I don’t understand the Koszul condition.

I don’t yet appreciate infinity-algebras.

I don’t really understand Poisson structures: Why do they automatically arise from action principles? Why do they necessarily emerge in computing path integrals? Why should I care about their deformation quantizations?

I don’t understand thermal physics - energy, entropy, enthalpy, and all that. Such basic things are there that it is really embarrassing that I don’t understand the constraints my air-conditioner is bound by.

— From Feynman’s Lectures on Physics: “equal volumes of gases, at the same pressure and temperature, contain the same number of molecules”; $N_0 = 6.022 \times 10^{23}$ as in (1 mole)$=12g$ of $^{12}C$. $P = F/A$. $dW = -PdV$. $PV = \frac{2}{3}N(\frac{1}{2}mv^2) = \frac{2}{7}U \ldots = NK(T)$. With $\gamma - 1 = \frac{2}{3}$, $PV^\gamma = C$. In gas mixtures, $\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2$ (messy!). $\frac{1}{2}mv^2 = \frac{k}{m}$ with $k = 1.38 \times 10^{-23} J/degree$ (J = joule = newton metre = watt second).

— From Bamberg-Sternberg: First law of thermodynamics: $\alpha + \omega = dU$, with $\alpha$: heat 1-form, $\omega$: work 1-form, $U$: internal energy. Second law of thermodynamics: $\alpha = TdS$, with $T$: temperature, $S$: entropy.

— From Schroed: $1 cal = 10^{-3}$ food calorie $= 4.186J \sim heat$ to raise 1g of water by 1°C.

— See also Lieb-Yngvason.

I don’t understand supersymmetry.

I don’t understand renormalization theory. Minor point: it would be great if I could present the renormalization of associators/vertices as a special case.

I don’t understand the Mostow rigidity theorem.

I’m not as comfortable with special relativity as I want to be.

I don’t really understand general relativity.

I don’t understand love and sex.

I don’t fully understand the $h$-cobordism theorem. Perhaps follow Milnor’s lecture notes?

**Def.** An $h$-cobordism is a cobordism in which the boundary inclusions are deformation retractions.

**Thm.** In Diff, PL, or Top, a simply-connected $h$-cobordism between simply-connected ($n \geq 5$)-manifolds is trivial.

I don’t really understand Faddeev-Popov and/or BRST.

I don’t understand the Batalin-Vilkovisky formalism.

— Mnëv’s example. “Space of fields” $M = R^3_{xyz} \times S^1_z$; “classical action” $S_{cl} := \frac{1}{2} \gamma^2$; “Gauge symmetry” $E := \text{span} (\partial_y, \partial_x + t y \partial_c)$, integrable on $EL = \{ t = 0 \}$ surface but not on $M$, $S_{cl}$ is invariant. $M/E$ is not $T_2$ and $\int_{M/E} e^{-S}$ makes no sense.

— BV space of fields $F = T^*[1](\mathbb{R}^2[1] \times M)$ with coords $c_{1,2}$ (ghost number 1), $t, x, y, z$ (g.n. 0), $i^t, x^i, y^i, z^i$ (g.n. -1) and $c_{1,2}$ (g.n. -2). The BV action is $S = \frac{1}{2}\gamma^2 + c_{1,2} + c_2(x^i + ty^i) + c_1 c_2 i^t z^i$; satisfies QME & consistent with $S_{cl}$ and $E$.

— Losev: For $\omega \in \Omega^{n-1}(M^n)$, $\int_{\{ t = 0 \}} \omega = \int_{T^*M \mathbb{R}^{n+1}} \omega e^{-d(f_A)}$.

— Further: old paper by Schwarz; arXiv:0812.0464 by Albert, Bleile, Fröhlich; notes by Kazhdan; thesis by Gwilliam; notes by Ens.

I still don’t understand the BF TQFT. From Cattaneo-Rossi’s arXiv:math-ph/0210037 Wilson Surfaces: $A \in \Omega^1(\mathbb{R}^4, g)$,

$$S(A, B) := \int \langle B, F_A \rangle,$$

$G := \exp \Omega^0(\mathbb{R}^4, g)$ is (u-)gauge transformations, $(g, \sigma) \in \tilde{G} := G \ltimes \Omega^1(\mathbb{R}^4, g^*)$ acts by

$$A \rightarrow A^g \cdot B \rightarrow B^{g, \sigma} := Ad^g_{\frac{1}{2}} B + dA_{\frac{1}{2}} \sigma.$$

With $f: \mathbb{R}^2 \rightarrow \mathbb{R}^4, \xi \in \Omega^0(\mathbb{R}^2, g), \beta \in \Omega^1(\mathbb{R}^2, g^*)$, set

$$O(A, B, f) := \int \Omega \xi \Omega \beta \exp \left( \frac{i}{\hbar} \int \Omega \xi d f_A \beta + f^* B \right).$$

I forgot too much of what I used to know about Lie theory. From Humphreys: Weyl’s formula: For $\lambda \in \Lambda^+$,

$$ch_A \ast \sum_{\sigma \in \mathcal{W}} (-t)^{\sigma(\lambda)} = \sum_{\sigma \in \mathcal{W}} (-t)^{\sigma(\lambda + \delta)}.$$
Easy from multi-linearity and anti-symmetry if \((u_i)\) and \((v_j)\) are in a symplectic basis for \(\omega\).

- I don’t understand the Guillou-Gamma-Viro theorem.
- I don’t fully understand Habiro’s theory of clasps.
- I don’t understand Gröbner bases.
- I still don’t know a proof of the Milnor-Moore theorem. — Maybe “Spencer Bloch’s course on Hopf Algebras” or Kreimer’s thesis. Maybe search inside?
- I still don’t understand Vogel’s construction.
- I’m missing the key to equivariant cohomology, \(EG, BG\), and all that. — I need a framework for \(X_G := (X \times EG)/G\).
- I don’t understand fusion categories and subfactors.
  — Morrison’s drorbn.net/dbnvp/Morrison-140220

### I don’t understand group cohomology.

— Pensieve: 2013-02: \(G\) group; \(M\) a \(G\)-module; \(C^n(G, M) := \{\varphi: G^n \to M\};

\[
(d\varphi)(g_1, \ldots, g_{n+1}) := g_1 \varphi(g_2, g_3, \ldots, g_{n+1}) + \sum_{i=1}^{n} (-)^i \varphi(g_1, \ldots, g_ig_{i+1}, \ldots) + (-)^{n+1} \varphi(g_1, \ldots, g_n).
\]

For \(M = \mathbb{K}\): \(H^n = H^n(K(G, 1))\). \(H^1 = \text{Hom}(G, \mathbb{K})\). \(H^2\) counts central extensions by \(\mathbb{K}\).

### I don’t understand the basics of three-dimensional topology: the loop and sphere theorems, JSJ decompositions, etc.

From Hatcher’s notes:

**Definition.** \(M\) prime: \(M = P\#Q \Rightarrow (P = S^3) \lor (Q = S^3)\). \(M\) irreducible: an embedded 2-sphere in \(M\) bounds a 3-ball. (Irreducible \(\Rightarrow\) Prime).

**Theorem (Alexander, 1920s).** \(S^3\) is irreducible.

**Proof.** Study the change to the “canonical closure” of a cropped embedded \(S^2\) under the following cases:

**Theorem.** Orientable, prime, not irreducible \(\Rightarrow S^2 \times S^1\). Nonorientable? Also \(S^2 \times S^1\) (Klein 3D).

**Theorem.** Compact connected orientable 3-manifolds have unique decomposition into primes.

**Proof.** Given a system of splitting spheres (sss) and a \(\theta\)-partition of one member, at least one part will make an sss. An sss can be simplified relative to a fixed triangulation \(\tau\): only disk intersections with simplices; circle and single-edge-arc intersections with faces of \(\tau\) can be eliminated. The size of an sss is bounded by \(4|\tau| + \text{rank } H_1(M; \mathbb{Z}/2)\) and hence prime-decompositions exist.

- Uniqueness. \(\square\)

Nonorientable \(M\)? Same but \(M\#(S^2 \times S^1) = M\#(S^2 \times S^1)\).

**Theorem.** If a covering is irreducible, so is the base. ([Ha] proof is fishy).

**Examples.** Lens spaces, surface bundles \(F \to M \to S^1\) with \(F \neq S^2,\mathbb{RP}^2\). Yet \(S^1 \times S^2/(x, y) \sim (\bar{x}, y) = \mathbb{RP}^3 \times \mathbb{RP}^3\), a prime covers a sum.

**Definition.** \(S \subset M^3\) a 2-sided surface, \(S \neq S^2, S \neq D^2\). Compressing disk for \(S\) is a disk \(D \subset M\) with \(D \cap S = \partial D\). If for every compressing \(D\) there’s a disk \(D' \subset S\) with \(\partial D' = \partial D\), \(S\) is incompressible.

**Claims.** \(\pi_1(S) \to \pi_1(M) \Rightarrow S\) incompressible. \(\bullet\) No incompressibles in \(\mathbb{RP}^3/S^3\). \(\bullet\) In irreducible \(M^3\), \(T^2\) is 2-sided incompressible iff \(T\) bounds a \(D^2 \times S^1\) or \(T\) is contained in a \(B^3\). \(\bullet\) A \(T^2\) in \(S^3\) bounds a \(D^2 \times S^1\) on at least one side. \(\bullet S \subset M\) irreducible \(\Rightarrow (M\text{ irreducible iff } M\lfloor S\text{ irreducible})\). \(\bullet S\) a collection of disjoint incompressibles or disks or spheres in \(M\), \(T \subset M\lfloor S\). Then \(T\) is incompressible in \(M\lfloor S\).

From Hempel’s book:

**Dehn’s Lemma** (Dehn 1910 (wrong), Papakyriakopoulos 1950s). \(M\) a 3-manifold, \(f: B^2 \to M\) s.t. for some neighborhood \(A\) of \(\partial B^2\) in \(B^2\) the restriction \(F_A\) is an embedding and \(f^{-1}(f(A)) = A\). Then \(f|_{\partial B^2}\) extends to an embedding \(g: B^2 \to M\).

**The Loop Theorem** (Stallings 1960, implies Dehn’s lemma). \(M\) a 3-manifold, \(F\) a connected 2-manifold in \(\partial M\), \(\ker(\pi_1(F) \to \pi_1(M)) \neq N \neq \pi_1(F)\). Then there is a proper embedding \(g: (B^2, \partial B^2) \to (M, F)\) s.t. \([g]|_{\partial B^2} \notin N\).

**The Sphere Theorem.** \(M\) orientable 3-manifold, \(N\) a \(\pi_1(M)\)-invariant proper subgroup of \(\pi_2(M)\). Then there is an embedding \(g: S^2 \to M\) s.t. \([g] \notin N\).
Redeemed Confessions.

- I don’t understand Galois theory, for real. Abstractness is fun, but Galois surely understood everything in very concrete terms. I wish I did too. — youtu.be/RhpVSV6iCko and then drorbn.net/dbnvp/AKT-140314.php do the job!