t_{23}, t_{34})\varphi(t_{45}, t_{51}) = 1.$$

151209 Are there solutions of R4 (+more?) in $\langle a_{12}, a_{21} \rangle$? No?

150924 Schneps in Les Diablerets: For $f \in FL(x, y)$, $\pi_y(f)$ proj. on words ending with y , $f_* := \pi_y(f) - \sum \frac{(-)^n}{n} (f | x^{n-1}y)y^n$ rewritten in $y_i := x^{i-1}y$. $\partial s := \{f: \Delta_*(f_*) = f_* \otimes 1 + 1 \otimes f_*\}$, with $\Delta_*(y_i) := \sum_{k+l=i} y_k \otimes y_l$ (group version in [BBS:Schneps-151209](#)).

BBS:Ens-150923: Write $u, v \in FA(x, y)y$ in $y_i := x^{i-1}y$ and set $St(1, u) = St(u, 1) = 1$, $St(y_i u, y_j v) = y_i St(u, y_j v) + y_j St(y_i u, v) + y_{i+j} St(u, v)$. Then $\partial s = \{f \in FL_{\geq 3}(x, y): (f | St(u, v)) = 0\}$, where not both u and v are powers of y . For $f \in \partial s$ set $F(x, y) = f(-x-y, -y) = xF^x + yF^y$, $G(x, y) = \sum_{i \geq 0} \frac{(-)^i}{i!} \partial_x^i (F^x) y x^i$. Then $f \mapsto D_{F,G} = \partial s \hookrightarrow \text{frv}_2$.

151003a **BBS:Ens-151002:** In $FA(u_d)$ with $\deg u_d = d$, if $\exp \sum u_d = \sum y_k$ with $\deg y_k = k$, then $\Delta y_k = \sum_{i+j=k} y_i \otimes y_j$.

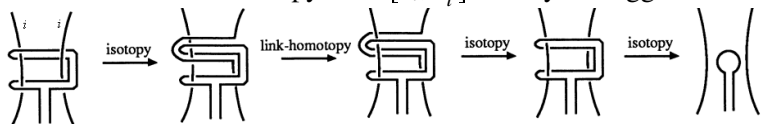
151126 **A** a Hopf algebra, b a primitive derivation $b \parallel \square = \square \parallel (b \otimes 1 + 1 \otimes b)$, $B := \{D \in A: (bD = 0) \wedge (\square D = D \otimes 1 + 1 \otimes D)\}$. Characterize the subalgebra $\langle B \rangle$ generated by B .

151118c Wikipedia: Schur multiplier: "A projective representation of G can be pulled back to a linear representation of a central extension C of G ".

151118b Hopf: F free, $G = F/R$, $H_2(G, \mathbb{Z}) \cong (R \cap [F, F])/[F, R]$.

151118a Hillman's Alg. Inv. of Links, pp. 238: "Cochran, Orr conjectured that if all Milnor invariants of length $< r$ vanish then all to length $2r$ are well-defined".

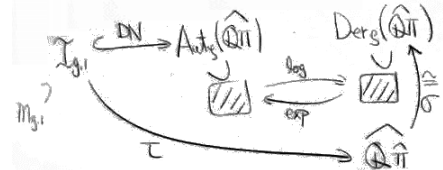
151110 The Milnor homotopy trick $[x_i, x_i^g] = 1$ by Habegger-Lin:



151012 The Goldman Lie algebra, which is its group?

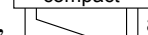
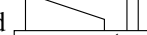
151006 Kuno@LD15:

(missing: the archetypical model for " σ is an isomorphism")



151003b The Alexander

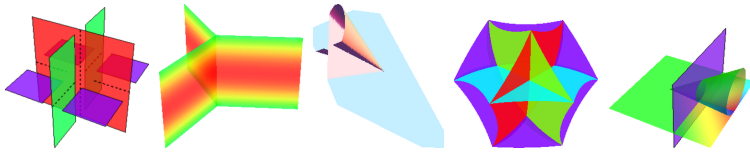
quandle: $t: A \rightarrow A$ an automorphism of an Abelian group, $a \uparrow b := ta + (1-t)b$.

131122c Overhand/underbelly,  and , abstain.

Sep. 2015, stronger: "4-end bottom tangles are unparenthesized"!

150925 König's lemma: in an infinite connected graph with finite valencies there's an infinite simple path.

140123 ≥ 47 4D hardware pieces at [2013-12/4DHardware/](#):



131106a $XII \Leftrightarrow \text{FiC} \Leftrightarrow \text{“bra-cobra”} \Leftrightarrow \text{“involutive”}$ (Chas, [arXiv:math/0105178](#)) \Leftrightarrow “infinitesimal of $S^2 = 1$ ”. Ševera in 2015 Les Diablerets talk: this globalizes.

150829 Massuyeau: Passi, Passman: over \mathbb{Q} , dimension=LCS.

150806 Two $1/3$ rotations $\rho_{3|3'}$ on $\mathcal{W}(\uparrow_x \uparrow_y)$: $S^y // \Delta_{yz}^y // m_x^{xz|zx} // \sigma_{yx}^{xy}$. In general, $\text{Aut}(FG_n) \subset \mathcal{A}^v(\uparrow^n)$ and $\text{Out}(FG_n) \subset \mathcal{A}^u(\uparrow^n)$.

150804 **TIL**. ctrl-alt-F1 through ctrl-alt-F7, pstree.

150729 Habiro ring: $\widehat{\mathbb{Z}[q]} := \varprojlim \mathbb{Z}[q]/(q)_n$ w/ $(q)_n := \prod_{i=1}^n (1 - q^i)$.

150719b Odd quandle-from-group: $a \uparrow b := ba^{-1}b$.

150719a Gordon on Wada: With $\pi(K) := \begin{matrix} z & \nearrow & x \\ & x & \searrow & y \end{matrix} \rightarrow x^{-1}zx^{-1}y = 1$, $\pi(K) \cong \pi_1(\Sigma_2(K)) * \mathbb{Z}$.

150205 Abe-Tagami [arXiv:1502.01102](#), Gompf-Scharlemann-Thompson [arXiv:1103.1601](#): slice-ribbon counterexamples?

150624 “Set function $\varphi: G \rightarrow H$ is affine” means $\varphi(I_G^n) \subset I_H^n$. Makes a category. Group morphisms and translations are affine. $\varphi_i: G_i \rightarrow H$ affine $\Rightarrow \varphi_1\varphi_2$ affine, so “sorting” on FG is affine, $Id: G \times H \rightarrow G \ltimes H$ is affine. If $G \ltimes H$ is almost-direct, $Id: G \ltimes H \rightarrow G \times H$ is affine, so combing braids is bi-affine.

150517 $G =: G_1$ a group, $G_{n+1} := (G, G_n)$, $\pi_n: G_n \rightarrow L_n := G_n/G_{n+1}$. Given affine sections $\varphi_n: L_n \rightarrow G_n$ let $\zeta: G \rightarrow \hat{L} := \prod L_n$, the “LCS-expansion using φ .” (“group-PBW for φ_* ”), by $\zeta_1 := \pi_1$ and $\zeta_n(g) := \pi_n(\varphi_{\zeta_{<n}}(g)^{-1}g)$ where $\varphi(\lambda_1, \lambda_2, \dots) := \varphi_1(\lambda_1)\varphi_2(\lambda_2) \dots$. Then $\zeta_{<n}(h) = 0$ iff $h \in G_n$ and $g = \varphi\zeta_{<n}(g)$ in G/G_n . Is ζ_n of type n ?

150522 Automatic structure on a group G : A set of semigroup generators, “acceptor” automaton M on A accepts L s.t. $\pi: L \twoheadrightarrow G$, for each $x \in A \cup \{e\}$ “multiplier” automaton M_x on $(A, A) = (A \times \$) \times (A \times \$) \setminus (\$, \$)$ accepting (u_1, u_2) iff $\exists v_i \in L$ with $u_i \in v_i \$$ and $\pi(v_2) = \pi(v_1 x)$.

150609c Budney’s [arXiv:math/0309427](#): Long knot space is $\bigsqcup_{n=0}^{\infty} (C_2(n) \times \mathcal{P}^n)/S_n$, with $C_2(n)$ the space of n little 2-cubes and \mathcal{P} the set of prime knots, with $[0, 1]_{t,s}^2$ parameterizing arc length (t) and scale (s).

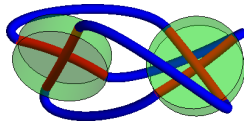
150609b **BBS:Lambrechts-150603**: $H_*(\text{Emb}(\mathbb{R}, \mathbb{R}^{\geq 4})) \cong H_*(\text{a graph complex})$. **BBS:Lambrechts-150603** Goodwillie-Sinha: At $n \geq 4$, $\text{holim}_{p \rightarrow \infty} \overline{\text{Conf}}^f(p, \mathbb{R}^n) \simeq \text{Emb}^f(\mathbb{R}, \mathbb{R}^n)$. Naive? Intuition?

150609a Is $\pi_{>0}$ ever useful to understand π_0 ?

150608 A PogForm in **2015-06**:

150502 **BBS:Martins**: • A Crossed Module (CM, e.g. [arXiv:0801.3921](#)) models $\partial: \pi_2(X, A) \rightarrow \pi_1(A)$: a group homomorphism $\partial: E \rightarrow G$ with an action $\triangleright: G \curvearrowright E$ s.t. (1) $\partial(g \triangleright e) = g(\partial e)g^{-1}$, (2) $(\partial e) \triangleright f = e f e^{-1}$ (contains $\pi_1 := \text{coker } \partial$, $\pi_2 := \ker \partial$, and the Postnikov k -invariant in $H^3(\pi_1, \pi_2)$ when $A = X^1$; equivalent to a “2-group”). There are homotopies of CMs, the free CM over a set-to-group $\partial_0: C \rightarrow G$, quotients of CMs, the “actor” CM $G \rightarrow \text{Aut}(G)$. Whitehead (JHC): $\pi_2(X^2, X^1)$ is the free CM over the attaching maps.

• A Differential Crossed Module (DCM, Baez-Crans [arXiv:math/0307263](#), Cirio-Martins [arXiv:1309.4070](#)) is a Lie algebra morphism $\partial: \mathfrak{h} \rightarrow \mathfrak{g}$ with an action $\triangleright: \mathfrak{g} \curvearrowright \mathfrak{h}$ by derivations, s.t.



(1) $\partial(g \triangleright h) = [g, \partial h]$, (2) $(\partial h) \triangleright h' = [h, h']$. Assign CM to DCM by $G := \{e^\gamma: \gamma \in \mathfrak{g}\}$, $H := \{e^\eta: \eta \in \mathfrak{h}\}$, $\partial: e^\eta \mapsto e^{\partial \eta}$, $e^\gamma \triangleright e^\eta := e^{e^\gamma \triangleright \eta}$. There’s an analytic $\{CM\} \rightarrow \{DCM\}$; not yet algebraic. Use Rker, $s, t: E \rtimes G \rightarrow G$ (**BBS:Martins-150501**)?

• There’s a DCM $\mathcal{GL}(\mathcal{V}) = (\mathfrak{gl}_1(\mathcal{V}) \rightarrow \mathfrak{gl}_0(\mathcal{V}))$ for a chain complex \mathcal{V} .

• Braided surfaces have $(\circ) \rightarrow (\times)$ (see Khovanov-Thomas [arXiv:math/0609335](#)).

• **BBS:Martins-150501**: A CM $\pi_{12}(K^c)$ for virtual 2-knots.

• **BBS:Martins-150501**: Rker for Hopf morphisms.

150417 \LaTeX displayed equations: `equation(*)`, `align(*)` `BT(&T)*\...\text{E}`, `multline(*)`, `split` (inner for displayed, `BT&T*\...\text{E}`), `aligned` (inner `align`). Related: `\label`, `\tag`, `\nonumber`, `\notag`. [**WB**→].

150422 Lambert’s dreaded W function: $y = xe^x \Leftrightarrow x = W(y)$.

150412 Deriving Gassner: In **2015-04/OneCo.pdf**.

150409 2Dv: In **2015-04/OneCo.pdf**.

150107 $\square: \mathcal{A}(G) \rightarrow \mathcal{A}(G) \otimes \mathcal{A}(G)$ wrong sketch: • If V is doubly filtered, the associated graded of the diagonally-associated single filtration of V is isomorphic to the diagonal single-gradation of the associated doubly-graded of V . **False**. Take $V = \mathbb{Q}\langle x, y \rangle$, $F_{0,0} = F_{1,0} = F_{0,1} = V$, $F_{2,0} = \langle x \rangle$, $F_{1,1} = F_{0,2} = \langle y \rangle$. Then $0 + \langle [x] \rangle = V_{1,0} \oplus V_{0,1} \neq V_1 = 0$. • $\mathcal{A}(G \times H) \cong \mathcal{A}(G) \otimes \mathcal{A}(H)$ as the associated single filtration of the double filtration of $\mathbb{Q}(G \times H)$ is its single filtration. • $g \mapsto (g, g)$ induces $\square: \mathcal{A}(G) \rightarrow \mathcal{A}(G \times G) \cong \mathcal{A}(G) \otimes \mathcal{A}(G)$.

140723 w-meaning for $\sigma_{ij} \mapsto \begin{pmatrix} 1 - t_j & 1 \\ t_i & 0 \end{pmatrix}$? u-meaning for $\sigma_{ij} \mapsto \begin{pmatrix} 1 - t_i & 1 \\ t_i & 0 \end{pmatrix}$? Using the “other” Artin rep. **BBS:Dalvit-150318**?

150307 Georgetown vocabulary: control theory, zinbieL algebra, Fliess operators, shuffle algebra, dendriform algebra.

150227 Infinitesimal $G = \langle X_i \mid R_j \rangle$ definitions [**Br**→], [**DPS**→].

• Pro-unipotent? • Malcev completion: $\text{Mal}(G) := \varprojlim \mathbb{Q} \otimes_{\mathbb{Z}} (G/G^{(n)})$. • $\text{gr } G := \mathbb{Q} \otimes_{\mathbb{Z}} \bigoplus G^{(n)}/G^{(n+1)}$. • Malcev Lie algebra: roughly, $\text{mal}(G) := \hat{F}L(x_i)/(\log R_j)$, with $x_i := \log X_i$. Is filtered. • 1-formal: $\text{mal}(G)$ isomorphic as filtered to a quadratic Lie algebra. • Holonomy Lie algebra of X : \sim quadratic generated by H_1 modulo $\text{im } H_2$.

150224a Surface braids: Bardakov, Bellingeri, Birman, Funar, Gervais, Gonzalez-Meneses, Guaschi, Juan-Pineda.

141226a With Dalvit: for $(+, +) \neq (s_1, s_2) \in \{\pm\}^2$, is there $\phi \in \text{Aut}(FG(x, y))$ s.t. $\phi(y^{-1}xy) = y^{-s_1}xy^{s_1}$, $\phi(x^{-1}yx) = x^{-s_2}yx^{s_2}$? Sela: $\text{Out}(FG_2) := \text{Aut}(FG_2)/\text{Inn}(FG_2) = \text{Aut}(FG_2/[FG_2, FG_2]) = GL_2(\mathbb{Z})$, hence easily not. Chterental: That’s easily within “Whitehead’s algorithm”. Why bother? Otherwise the 4 distinct handshake w-links of **BBS:Dalvit-140617** could be equal as 2-knots contradicting Satoh’s conjecture & showing that Z^w doesn’t extend via BF.

150219 Jones ribbon conditions from the Oberwolfach-1405 AKT?

150206 Study annular braids / tangles. Canonical forms?

131104 Humbert’s thesis pp 22: The relations of t_n^1 : $[v_i, w_j] = \langle v, w \rangle t_{ij}$, $[v_i, t_{jk}] = 0$, $[x_i, y_i] = -\sum_{j \neq i} t_{ij}$. Imply centrality of $\sum_j v_j$ and $t_{ij} = t_{ji}$, $[v_i + v_j, t_{ij}] = 0$, $[t_{ij}, t_{kl}] = 0$, and $[t_{ij}, t_{ik} + t_{jk}] = 0$. Canonical forms?

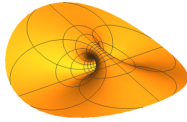
150217 Enriquez: B_n^1 is $\langle \sigma_i, X_1^\pm \rangle \bmod (\sigma_1^{\pm 1} X_1^\pm)^2 = (X_1^\pm \sigma_1^{\pm 1})^2$, $[X_1^\pm, \sigma_i] = 1$ for $i \geq 2$, $[X_1^-, (X_2^+)^{-1}] = \sigma_1^2$, $X_1^\pm \cdots X_n^\pm = 1$, and braid relations, where $X_{i+1}^\pm = \sigma_i^{\pm 1} X_i^\pm \sigma_i^{\pm 1}$.

150210 Reidemeister-Schreier: 1. $H < G \leadsto$ groupoid $H \backslash G$ with objects cosets $H\gamma$, morphisms $(H\gamma, g): H\gamma \rightarrow H\gamma g$, and compositions $(H\gamma, g_1) \parallel (H\gamma g_1, g_2) := (H\gamma, g_1 g_2)$. With this, $H = \text{Aut}(He)$. 2. If $G = \langle X: R \rangle$, $H \backslash G$ is presented with $X \times (H \backslash G)$ generators and $R \times (H \backslash G)$ relations. 3. There's a same-size presentation of $\text{Aut}(He)$.

150208 Kohno knew elliptic KZ in 1996.

150201a In 2015-01:

141204a **Prob.** Find a simple description of simple 2-knots. Done in Kawauchi's *A Chord Diagram of a Ribbon Surface-Link?*



150131 Katz 5.1: $R\mathcal{A}^s(|_h \uparrow^n) \hookrightarrow \mathcal{A}^s(|_h \uparrow^n)/C$. R : nothing on last strand. $|_h$: a handle line.

150130b Katz 5.2: $L\mathcal{A}_1^{s\wedge}(\uparrow^n) \cong \mathcal{A}_1^{s\wedge}(\uparrow^n)/C$. \square_1 : elliptic. \wedge : strutless. s : skeleton-connected. L : only lonely vertices on last strand. C : closed surface.

150130a **Q.** Why is PB^g related to non-tangential differential operators on $\text{Fun}(\mathfrak{g}^g)$?

141224 Katz points (BBS: [Katz-141224](#), [arXiv:1412.7848](#)): • Cheptea-Habiro-Massuyeau's [arXiv:math/0701277](#) has a Clifford-like relation in sec. 8 (earlier, in Habiro's [arXiv:math/0001185](#), fig. 48). • LMO for Lagrangian cobordisms partially interprets leg-gluing in \mathcal{B} . • \mathcal{B}^g -grading: # of trivalent vertices (excluding univalents).

150123 $\text{gr}(PB_n^0 \rightarrow PB_n^1)$ is 0: $\mathcal{A}^{pb,0} \rightarrow \mathcal{A}^{pb,1}$ for a degree mismatch. Likely $[PB_n^1, PB_n^1] \not\supseteq PB_n^0$.

150121 Quillen: $\mathcal{U}(\mathbb{Q} \otimes \text{gr } G) \cong \text{gr } \mathbb{Q}G$, where $\text{gr } G$ uses lower central series, and $\text{gr } \mathbb{Q}G$ uses the augmentation ideal.

150112 Whitney's trick, loosely: In high dimensions at $\pi_1 = 0$, algebraic intersection numbers have precise geometric realizations.

141221 A-S super-CS: (uncertainties **highlighted**) ($d = \theta^\mu \partial_\mu$)

$$\mathcal{A}(\theta) = c + \theta^\mu A_\mu + \theta^\mu \theta^\nu \epsilon_{\mu\nu\rho} \partial_\rho^{(0)} \bar{c} + \theta^\mu \theta^\nu \theta^\rho \epsilon_{\mu\nu\rho} \phi,$$

$$SCS(\mathcal{A}) = \int dx d\theta \text{tr} \left(\frac{1}{2} \mathcal{A} \cdot d^{(0)} \mathcal{A} + \frac{1}{6} \mathcal{A}^3 \right).$$

150106 Przytycki, Sikora's [arXiv:math/0007134](#) "Chinese Rings", [Wikipedia: Baguenaudier](#):

141226c Presentations of $[FG, FG]$, $[FL, FL]$?

141226d **Q.** If $G \leadsto \text{gr } G = \bigoplus I^n / I^{n+1}$ is understood, is $\text{gr}_2 G := \bigoplus I^{2n} / I^{2(n+1)}$ interesting? (A. Likely not.).

141209 **Plan.** Understand simple circle-pair diagrams, then attempt to generalize to ones with intersections. *Diagrams*: Planar multiple paired oriented circles, AS in said orientation. *Relations*: subdivision, $4T_{1,2}$ as in [BBS: Dalvit-141212](#). Relation with \mathcal{A}^w at [BBS: Dalvit-141217](#).

141208 **Proj.** Milnor/trees / Alexander/MVA / π_1 for 2-knots with boundary.

140210 **Proj.** A quick paper on a quick combinatorial construction of the wheels invariant following [Talks: Hamilton-1412](#).

141204b **Proj.** Write up "combinatorial KV" \Rightarrow "convolutions".

141127a **Q.** Are intersection graphs mod $4T$ the gr of something?

141127b Repeat talks: Watch previous video, repartition handout.

141114b **Proj.** Exposition of Enriquez' solution of YB.

141113a Boden: Brandenbursky has 2 Alexander polys on $v\mathcal{K}$.

140726 Boden's VG_K : $[s, q] = 1$ and $\begin{array}{c} z \nearrow w \\ x \searrow y \end{array} \rightarrow \begin{array}{c} z = xysx^{-1}s^{-1} \\ w = sxs^{-1} \end{array}$ $\begin{array}{c} z \nearrow w \\ y \searrow x \end{array} \rightarrow \begin{array}{c} z = s^{-1}x^{-1}syx \\ w = s^{-1}xs \end{array}$ $\begin{array}{c} z \nearrow w \\ x \searrow y \end{array} \rightarrow \begin{array}{c} z = q^{-1}yq \\ w = qxq^{-1} \end{array}$ At $q = 1 \neq s$, not basis-conjugating. At $q = s$, OC holds. At $s = 1$ this is Manturov's $v\mathcal{B}_n \rightarrow \text{Aut}(F(x_1, \dots, x_n, q))$:

$$\sigma_i \mapsto \begin{cases} x_i \mapsto x_i x_{i+1} x_i^{-1} \\ x_{i+1} \mapsto x_i \end{cases} \text{ or } \begin{cases} x_i \mapsto x_i q^{-1} x_{i+1} q x_i^{-1} \\ x_{i+1} \mapsto q x_i q^{-1} \end{cases}$$

$$\tau_i \mapsto \begin{cases} x_i \mapsto q x_{i+1} q^{-1} \\ x_{i+1} \mapsto q^{-1} x_i q \end{cases} \text{ or } \begin{cases} x_i \mapsto x_{i+1} \\ x_{i+1} \mapsto x_i \end{cases}$$

141102a Zung's visit: • We don't fully understand Configuration Space Integrals (CSI) for curves in the punctured plane. • It's likely that every CSI has a Gauss Diagram Formula (GDF), as winding numbers are computable as intersection numbers. • Degree n GDFs are FT of type $\frac{3}{2}n$, with simple Weight Systems (WS). Likely they are not determined by their WS. • The Merkov quotient of the Feynman-diagram space \mathcal{A}^t is: * internal trivalent vertices vanish. * "Split" arrow-exchange relation. • Is there a δ like in KBHs?

140923 Manturov's rep. for $P\mathcal{B}_n$: $\sigma_{ij} \mapsto \begin{cases} x_i \mapsto q x_i q^{-1} \\ x_j \mapsto x_i^{-1} q^{-1} x_j q x_i \end{cases}$ (and

hence there's a map $P\mathcal{B}_n \rightarrow P\mathcal{B}_{n+1}$).

141102b Assaf's riddle: k kids share a loot of n indivisible candies. The first proposes a split; if not accepted by a strict majority, she leaves and the second proposes, etc. How is the loot split?

131213a **Proj.** G-FT invariants of plane curves, [2014-01/PlaneCurves.pdf](#).

140821 Fiedler: There may exist a "new" non-oracle map $\mathcal{K} \rightarrow \mathcal{Z}\mathcal{K}$. (*) Poly-time, multi-local, low profile, high rank.

140909 **Question.** Is 2-component 2D linking in 4D non-trivial, modulo subdivision and melding? A. Likely trivial.

131112b A map $\mathcal{A}^w(\uparrow \uparrow) \rightarrow \mathcal{A}^u(\uparrow \otimes)$ arises in deducing wheeling from the full Duflo (not α^{-1} for α is not well-defined!); There's a pairing $\mathcal{A}^w(\uparrow \uparrow) \otimes \mathcal{A}^u(\uparrow) \rightarrow \mathcal{A}^u(\uparrow)$. Topological meaning?

140831 **Proj.** Paper: "Why I care about virtual knot theory?".

140731 **Proj.** Make the polynomiality of B_n ridiculously easy.

140725 Two permutations to the virtual braid: $S_n \xleftarrow{s} P\mathcal{B}_n \xrightarrow{s} S_n$ via $(e, \tau_i, (ij)) \xleftarrow{s} (\sigma_i, \tau_i, \sigma_{ij}) \xrightarrow{s} (\tau_i, \tau_i, e)$.

140708 Fox $\partial_i: F_n \rightarrow \mathbb{Z}F_n$: $\partial_i x_j = \delta_{ij}$, $\partial_i(uv) = \partial_i u + u \partial_i v$ (a 1-cocycle). Gassner: $b \mapsto \pi \partial_j b(x_i)$, with $\pi: \mathbb{Z}F_n \rightarrow \mathbb{Z}\mathbb{Z}^n$ the Abelianization.

140622 Bureau: $\Sigma := D^2 \setminus \{n \text{ pts}\}$, $p: \tilde{\Sigma} \rightarrow \Sigma$ its \mathbb{Z} -cover w/ basic deck transformation t , $1 \in \partial \Sigma$ a basepoint, $\tilde{1} \in \tilde{\Sigma}$ a lift, $1^* = p^{-1}(1)$ all lifts, $\tilde{H} := H_1(\tilde{\Sigma}, 1^*; \mathbb{Z})$ is a $(\Lambda := \mathbb{Z}[t^{\pm 1}])$ -module. $Bu: B_n \rightarrow \text{Aut}_\Lambda(\tilde{H})$ is Bureau.

140721 **Proj.** Hilden braids: expansions, the a/α -map, tangles?

140716 The $\bar{\mu}$ invariants are (homology-) concordance invariant.

140713 **Proj.** Low v -algebra: Lie bi-algebras & arrow diagrams.

140604 In \mathbb{R}^4 , framing a hoop is whatever makes tubing well defined, framing a balloon is whatever makes doubling well defined, and framing a vertex is the interaction between the two.

140424 Mathematica-WikiLink re-implementation: [CreateWikiConnection](#), [WikiUserName](#), [WikiGetPageText](#), [WikiSetPageText](#), [WikiSetPageTexts](#), [WikiUploadFile](#).

140422a The Kontsevich propagator $d\phi$.

140422b Find $\int_{\mathbb{C}_z} \bigwedge_{i=1}^n d \operatorname{Arg}(z - z_i) \in \Omega^{n-2}(\mathbb{C}_z^n)$.

140422c Khovanskii's "On a Lemma of Kontsevich" proves Kontsevich's vanishing lemma in 3 pages.

140309 **Proj.** Low degree BF.

140419 **Proj.** Too many definitions of the Alexander polynomial.

140417a Kervaire: G is an $(n \geq 3)$ -knot group iff it is f.p., normally generated by one element, and $H_{1,2}(G) = (\mathbb{Z}, 0)$.

140417b Does every simple decker set come from a (ribbon) 2-link? Is a 2-link group a LOF group? A decker group a 2-link group?

140413 Are there "spherical w-braids"?

140316 **Proj.** What are all internal quotients of FL (compare "PI-Rings")? Which are of polynomial growth?

140325 Monty Hall: A prize is in 1 of 3 envelopes. You choose one, an oracle shows another to be empty. Will you switch? Deliberate oracle: Yes. Chance oracle: No. What makes it so confusing?

140318 **Proj.** Study Vogel's weight system in the context of \mathcal{A}^v .

140304 Itai's iterated mean value theorem: with $(\delta f)(x) := f(x + 1) - f(x)$, $\forall x_0, n \exists x \in [x_0, x_0 + n]$ s.t. $(\delta^n f)(x_0) = (\partial^n f)(x)$. Pf. With $\chi := 1_{[0,1]}$, $\delta = \chi \star \partial$ hence $\delta^n = \chi^{*n} \star \partial^n$ and as $\|\chi^{*n}\|_{L^1} = 1$, $(\delta^n f)(x_0)$ is bound between the extremals of $(\partial^n f)(x)$.

140302 Assaf: The fundamental group and the fundamental groupoid of a path-connected space are naturally equivalent.

140228b What's the relation between quandle cocycles and 2-knots?

140227 Itai: For at least some quadruples of lines in \mathbb{R}^3 , there are at least two lines that intersect all of them.

140106 Is the \vee -invariant of gnots trivial on 2-knots? Is there a multiplicative Alexander duality? Alexander: $X \subset S^n$ compact, locally contractible $\Rightarrow H^q(X) \simeq H_{n-1-q}(X^c)$.

140218a Vienna vocabulary: cobordism hypothesis, WKB approximation, Fukaya category, gerbes, fusion categories, differential cohomology.

140218b **Proj.** Clean and write up the shielding story.

140218c **Proj.** A note on how DG arises in the context of KBHs.

140217 nLab: C monoidal category. Its Drinfel'd centre is the BMC with objects pairs (X, β) of $X \in C$ and natural isomorphism $\beta_- : X \otimes (-) \rightarrow (-) \otimes X$ such that $\forall Y, Z \in C$, $\beta_{Y \otimes Z} = (I_Y \otimes \beta_Z) // (\beta_Y \otimes I_Z)$, with $\operatorname{Hom}((X, \beta), (X', \beta')) := \{f \in \operatorname{Hom}(X, X') : \forall Z, \beta_Z // (I_Z \otimes f) = (f \otimes I_Z) // \beta'_Z\}$, $(X, \beta) \otimes (X', \beta') := (X \otimes X', (I_X \otimes \beta') // (\beta \otimes I_{X'}))$, and $R_{(X, \beta), (X', \beta')} := \beta_{X'} (?)$.

140211 Ogasa's *Local Move Identities*... — some skein relations for high-dimensional Alexander.

140120 How exactly do normal Euler numbers relate to branch points? Can the latter be avoided?

140130 Satoh's w-knot has the same π_1 and the same Z -polynomial (Sawollek) as the trefoil.

140115 Chterental: wB_n acts faithfully on "virtual curve diagrams", and with run-length compression, this is describable in poly time.

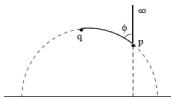
140126 Is tube-bypass an unknotting operation for 2-knots?

140117 Carter: A spun Hopf link with an additional orthogonal plane running once above and once below it makes a knotted $2T^2 + S^2$ with 4 triple points.

140116 Using LMO, FT invariants of links in S^3 extend to links in arbitrary $\mathbb{Q}HS$. A simple description? \rightarrow p9:190321

140114 Gavish: "Singular value decompositions".

140113 Many papers by Seiichi Kamada.



131229 Smale ('57): Long immersions $\mathbb{R}^k \hookrightarrow \mathbb{R}^m$ are classified by $\pi_k(V_{m,k})$ where $V_{m,k}$ is the Stiefel manifold (linear embeddings $\mathbb{R}^k \hookrightarrow \mathbb{R}^m$). Paechter (I, '56): $\pi_2(V_{4,2}) = \mathbb{Z}$.

131219 $Gr(\mathbb{R}^2 \hookrightarrow \mathbb{R}^4) = S^2 \times S^2$. Yael: • There's a $\mathbb{C}P^2 = S^2$ of complex lines in $\mathbb{C} \times \mathbb{C}$ and in $\mathbb{C} \times \bar{\mathbb{C}}$. • It is the product of the moduli $C(\mathbb{R}^4) \times C(\bar{\mathbb{R}}^4)$ of metric complex structures on $\mathbb{R}^4 / \bar{\mathbb{R}}^4$. For $(I, \bar{I}) \in C(\mathbb{R}^4) \times C(\bar{\mathbb{R}}^4)$ there is a unique $\mathbb{R}^2 \hookrightarrow \mathbb{R}^4$ which is complex relative to both, and a given $P = \mathbb{C} = \mathbb{R}^2 \hookrightarrow \mathbb{R}^4$ determines two metric complex structures on $\mathbb{R}^4 / \bar{\mathbb{R}}^4$ by multiplication by i on P and by $\pm i$ on P^\perp . Finally $C(\mathbb{R}^4) = SO(4)/U(2) = \{\text{left multiplications } L_u \text{ by unit imaginary quaternions } u\} = S^2$ and $C(\bar{\mathbb{R}}^4) = \{R_v\}_{v \in S^2 \subset \mathbb{R}^3 \subset \mathbb{H}}$. • $P(u, v) = \operatorname{span}(u + v, uv - 1)$ or $\operatorname{span}(u - v, uv + 1)^\perp$ and for orthonormal (α, β) , $\operatorname{span}(\alpha, \beta) \mapsto (\beta \bar{\alpha}, \bar{\alpha} \beta) = ((\alpha \wedge \beta)^+, (\alpha \wedge \beta)^-)$, the last using the self-dual and anti-self-dual projections $\Lambda^2 \rightarrow \Lambda^{2\pm}$.

131218 Bjørndahl: N prisoners each wears ∞ -many b/w hats. Simultaneously each needs to point at a black hat on her head. How can they maximize the chance that they will *all* get it right?

131217 Goryunov's *finite order* ... $J^+ \dots$: Generic smooth plane curves, allowing triple points and opposite self-intersections, map to knots in the solid torus $ST^*\mathbb{R}^2$ inducing an isomorphism of projectivizations.

131213b Stallings' theorem: $h: A \rightarrow B$ a group homomorphism w/ $h: H_1(A) \simeq H_1(B)$ and $h: H_2(A) \twoheadrightarrow H_2(B)$. Then $h: A/A_n \simeq B/B_n$, where A_n, B_n denote lower central series (+ more...).

131213c Getzler (bbs): Homotopy 2-types are determined by the action $\pi_1 \curvearrowright \pi_2$ and a class in $H^3(\pi_1, \pi_2)$.

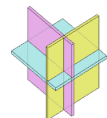
131126b Anton: is there a triality for solutions of the KV equation? — Yes, 2013-11/DoubleTree/TrialityComputations.nb. Minor: does α intertwine the triality of sder_2 with that of $\operatorname{tder}_2 \ltimes \operatorname{tr}_2$? — 2013-12/: Most likely not.

131211 Is the nilpotent completion of the fundamental group of a gnot complement always free-nilpotent? — No; Abelian it is for $\mathbb{R}^2 \times \{0\} \cup \{0\} \times \mathbb{R}^2$.

131122a **Proj.** Figure out the bubble-wrap-finite-type invariants of *all* knotted objects in \mathbb{R}^4 .

131202a From *Virtual 2-Knots* by Schneider:

In a virtual 2-knot diagram, these are also allowed:



pure virtual triple point



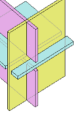
mixed triple point



welded triple point



unwelded triple point



woven triple point

...but these are forbidden:

In light of "virtual doodles", perhaps this should be modified?

131205a Are there 3 embedded surfaces in \mathbb{R}^4 so that any 3 immersed handlebodies bounding them have a common point?

131205b Cimasoni: Levine's *Poly. Inv. of Knots of Codimension 2*.

131204 Coboundary: $\delta(x) = [r, \Delta(x)]$, with invariant $r + r^{21}$.

131202b Farber's *Noncommutative Rational Functions and Boundary Links*, continued Retakh, Reutenauer, Vaintrob, [arXiv:math/0004112](https://arxiv.org/abs/math/0004112).

131130c Yanagawa ('69): Ribbon 2-knot K is trivial iff $\pi_1(K) \simeq \mathbb{Z}$.

131130d Meilhan: 2-knots papers by Yajima ('62, '64), Yanagawa ('69'), Omae ('71).

131126a Crainic [arXiv:math/0403266](https://arxiv.org/abs/math/0403266) on Homological perturbations: A *Homological Homotopy Equivalence (HHE)* is a pair of com-

plexes with quasi-isomorphisms $(L, b) \xrightleftharpoons[p]{i} (M, b)$, with a homotopy h between $1 = 1_M$ and ip , so $ip = 1 + bh + hb$. A *perturbation* is $\delta: M \rightarrow M$ with $\deg b = \deg \delta$ and $(b + \delta)^2 = 0$; it is *small* if $(1 - \delta h)^{-1}$ exists. **Claim.** $(L, b_1) \xrightleftharpoons[p_1]{i_1} (M, b + \delta)$ is again an HHE, with $A := (1 - \delta h)^{-1}\delta$, $b_1 := b + pAi$, $i_1 := i + hAi$, $p_1 := p + pAh$, and with $h_1 := h + hAh$.

131212 Cimasoni: There is a 1-double-point gnot.

131209 Cimasoni: There is a natural smoothing of 2-gnots.

131122b Moskovich, [arXiv:math/0211223](https://arxiv.org/abs/math/0211223): On the right, ϕ and β pair to an integer. Indeed $D^2 \ni x \mapsto \beta(x)^\perp$ is a circle bundle on D^2 which must be trivial, inducing a trivialization of the circle bundle $S^1 \ni x \mapsto \beta(x)^\perp$. But ϕ/e_2 is a section of that bundle, hence an integer.

$$\begin{array}{ccc} D^2 & \xrightarrow{\beta} & SO(3) \\ \uparrow \phi & \searrow & \downarrow e_1 \\ S^1 & \xrightarrow{\gamma} & S^2 \end{array}$$

131121 Burke, Koytcheff: [arXiv:1311.4217](https://arxiv.org/abs/1311.4217), *A colored operad for string link infection*.

131114 Bar-Hillel's Simpson's paradox: In Israel in every age bracket death rates for Arabs are higher than for Jews, yet overall death rates for Jews are higher.

131111 Massuyeau (bbs). Given an algebra A and $N \geq 1$, \exists commutative algebra A_N s.t. \forall commutative algebra B ,

Indeed $\text{Hom}_{\text{Alg}}(A, \text{Mat}_N(B)) \simeq \text{Hom}_{\text{C-Alg}}(A_N, B)$.

$$A_N \simeq \frac{\langle a_{ij} : a \in A, 1 \leq i, j \leq N \rangle}{(a + \lambda b)_{ij} = a_{ij} + \lambda b_{ij}, 1_{ij} = \delta_{ij}, (ab)_{ik} = \sum_j a_{ij} b_{jk}}.$$

131110 Massuyeau (bbs, eprints), after Van den Bergh: A double bracket in an algebra A is $\llbracket -, - \rrbracket: A \otimes A \rightarrow A \otimes A$ s.t. (1) $\llbracket b, a \rrbracket = -\llbracket a, b \rrbracket^{op}$. (2) $\llbracket a, b_1 b_2 \rrbracket = (b_1 \otimes 1) \llbracket a, b_2 \rrbracket + \llbracket a, b_1 \rrbracket (1 \otimes b_2)$. It is Poisson if

$$\llbracket -, -, - \rrbracket := \text{diagram 1} + \text{diagram 2} + \text{diagram 3} = 0$$

131107 Enriquez/EllipticAssociators: with $\bar{e}(z) := \frac{\text{ad } z}{e^{\text{ad } z} - 1}$,

$$\begin{aligned} (\mu, \Phi) &\mapsto A = \Phi(-\bar{e}(x)y, t^{12}) e^{-\mu \bar{e}(x)y} \Phi^{-1}(-\bar{e}(x)y, t^{12}), \\ B &= e^{\mu t^{12}/2} \Phi(\bar{e}(-x)y, t^{21}) e^x \Phi^{-1}(-\bar{e}(x)y, t^{12}). \end{aligned}$$

131109 C. Frohman, A. Nicas, *The Alexander Polynomial via topological quantum field theory*, Differential Geometry, Global Analysis, and Topology, Canadian Math. Soc. Conf. Proc. **12**, Amer. Math. Soc. Providence, RI, (1992) 27–40.

131106b Massuyeau (bbs, eprints, easy): \exists “symplectic expansion” — a group-like expansion $Z: FG(x_i, y_i) \rightarrow FA(\bar{x}_i, \bar{y}_i)$ with $Z(\prod_i [x_i, y_i]) = \exp(-\sum_i [\bar{x}_i, \bar{y}_i])$. Thus surface groups are quadratic and have homomorphic expansions.

131027a Cattaneo: “BV is the ‘right’ de-Rham differential on supermanifolds.”

131027b The Hilbert basis theorem: An ideal in the ring of multivariable polynomials over a Noetherian ring is finitely generated.

Pf. Enough, R Noetherian \Rightarrow any $I \subset R[x]$ is finitely generated. Let $p_n \in I \setminus \langle p_1, \dots, p_{n-1} \rangle$ be of minimal degree. As R is Noetherian, for large N the leading coefficient of p_N is a combination of previous leading coefficients, so it can be killed off contradicting the minimality of p_N . \square Can be made constructive using Gröbner bases.

131020 Artin-Wedderburn: A semi-simple ring is uniquely (up to a permutation) isomorphic to a product of finitely many finite matrix rings over division rings.

131007c I don't understand Ševera's $(A \otimes A) \otimes A \xrightarrow{\Phi} A \otimes (A \otimes A)$

$$\begin{array}{ccc} (A \otimes A) \otimes A & \xrightarrow{\Phi} & A \otimes (A \otimes A) \\ \downarrow m \otimes 1 & & \downarrow 1 \otimes m \\ A \otimes A & \xrightarrow{m} & A \end{array}$$

131007b From 2013-10/Swinging:

$$s = \text{diagram 1} = \text{diagram 2} = \text{diagram 3}$$

131007a Monoblog starts.

Archived Items.

251212 **Q.** Are there cleaner $F_{1,2,3}$'s solving the upright R-moves merely up to coboundaries? (likely meaningless).

201109 There is a “crossing change” construction of Seifert surfaces, and a construction starting from an immersed bounding disk.

240205 **Q.** Is there a (poly-)computable functor from measured v.s. to Sets, extending Gaussian integration?

250701 **UGP:** Implement Fiedler's [arXiv:2506.17738](#).

241105b **UGP:** Implement many families of knots, compute their θ .

241127 **Proj.** Write Θ in Sage.

250830 Is the residue invariant 0? $F_1 = (g_{jj} - g_{jj+})(2g_{jj+} - g_{ii} - g_{jj})$

$$F_2 = (g_{j_0 j_1} - g_{j_0 j_1^+})(g_{j_1 j_0} - g_{j_1 j_0^+}) \cap (i_0 \rightarrow j_0, i_1 \rightarrow j_1).$$

250624c **Q.** Do cyclic words inject into their profiles?

231105b Is “almost classical” an appropriate tag for w objects with $\text{div} = 0$ (KV solutions, horizontal chord associators)?
→p3:231105a (After archiving: No.)

210511 **Chal.** Reconstruct a category like $\{\text{Hom}(V^{\otimes A} \rightarrow V^{\otimes B})\}$ from a “forward contraction algebra” akin to $\{(V^*)^{\otimes A} \otimes V^{\otimes B}\}$ for (some) ∞D vector spaces V . **DoPeGDO** Sol'n: L/G variables are chronologically ordered. In Q , LL terms are small, and also LG terms if L preceeds G. Allow only these constrained LG contractions.

210421 **T/F?** $\sqrt{G}^{-1} \Gamma \sqrt{G} = \Gamma^T$.

170913a A pushforwards challenge in [Pushforwards.pdf](#).

230612a 20m talk: “Rooting the BKT for FTI”. Key: $X \subset \underline{n}^d$, $|X| = P$. Using dyadic decompositions, in time $\sim P$ can set a database of size $\sim P$ so $|X \cap R|$ can be computed in time ~ 1 for every rectangle $R \subset \underline{n}^d$.

230109 **Def.** Given a v.s. V , a Partial Quadratic (PQ) Q on V is a symmetric bilinear form Q on a subspace $\mathcal{D}(Q) \subset V$. For $U \subset \mathcal{D}(Q)$, denote $\text{ann}_Q(U) := \{v \in \mathcal{D}(Q) : Q(U, v) = 0\}$.

Def. $Q_1 + Q_2$ is with $\mathcal{D}(Q_1 + Q_2) = \mathcal{D}(Q_1) \cap \mathcal{D}(Q_2)$.

Def. Given a linear $\psi: V \rightarrow W$ and a PQ Q on W , the pullback is $(\psi^* Q)(v_1, v_2) = Q(\psi v_1, \psi v_2)$ with $\mathcal{D}(\psi^* Q) = \phi^{-1}(\mathcal{D}(Q))$.

Def. Given $\phi: V \rightarrow W$ and a PQ Q on V the pushforward $\phi_* Q$ is with $\mathcal{D}(\phi_* Q) = \phi(\text{ann}_Q(\mathcal{D}(Q) \cap \ker \phi))$ and $(\phi_* Q)(w_1, w_2) = Q(v_1, v_2)$, where v_i are s.t. $\phi(v_i) = w_i$ and $Q(v_i, \text{rad } Q|_{\ker \phi}) = 0$.

Thm(?). ψ^* and ϕ_* are well-defined and functorial, and if $\alpha // \beta = \gamma // \delta$, then $\gamma^* // \alpha_* = \delta_* // \beta^*$. ψ^* is additive but ϕ_* isn't. $\phi^* // \phi_*$ is restriction to $\text{im } \phi$. $\phi_* // \phi^*$ is ?.

Thm(?). Over \mathbb{R} , given $\phi: V \rightarrow W$ and PQs Q on V and C on W ,
 $\text{sign}_V(Q + \phi^* C) = \text{sign}_{\ker \phi}(\iota^* Q) + \text{sign}_W(C + \phi_* Q)$

($\Leftrightarrow \text{sign}_V(Q) = \text{sign}_{\ker \phi}(\iota^* Q) + \text{sign}_W(\phi_* Q)$ (no C , no $+$)).

230109 **Def.** Given a v.s. V , a Partial Quadratic (PQ) on V is $Q = (D = \mathcal{D}(Q) \subset W, L = \mathcal{L}(Q): D \rightarrow D^*)$ with $L = L^*$. Write $Q(v_1, v_2) = L(v_1)(v_2)$.

Def. Given a linear $\psi: V \rightarrow W$ and a PQ $Q = (D = \mathcal{D}(Q) \subset W, L = \mathcal{L}(Q): D \rightarrow D^*, L = L^*)$ on W , the pullback $\psi^* Q$ is (D', L') with $D' = \phi^{-1}(D)$ and $L' = \psi // L // \psi^*$, so $(\psi^* Q)(v_1, v_2) = Q(\psi v_1, \psi v_2)$.

Def. If Q_i are PQs on V , $Q_1 + Q_2$ is with $\mathcal{D}(Q_1 + Q_2) = \mathcal{D}(Q_1) \cap \mathcal{D}(Q_2)$.

Def. Given a linear $\phi: V \rightarrow W$ and a PQ $Q = (D, L)$ on V the pushforward $\phi_* Q$ is $Q' = (D', L')$ with $D' = \phi((L(\text{rad } Q|_{D \cap \ker \phi}))^\perp)$ and $Q'(w_1, w_2) = Q(v_1, v_2)$, where v_i are s.t. $\phi(v_i) = w_i$ and $Q(v_i, \text{rad } Q|_{\ker \phi}) = 0$.

Thm. Pullbacks / pushforwards are well-defined and functorial.

Thm(?). Over \mathbb{R} , given $\phi: V \rightarrow W$ and PQs Q on V and C on W ,
 $\text{sign}_V(Q + \phi^* C) = \text{sign}_{\ker \phi}(\iota^* Q) + \text{sign}_W(C + \phi_* Q)$.

220923 **Q.** For balanced Heisenberg exponentials, does p -scattering determine x -scattering?

210101 **Proj.** Implement a PD2ThinMorse with guaranteed bounds and/or using simulated annealing.

211210 **Proj.** Complexity by ropelength?

180820 $\rho_1 = t(P|_{e, f \rightarrow 0} - t\omega' \omega^3) / (t-1)^2 \omega^2$ and

$$P = A^2 \frac{(t-1)^3 \rho_1 + t^2(2\nu w + (1-t)(1-2c))AA'}{(1-t)t}.$$

210302 **Q.** (w/ Abbasi) Does every contractible but not manifestly contractible curve in the annulus have an odd self-intersection?

170829a **Do.** Find Duflo in Goldman-Turaev.

190108 **Do.** With $P = \sum a_{mn} z^m \zeta^n$, compute $\langle\langle \epsilon P \rangle\rangle := \log \langle \exp \epsilon P \rangle$.

150416 Chterental: Is there a Melvin-Morton statement for v-knots?

181031 **Proj.** Verify Kashaev's conjecture @[arXiv:1801.04632](#), re. Tristram-Levine signatures.

170321 For NOE1 with $\Lambda \rightarrow 0$, are there interesting R 's?

170320b **Proj.** k -co inductive constructions.

210318a Riba Garcia's [talk](#), “Invariants of Rational Homology 3-Spheres and the Mod p Torelli Group”:

210211a-old Halacheva (\sim): $\mathcal{A}(X) := \bigoplus_k \text{End}(\Lambda^k X)$ is a traced meta-monoid with $m_z^{xy}(A) := (z \rightarrow y) // (e_x // i_x // A // e_y // i_y - e_x // A // i_y) // (x \rightarrow z)$ and $\text{tr}_x(A) := e_x // i_x // A // e_x // i_x - e_x // A // i_x$. Contains Γ (w/ fixed colours) via $\Upsilon: (\omega, M) \mapsto \omega \Lambda^*(M)$. Predict \mathcal{A} from Γ ? Interpret \mathcal{A} in $ybox$? Related to super-algebras? Raise \mathcal{A} to meta-Hopf? Understand $\text{im}(\Upsilon)$?

180311 **Do.** With strongly docile L and Λ , compute $\log \langle e^L | e^\Lambda \rangle$ without exponentiating.

171029 **Do.** Solve $\hbar^{-1}(1 - e^{\hbar(t-2a\epsilon)}) = g(a-1, z) + (-e^{\hbar} - (t-2a\epsilon)\partial_z + \epsilon z \partial_z^2)g(a, z)$.

170309 →p19:170309 Then [BBS:AKT17-170317](#): $e^{\alpha w} e^{\beta u} = e^{a u} e^{d(b-2\epsilon c)} e^{b w}$ with $\gamma = 1 - \alpha\beta\epsilon$, $a = \beta/\gamma = \beta + \dots$, $b = \alpha/\gamma = \alpha + \dots$, $d = \epsilon^{-1} \log \gamma = -\alpha\beta + \dots$, so $\bigcirc(wu: e^{\alpha w + \beta u}) = \bigcirc(ucw: e^{a u + b w + d(b-2\epsilon c)}) = \bigcirc(ucw: e^{\lambda_\epsilon(\alpha\beta)} e^{\alpha w + \beta u - \alpha\beta b}) = \bigcirc(ucw: e^{\lambda_\epsilon(\partial_w, \partial_u)} e^{\alpha w + \beta u - \alpha\beta b})$ so $\bigcirc(wu: e^{\alpha w + \beta u + \delta u w}) = \bigcirc(ucw: e^{\delta \partial_\alpha \partial_\beta} e^{\lambda_\epsilon(\partial_w, \partial_u)} e^{\alpha w + \beta u - \alpha\beta b}) = \bigcirc(ucw: e^{\lambda_\epsilon(\partial_w, \partial_u)} v e^q) = \bigcirc(ucw: e^{\Lambda_\epsilon v e^q})$, with $v = (1 + b\delta)^{-1}$, $q = v(\alpha w + \beta u + \delta u w - \alpha\beta b)$ and $\Lambda_\epsilon \in \mathbb{Q}(w, u, b, c, \alpha, \beta, \delta) \ll \mathbb{E}$.

170522 What is the “sensical” sub-meta-object of

$(\mathcal{U}(b_+), m, \Delta, S, P, R)$?

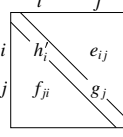
180528a Is there an operation-uniformizing “bottom tangles in handlebodies” theory for (rotational) virtuals similar to Habiro-Massuyeau?

131112a The diamond lemma: If \rightarrow is a connected Noetherian binary relation (Noetherian: an infinite $a_1 \rightarrow a_2 \rightarrow \dots$ is ultimately constant), and if whenever $a \rightarrow b$ and $a \rightarrow c$ there is d with

$b \Rightarrow d$ and $c \Rightarrow d$ where \Rightarrow is the reflexive transitive closure of \rightarrow , then $\exists! m \forall a a \Rightarrow m$.

200204b Talk. Over then Under Tangles. **Abstract.** Brilliant wrong ideas should not be buried and forgotten. Instead, they should be mined for the gold that lies underneath the layer of wrong. In my talk I will explain how “over then under tangles” lead to an easy classification of knots, and under the surface, also to some valid mathematics: ...

170126c In $gl_n^{\epsilon} : [\nabla, \nabla] = \nabla, [\triangle, \triangle] = \epsilon \triangle, [\triangle, \nabla] = \triangle + \epsilon \nabla$, so with $h_i = h'_i - \epsilon g_i, [h_i, \cdot] = 0, [g_i, g_j] = 0, [e_{ij}, e_{kl}] = \delta_{jk} e_{il} - \delta_{il} e_{kj}, [f_{ij}, f_{kl}] = \epsilon(\delta_{jk} f_{il} - \delta_{il} f_{kj}), [e_{ij}, f_{kl}] = \delta_{jk}(\epsilon \delta_{i < l} e_{il} + \delta_{i > l} f_{il}) - \delta_{li}(\epsilon \delta_{k < l} e_{kj} + \delta_{k > j} f_{kj}) + \delta_{jk} \delta_{li}((h_i - h_j)/2 + \epsilon(g_i - g_j)), [g_i, e_{jk}] = (\delta_{ij} - \delta_{ki}) e_{jk}, [g_i, f_{jk}] = (\delta_{ij} - \delta_{ki}) f_{jk}, \deg(\epsilon, h_i, f_{ij}, g_i, e_{ij}) = (1, 1, 1, 0, 0)$. Verification in [2017-02/glne.nb](#). Order n symmetry in [2020-01/glne.nb](#).



171012 Talks/LesDiablerets-1708, esp. [PBWDemo.nb](#), verifications [2017-10/Phi2CR-Classical.nb](#): In $\hat{\mathcal{U}}(g^{\epsilon}) = \langle t, y, a, x \rangle / ([t, \cdot] = 0, [a, x] = x, [a, y] = -y, [x, y] = t - 2\epsilon a)$, we have $\prod_{i=1}^2 \mathbb{E}^{\tau_i t} \mathbb{E}^{\eta_i y} \mathbb{E}^{\alpha_i a} \mathbb{E}^{\xi_i x} = \mathbb{E}^{\tau t} \mathbb{E}^{\eta y} \mathbb{E}^{\alpha a} \mathbb{E}^{\xi x}$, with

$$\tau = \tau_1 + \tau_2 - \frac{\log(1 - \epsilon \eta_2 \xi_1)}{\epsilon} = \tau_1 + \tau_2 + \eta_2 \xi_1 + \frac{\epsilon}{2} \eta_2^2 \xi_1^2 + \dots,$$

$$\eta = \eta_1 + \frac{\mathbb{E}^{-\alpha_1} \eta_2}{(1 - \epsilon \eta_2 \xi_1)} = \eta_1 + \mathbb{E}^{-\alpha_1} \eta_2 + \epsilon \mathbb{E}^{-\alpha_1} \eta_2^2 \xi_1 + \dots,$$

$$\alpha = \alpha_1 + \alpha_2 + 2 \log(1 - \epsilon \eta_2 \xi_1) = \alpha_1 + \alpha_2 - 2\epsilon \eta_2 \xi_1 + \dots,$$

$$\xi = \frac{\mathbb{E}^{-\alpha_2} \xi_1}{(1 - \epsilon \eta_2 \xi_1)} + \xi_2 = \mathbb{E}^{-\alpha_2} \xi_1 + \xi_2 + \epsilon \mathbb{E}^{-\alpha_2} \eta_2 \xi_1^2 + \dots$$

181024 With Ens. The CD_a universe $\mathcal{U} = FA \langle H_k, R_k, \dots, Z_k^i, \dots \rangle^* \mathbb{Q}S_*$ has a strand-filtration \mathcal{F}_n , a Vassiliev degree $\deg \geq 0$, a homological degree $ht \geq 0$, an ht-odd differential δ with $\deg \delta = 0$, $ht \delta = -1$, an endomorphism c with $c\mathcal{F}_n \subset \mathcal{F}_{n+1}$ and $\deg c = ht c = 0$ and

- $S_* := \bigcup_{n \geq 0} S_n$ with $(\deg, ht) = (0, 0)$, $S_n \subset \mathcal{F}_n$, and $c : S_n \rightarrow S_{n+1}$ via $(c\sigma)_1 = 1$ and $(c\sigma)_{i \geq 1} = \sigma_{i-1} + 1$.
- For U any H, R , or Z , $U_k = c_k^k U_0 =: c_k^k U$.
- $H \in \mathcal{F}_1$ with $(\deg, ht) = (1, 0)$. Let $t_{0i} := (1i)H(1i)$, let $t_{12} := H_1 - (12)H(12)$ and at $j > i > 0$ let $t_{ij} = (1i)(2j)t_{12}(2j)(1i)$.
- $R \in \mathcal{F}_2$ with $(\deg, ht) = (1, 1)$, $\delta R = \dots$
- ...

Claim/goal: $H_0(\mathcal{U}, \delta) \cong S_* \rtimes DK_{\{0\} \sqcup *}$ and $H_1(\mathcal{U}, \delta) = 0$.

190115 The monoidal category with objects \mathbb{N} generated by $\sigma \in \text{Aut}(2)$ with $1_1 \otimes \sigma \otimes 1_1 = \sigma \otimes 1_1 \otimes \sigma$, possibly with $\sigma^2 = 1_2$.

171104 Roland: Solve $g(a, t)g(-a - 1, -t) = P(a, t)$.

170725 (Wrong, see 2017-10/Phi2CR.nb) 2017-07/Multi-beta-yax.nb: In $\mathcal{U}_{\gamma^{-1}, \gamma\beta}$ where $q = \mathbb{E}^{\beta}$, $\prod_{i=1}^2 \mathbb{E}^{\eta_i y} \mathbb{E}^{\alpha_i a} \mathbb{E}^{\xi_i x} = \mathbb{E}^{\eta y} \mathbb{E}^{\alpha a} \mathbb{E}^{\xi x} \mathbb{E}^{\tau t}$, with

$$\eta = \eta_1 + \eta_2 e^{-\gamma \alpha_1} - \beta \gamma \eta_2^2 \xi_1 e^{-\gamma \alpha_1} + \dots = \eta_1 + \delta \eta_2 e^{\beta - \alpha_1 \gamma}$$

$$\alpha = \alpha_1 + \alpha_2 + 2\beta \eta_2 \xi_1 + \dots = \alpha_1 + \alpha_2 - 2(\beta + \log \delta) / \gamma$$

$$\xi = \xi_1 e^{-\gamma \alpha_2} + \xi_2 - \beta \gamma \eta_2 \xi_1^2 e^{-\gamma \alpha_2} + \dots = \delta \xi_1 e^{\beta - \alpha_2 \gamma} + \xi_2$$

$$\tau = -\eta_2 \xi_1 + \beta \eta_2 \xi_1 (\gamma \eta_2 \xi_1 + 1) / 2 + \dots = (\beta + \log \delta) / (\beta \gamma)$$

$$\text{and } \delta := ((e^{\beta} - 1) \gamma \eta_2 \xi_1 + e^{\beta})^{-1} = 1 - (1 + \gamma \eta_1 \xi_1) \beta + \dots$$

170805 With $\Phi = (\phi_j(\alpha_i))$ and $Z = \zeta(\partial_{\alpha_i})$, set $\Phi_* Z := \mathbb{E}^{\sum \partial_{\beta_j} \phi_j(\partial_{\alpha_i})} \zeta(a_i) \Big|_{a_i=0}$. **Do.** With $(a_i, y_i, x_i, t_i) := (\partial_{\alpha_i}, \partial_{\eta_i}, \partial_{\xi_i}, \partial_{\tau_i})$,

compute/implement $\Phi_* Z$, with

$$Z = \omega \exp \left(\sum \lambda_{ij} t_i a_j + \sum q_{ij} y_i x_j + \epsilon P_0 \right),$$

$$\lambda_{ij} \in \mathbb{Z}, \omega, q_{ij} \in R := \mathbb{Q}(T_i = \mathbb{E}^{t_i}), P_0 \in R[a_i, y_i, x_i], \text{ and}$$

$$\Phi^*(\bar{\alpha}_i) = \sum \psi_{ij}^1 \alpha_j + \epsilon P_1,$$

$$\Phi^*(\bar{\eta}_i) = \sum \psi_{ij}^2 \eta_j + \epsilon P_2,$$

$$\Phi^*(\bar{\xi}_i) = \sum \psi_{ij}^3 \xi_j + \epsilon P_3,$$

$$\Phi^*(\bar{\tau}_i) = \sum \psi_{ij}^4 \tau_j + \sum \gamma_{ij} \eta_i \xi_j + \epsilon P_4,$$

$$\psi_{ij}^{1,4} \in \mathbb{Z}, \psi^{2,3} \in R, P_{1,4} \in \mathbb{Q}[x_i, y_i], P_{2,3} \in R[x_i, y_i], \gamma_{ij} \in R.$$

$$\mathbf{170713 KZ: } dH = H \sum_{i < j} \frac{dz_i - dz_j}{z_i - z_j} t^{ij}.$$

170610 (alt; main: [→p12:170625](#)) $\mathcal{U}_{h, \alpha\beta}$ conventions: $q = \mathbb{E}^{h\alpha\beta}$, $H = \langle a, x \rangle / ([a, x] = \alpha x)$ with

$$A = \mathbb{E}^{-h\beta a}, \quad xA = qAx$$

$$S(a, A, x) = (-a, A^{-1}, -A^{-1}x),$$

$$\Delta(a, A, x) = (a_1 + a_2, A_1 A_2, x_1 + A_1 x_2)$$

and dual $H^* = \langle b, y \rangle / ([b, y] = -\beta y)$ with

$$B = \mathbb{E}^{-h\alpha b}, \quad By = qyB$$

$$S(b, B, y) = (-b, B^{-1}, -yB^{-1}),$$

$$\Delta(b, B, y) = (b_1 + b_2, B_1 B_2, y_1 B_2 + y_2).$$

Pairing by $(a, x)^* = \hbar \langle b, y \rangle$ making $\langle a^j x^k, y^l b^j \rangle = \delta_{ij} \delta_{kl} i! [k]_q!$. Then $\mathcal{U} = H^* \otimes H^{op}$ with $(\phi f)(\psi g) = \langle f_1, \psi_1 \rangle \langle f_3, S \psi_3 \rangle (\phi \psi_2)(g f_2)$.

170513 (alt; main: [→p12:170625](#)) $\mathcal{U}_{\eta, \gamma}$ conventions: $A = \langle g, G = \mathbb{E}^{\eta g}, e \rangle / ([g, e] = \gamma e)$ with $S(g, G, e) = (-g, G^{-1}, -eG^{-1})$;

$$\Delta(g, G, e) = (g_1 + g_2, G_1 G_2, e_1 G_2 + e_2)$$

and dual $A^* = \langle h, H = \mathbb{E}^{\eta h}, f \rangle / ([h, f] = -\eta h)$ with $S(h, H, f) = (-h, H^{-1}, -H^{-1}f)$;

$$\Delta(h, H, f) = (h_1 + h_2, H_1 H_2, f_1 + H_1 f_2).$$

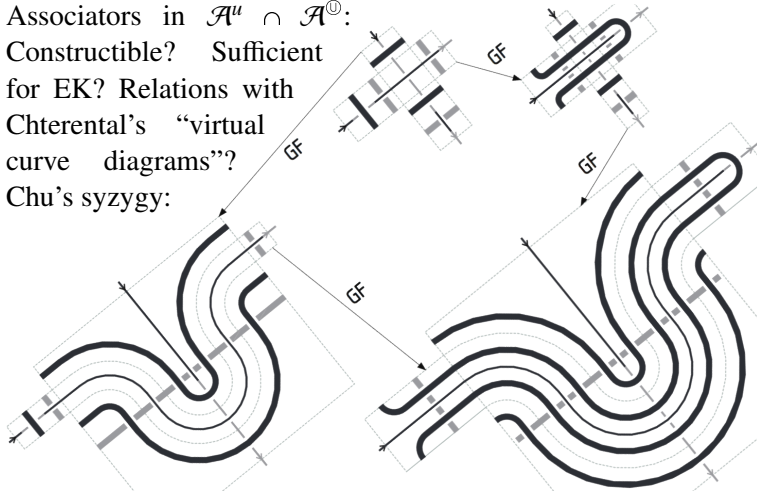
Pairing by $(g, e)^* = (h, f)$. Degrees by $\deg(\gamma, g, e, \eta, h, f) = 1$, so ops are degree non-decreasing except the basic pairing lowers 2 degrees.

170528 (alt; main: [→p12:170625](#)) $\mathcal{U}_{h, \epsilon}$ conventions: $A = \langle g, G = \mathbb{E}^{h\epsilon g}, e \rangle / ([g, e] = \hbar e)$ with $\Delta(g, G, e) = (g_1 + g_2, G_1 G_2, e_1 G_2 + e_2)$; $S(g, G, e) = (-g, G^{-1}, -eG^{-1})$ and dual $A^* = \langle h, H = \mathbb{E}^{h\epsilon h}, f \rangle / ([h, f] = -\hbar \epsilon h)$ with $\Delta(h, H, f) = (h_1 + h_2, H_1 H_2, f_1 + H_1 f_2)$; $S(h, H, f) = (-h, H^{-1}, -H^{-1}f)$. Pairing by $(g, e)^* = (h, f)$. Degrees by $\deg(\hbar, g, e, h, f) = 1$, so ops are degree non-decreasing except the basic pairing lowers 2 degrees.

170412a Title. The Dogma is Wrong. **Abstract.** It has long been known that there are knot invariants associated to semi-simple Lie algebras, and there has long been a dogma as for how to extract them: “quantize and use representation theory”. We present an alternative and better procedure: “centrally extend, approximate by solvable, and learn how to re-order exponentials in a universal enveloping algebra”. While equivalent to the old invariants via a complicated process, our invariants are in practice stronger, faster to compute (poly-time vs. exp-time), and clearly carry topological information. ¶ This is joint work with Roland van der Veen and continues work by Rozansky and Overbay.

170401a Project over-then-under “@-Tangles”. Closed under compositions; (v-)braids are @; non-braid @ tangles? Relations in @?

In $\mathcal{A}^{\textcircled{u}}$? Not all tangles are \textcircled{u} . Alexander properties; v-version. Associators in $\mathcal{A}^u \cap \mathcal{A}^{\textcircled{u}}$: Constructible? Sufficient for EK? Relations with Chterental's "virtual curve diagrams"? Chu's syzygy:



170108b **AKT-17 Reality // Plan: Gentle.** Course introduction (h1). Knots, Reidemeister moves and the Jones polynomial (h2-3). Tangles and a faster Jones program (h4). Tangles and meta-monoids (h5-6). Links, 3-manifolds, Seifert surfaces and genus, ribbon knots and "algebraic knot theory" (h7-8). The Alexander polynomial using Γ -calculus (h9-10). Finite type invariants and expansions (h11-14). / The relationship with metrized Lie algebras and PBW (h15-16). / The variants v , w , bv , and rv , and their expansions (h17-18). Lie bialgebras and solvable approximation (h19-20). **Brute.** Knots, algebras, YBE, CYBE, Lie algebras, universal enveloping algebras, formulas (h1) The Lie algebra \mathfrak{g}_0 , universal enveloping algebras and low degree computations (h2-3). Ordering symbols and commutation relations for \mathfrak{g}_0 (h4-5). The \mathfrak{g}_0 invariant (h6-7). The $\Delta\text{o}\gamma\text{o}\varsigma$ and \mathfrak{g}_1 computations (h8-10). // Morse knots and the \mathfrak{g}_1 invariant (h11). \mathfrak{g}_0 and \mathfrak{g}_1 as approximations of sl_2 , approximating sl_3 (h12). The sl_3^0 invariant (h13-14). The sl_3^1 invariant, fame, and glory (h15-16).

160513 **Q.** What's Fox-Milnor for links? \rightarrow p13:131130b.

170211a Gaussian pairing:

$$\left\langle \exp\left(\frac{xy}{2} + \sum_{i \in I} i \bullet\right) \mid \exp\left(\frac{zy}{2} + \sum_{j \in J} \bullet j\right) \right\rangle = \exp\left(\log\left(\frac{1}{1-xy}\right) \circ + \sum_{i \in I, j \in J} \frac{i \bullet j}{1-xy} + \sum_{i_1, 2 \in I} \frac{i_1 i_2}{1-xy} + \sum_{j_1, 2 \in J} \frac{x j_1 j_2}{1-xy}\right).$$

160314 **Proj.** Visualization of fibred knots. [Done, JB \rightarrow].

160403a Is \square on unipotent completions (page 7 of my GT1 paper) nonsense? Are Taylor expansions isomorphisms?

160321 **Prob.** Find a quadratic description for the Adjoint rep of $[\bar{a}_{ik}, \bar{a}_{jk}] = -[\bar{a}_{ij}, \bar{a}_{jk}] = \bar{a}_{ik} - \bar{a}_{jk}$, $[\bar{a}_{ij}, \bar{a}_{ji}] = \bar{a}_{ji} - \bar{a}_{ij}$.

141123 Qinhuangdao: Talk to me about China, America, Taiwan, economy, ecology, religion, democracy, censorship, and all else.

150201b A precise relationship between expansions for FG and "PBW bases" for same?

150401 Halacheva's meta-trace:

ω	c	S	
c	α	θ	$\frac{\Gamma::\text{tr}_c}{\mu:=1-\alpha} \frac{\mu\omega}{S} \mid \frac{S}{\Xi + \psi\theta/\mu}$
S	ψ	Ξ	

with

ω	a	b	S	
a	α	β	θ	$\frac{\Gamma::m_c^{ab}}{\mu:=1-\beta} \frac{\mu\omega}{S} \mid \frac{c}{\gamma + \alpha\delta/\mu} \frac{S}{\epsilon + \delta\theta/\mu}$
b	γ	δ	ϵ	
S	ϕ	ψ	Ξ	$\frac{\mu:=1-\beta}{T_a, T_b \rightarrow T_c} \frac{c}{\phi + \alpha\psi/\mu} \frac{S}{\Xi + \psi\theta/\mu}$

When exactly is it defined?

141211 **BBS: Alekseev-131108**, AT sec. 5.2: $F_1 \in \text{TAut}$ solving $\text{AT} \leftrightarrow F_t := F_1(tx, ty)$ with $F_0 = 1 \leftrightarrow u_t = \frac{dF_t}{dt} F_t^{-1}$ with $u_t = \frac{1}{t} u(tx, ty)$

$\leftrightarrow \text{tder} \ni u = (A, B)$ solving KV. What means $F_1(tx, ty)$?

140228a **Proj.** Associator computations using FreeLie'.

140203 **Project** "expansions and quadraticity for groups": definitions, relations with Hain / Mal'cev / Quillen / Vassiliev, torsion, semi-direct products, FG , PB , PB , PwB (and homotopy versions), elliptic / higher genus braids, mapping class groups, right-angled Artin groups, Stallings' theorem, knot groups (u, w, higher D), Hutchings-Lee. Flat braids after Merkov, Hilden braids, $[PB_n, PB_n]$ (also $u \rightarrow v, w$), $[G, G]$ in general, $\text{Aut}(FG_2)$, $\text{Aut}(FG_n)$, Torelli following Hain?

150107 **Paperlet.** "An algebraic characterization of the Taylor expansion".

141209



141129 Implement SeriesSolve:

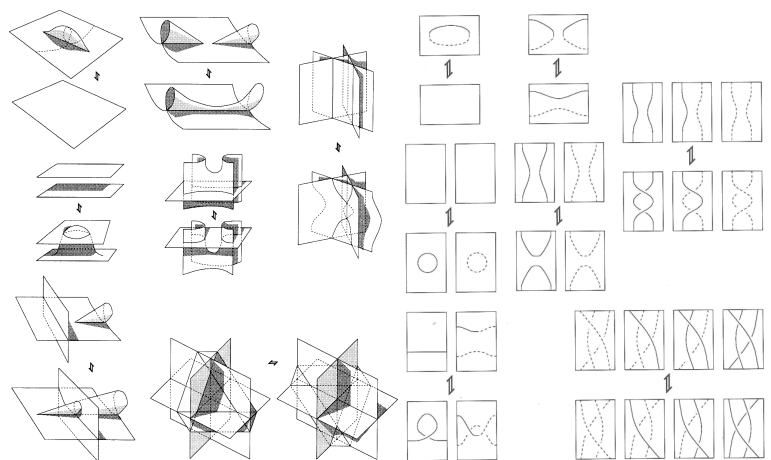
```
SeriesSolve[{
  α = LS[{ "1", "2" }, αs], β = LS[{ "1", "2" }, βs],
  γ = CWS[{ "1", "2" }, γs], κ = CWS[{ "1", "2" }, κs]
},
V = E_s[⟨1 → α, 2 → β⟩, γ];
h^-1 (ζ_s[R^*[2, 3] ** R^*[1, 3]] ** V ≡ V ** (ζ_s[R^*[1, 3]] // dA[1, 1, 2]))
&& V ** (V // dA[1] // dA[2]) ≡ de[1] ∪ de[2]
&& V ** (κ // dA[1, 1, 2]) // de[1] // de[2] ≡ κ ∪ (κ // dσ[1, 2])]
```

140627 **Proj.** A 3-page paper on Gassner and its unitarity.

140119 Lie-series Mathematica abstraction challenge (also 2014-01): **LieSeries**, **MakeLieSeries**, Crop, RandomLieSeries, \oplus , $c \cdot$, \equiv , \int , **b**, EulerE, adPower, **adSeries**, Ad, LieDerivation (also on CW, AW), $+$, $c \cdot$, DerivationPower, DerivationSeries, LieMorphism (also on CW, AW, \diamond and into $\langle \rangle$), StableApply, **BCH**, ASeries, MakeASeries, ι , σ , CWSeries, MakeCWSeries, RandomCWSeries, $+$, $c \cdot$, \equiv , \int , tr, div **JA**, $\langle \dots \rangle$, $+$, $c \cdot$, TangentialDerivation, **tb**, **I**, **I**⁻¹.

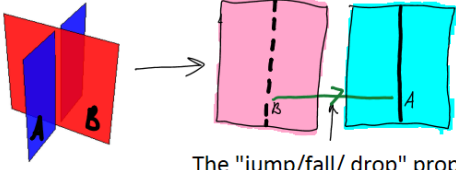
140213 Brochier's even associators to degree 9 at <http://abrochier.org/sage.php>.

140112 Carter-Saito: moves on decker curves:



140113 BF perturbation theory in ambient axial gauge (A-B propa-

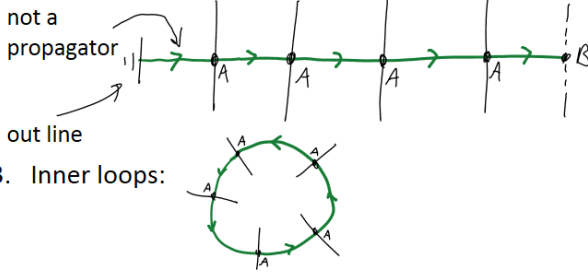
gator is t -vertical, B above A ; inner gauge unspecified):



The "jump/fall/drop" propagator

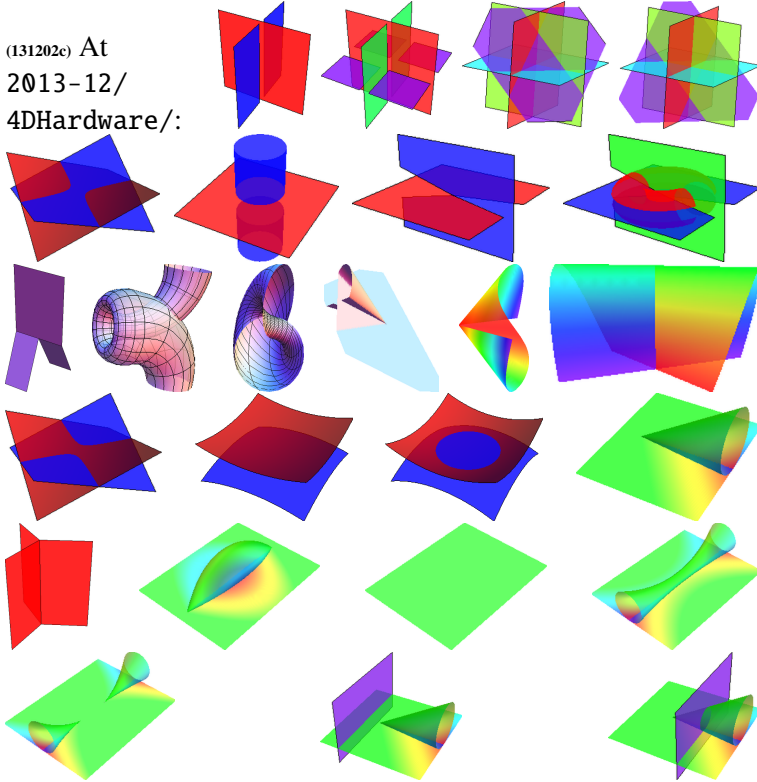
Feynman diagrams:

1. A "vertical B over A" "jump" propagator:
2. Inner chains: "biting worms"



3. Inner loops:

(131202c) At
2013-12/
4DHardware/:



Pensieve header: The Free Lie / Lazy Evaluation Abstraction Challenge.

```
MakeLieSeries[ser_Symbol, expr_] := (
  ser[] = Hold[MakeLieSeries[ser, expr]];
  ser[d_Integer] := ser[d] = Expand[expr /. w_LW // Deg[w] # d -> 0];
  LieSeries[ser]
);

AddLieSeries[ss___LieSeries] := AddLieSeries[ss] = Module[{ser},
  ser = Unique[AddLieSeries];
  ser[] = Hold[AddLieSeries[ss]];
  ser[d_Integer] := ser[d] = Plus @@ ({#[d] & /@ {ss}});
  LieSeries[ser]
];

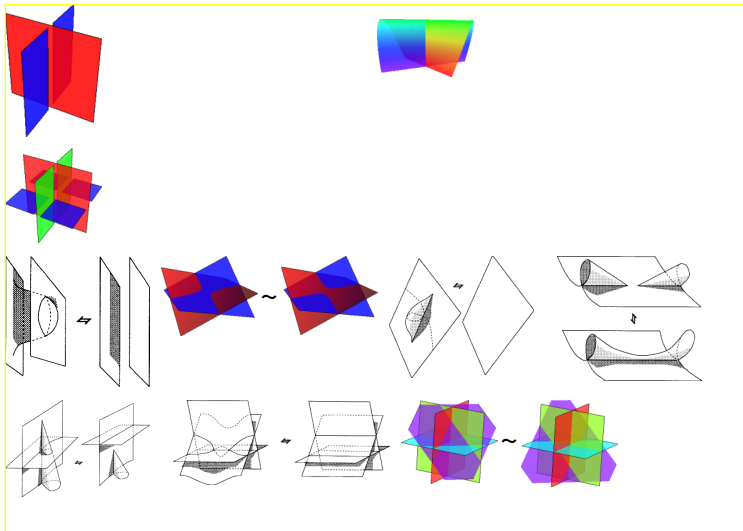
b[s1_LieSeries, s2_LieSeries] := b[s1, s2] = Module[{ser},
  ser = Unique[b];
  ser[] = Hold[b[s1, s2]];
  ser[d_Integer] := ser[d] = Sum[
    b[s1[k], s2[d-k]],
    {k, 1, d-1}
  ];
  LieSeries[ser]
];

LieDerivation[der_Symbol, rules_List] := (
  der[] = Hold[LieDerivation[der, rules]];
  (der[w_LW] // Deg[w] = 1) :=
    (der[w] = MakeLieSeries[w /. Append[rules, _LW -> 0]]);
  der[w_LW] := der[w] = Module[{x, y},
    {x, y} = LyndonFactorization[w];
    AddLieSeries[b[der[x], y], b[x, der[y]]]
  ];
  der[s_LieSeries] := der[s] = Module[{ser},
    ser = Unique[LieDerivationOnLieSeries];
    ser[] = Hold[der[s]];
    ser[d_] := ser[d] = Sum[
      der[s[k]][d],
      {k, 1, d}
    ];
    LieSeries[ser]
  ];
  der[as_ASeries] := Omitted;
  der[cws_CWSeries] := Omitted;
  der[expr_] [d_] :=
    Expand[expr /. {w_LW -> der[w][d], s_LieSeries -> der[s][d]}];
  LieDerivation[der]
);

BCHBase = Module[{bch},
  bch = Unique["BCHBase"];
  bch[] = Hold[BCHBase];
  bch[1] = {"x"} + {"y"};
  bch[d_Integer] := bch[d] = Expand[Plus[
    adSeries[E^(-ad), MakeLieSeries[{"y"}]][MakeLieSeries[{"x"}]][d],
    -adSeries[(1 - E^(-ad)) / ad - 1, LieSeries[bch]] [d]
  ] / d];
  LieSeries[bch]
];

JA[-1, ___] = MakeCWSeries[0];
JA[n_, y_LW, mu_LieSeries, ss_] := JA[n, y, mu, ss] = Module[
  {s, sm, ms},
  sm = ScaleLieSeries[s, mu];
  ms = StableApply[LieMorphism[{y -> Ad[ScaleLieSeries[1, sm]][LW[z]]}], mu];
  ms = ms // LieMorphism[{LW[z] -> y}];
  IntegrateCWSeries[
    AddCWSeries[
      JA[n-1, y, mu, s] // LieDerivation[{y -> b[ms, y]}],
      div[y, ms]
    ],
    {s, 0, ss}
  ];
];

JA[y_LW, mu_LieSeries] := JA[y, mu] = Module[{cws, s},
  cws = Unique[JA];
  cws[] = Hold[JA[y, mu]];
  cws[d_Integer] := cws[d] = JA[d-1, y, mu, s][d] /. s -> 1;
  CWSeries[cws]
];
```



131229 What are the two winding numbers for immersions $\mathbb{R}^2 \hookrightarrow \mathbb{R}^4$? Is every pair realized? Is there a Whitney-Graustein theorem?

131130a Meilhan: Levine: [arXiv:q-alg/9711007](https://arxiv.org/abs/q-alg/9711007) *A Factorization of the Conway Polynomial*. Then Tsukamoto, Yasuhara: [arXiv:math/0405481](https://arxiv.org/abs/math/0405481) *A factorization of the Conway polynomial and covering linkage invariants*.

131026 Time to make an “agenda browser”.

131017 If $\lambda_{\{ij\}} = 0$, $(\lambda_{ij} dx^i \wedge dx^j)^{n/2} = \sqrt{\det(\lambda_{ij})} \bigwedge_i dx^i$.