Pensieve header: The Lashings Matrix.

```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\APAI"];
```

```
In[•]:= Once[<< KnotTheory`; << Rot.m];
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

Loading Rot.m from http://drorbn.net/APAI to compute rotation numbers.
```

```
In[•]:= R1[s_, i_, j_] := s (gji (gj*,j + gj,j* - gij) - gii (gj,j* - 1) - 1 / 2);
ρ[K_] := ρ[K] = Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ρ1},
    {Cs, φ} = Rot[K]; n = Length[Cs];
    A = IdentityMatrix[2 n + 1];
    Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ( -T^s  T^s - 1
                                                                 0     -1 ))];
    Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
    G = Inverse[A];
    ρ1 = Sum_{k=1}^{n} R1 @@ Cs[[k]] - Sum_{k=1}^{2 n} φ[[k]] (gkk - 1 / 2);
    Factor@{Δ, Δ^2 ρ1 /. α_^+ :> α + 1 /. g_α_,β_ :> G[[α, β]]}];
```

```
In[•]:= CompareMatrices[A_, B_] := Grid[
  MapThread[Column@*List, {A, B} /. 0 → "", 2],
  Frame → All, ItemSize → All
 ]
```

```
In[•]:= DepthList[K_] := Module[{Cs, n, dd, k, s, i, j},
  Cs = Rot[K][[1]]; n = Length[Cs];
  dd = Table[0, 2 n + 1];
  For[k = 1, k ≤ n, k++, {s, i, j} = Cs[[k]]; dd[[{i + 1, j + 1}]] = {s, -s}];
  FoldList[Plus, dd]
 ]
```

```
In[•]:= DepthList /@ AllKnots[{3, 7}]
```

```
Out[•]=
{{0, 1, 0, 1, 0, 1, 0}, {0, 1, 2, 1, 0, 1, 2, 1, 0},
 {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0}, {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0},
 {0, 1, 2, 1, 0, 1, 0, 1, 2, 1, 0, 1, 0}, {0, 1, 2, 1, 0, 1, 0, 1, 2, 1, 0, 1, 0},
 {0, 1, 0, 1, 0, -1, 0, 1, 0, -1, 0, -1, 0}, {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0},
 {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0}, {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0},
 {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0}, {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0},
 {0, 1, 0, 1, 0, 1, 0, 1, 0, -1, 0, 1, 0, -1, 0}, {0, 1, 2, 1, 0, 1, 2, 1, 2, 1, 0, 1, 2, 1, 0}}
```

```
In[•]:= LashMat0[K_] := Module[{Cs, φ, n, A, α, β, k, s, i, j, Δ, G, k1, k2, s1, s2, i1, i2, j1, j2},
          {Cs, φ} = Rot[K]; n = Length[Cs];
          A = IdentityMatrix[2 n + 1];
          For[k = 1, k ≤ n, k++, {s, i, j} = Cs[[k]];
           A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}})];
          d = DepthList[K];
          Δ = T^((-Total[φ]-Total[Cs[[All,1]]])/2) Det[A];
          G = Inverse[A];
          Factor@Table[
            {s1, i1, j1} = Cs[[k1]]; {s2, i2, j2} = Cs[[k2]];
            G[[j1, i2]] - 0 G[[j1 + 1, i2]] -
             T^-d[[i2+1]] (G[[j1, i2 + 1]] - δ_{j1,i2+1}) + 0 T^-d[[i2+1]] (G[[j1 + 1, i2 + 1]] - δ_{j1+1,i2+1}),
            {k1, n}, {k2, n}]
         ]
        LashMat0[Knot[7, 4]] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0 \end{pmatrix}$$

```
In[•]:= LashMat[K_] := Module[{Cs, φ, d, n, A, α, β, k, s, i, j, Δ, G, k1, k2, s1, s2, i1, i2, j1, j2},
          {Cs, φ} = Rot[K]; d = DepthList[K]; n = Length[Cs];
          A = IdentityMatrix[2 n + 1];
          For[k = 1, k ≤ n, k++, {s, i, j} = Cs[[k]];
           A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}})];
          Δ = T^((-Total[φ]-Total[Cs[[All,1]]])/2) Det[A];
          G = Inverse[A];
          Factor[Δ Table[
              {s1, i1, j1} = Cs[[k1]]; {s2, i2, j2} = Cs[[k2]];
              G[[i1, j2]] - G[[i1 + 1, j2]] -
               T^d[[j2+1]] (G[[i1, j2 + 1]] - δ_{i1,j2+1}) + T^d[[j2+1]] (G[[i1 + 1, j2 + 1]] - δ_{i1+1,j2+1}),
              {k1, n}, {k2, n}]]
         ];
        Column[MatrixForm /@ {LM = LashMat[Knot[8, 17]], Factor[LM /. T → T^1 / LM^T]}]
```

Power: Infinite expression $\frac{1}{0}$ encountered.

Power: Infinite expression $\frac{1}{0}$ encountered.

Power: Infinite expression $\frac{1}{0}$ encountered.

General: Further output of Power::infy will be suppressed during this calculation.

Out[•]=

$$\begin{pmatrix} -\frac{(-1+T)\left(1-4T+8T^2-11T^3+8T^4-4T^5+T^6\right)}{T^3} & -\frac{(-1+T)^3\left(1-2T+4T^2-2T^3+T^4\right)}{T^3} & \frac{(-1+T)^2\left(-1+2T-4T^2+3T^3-3T^4+3T^5-2T^6+T^7\right)}{T^2} & \frac{(-1+T)^2(1} \\ 0 & \frac{(-1+T)\left(1-T+T^2\right)\left(-1+3T-2T^2+T^3\right)}{T^3} & -\frac{(-1+T)^2\left(-1+3T-5T^2+5T^3-3T^4+3T^5-T^6+T^7\right)}{T^3} & -(-1+T \\ 0 & -\frac{(-1+T)^2}{T^2} & \frac{(-1+T)\left(-1+T-3T^2+4T^3-3T^4+T^5\right)}{T} & \\ 0 & \frac{(-1+T)^2\left(1-T+T^2\right)^2}{T^3} & -\frac{(-1+T)^2\left(1-T+T^2\right)\left(1+T^2+T^3\right)}{T^2} & -\left(\\ 0 & -\frac{(-1+T)^2\left(1-3T+4T^2-2T^3+T^4\right)}{T^3} & \frac{(-1+T)^2\left(1-2T+3T^2-3T^3+4T^4-2T^5+T^6\right)}{T^2} & (- \\ 0 & \frac{(-1+T)^2}{T} & -\frac{(-1+T)^2\left(1-2T+5T^2-3T^3+T^4\right)}{T} & - \\ 0 & \frac{(-1+T)^3\left(1-T+T^2\right)}{T^3} & -\frac{(-1+T)^3\left(1+T^2+T^3\right)}{T^2} & \\ 0 & -\frac{(-1+T)^3\left(1-T+T^2\right)}{T^2} & \frac{(-1+T)^3\left(1+T^2+T^3\right)}{T} & \end{pmatrix}$$

$$\begin{pmatrix} 1 & \text{ComplexInfinity} & \text{ComplexInfinity} & \text{ComplexInfinity} & \text{ComplexInfinity} & (\\ 0 & 1 & \frac{-1+3T-5T^2+5T^3-3T^4+3T^5-T^6+T^7}{T} & -\frac{T^3\left(1-3T+4T^2-2T^3+T^4\right)}{\left(1-T+T^2\right)^2} & \frac{T^2\left(-1+T-T^2+3T^3-T^4+T^5\right)}{1-3T+4T^2-2T^3+T^4} & -\frac{-1+} \\ 0 & \frac{T}{-1+3T-5T^2+5T^3-3T^4+3T^5-T^6+T^7} & 1 & -\frac{(-1+T)T}{1+T^2+T^3} & \frac{T\left(1-T^2+T^3\right)}{1-2T+3T^2-3T^3+4T^4-2T^5+T^6} & - \\ 0 & -\frac{\left(1-T+T^2\right)^2}{T^3\left(1-3T+4T^2-2T^3+T^4\right)} & -\frac{1+T^2+T^3}{(-1+T)T} & 1 & -\frac{1-2T+4T^2-T^3+T^4}{(-1+T)T^2\left(2-T+T^2\right)} & \\ 0 & \frac{1-3T+4T^2-2T^3+T^4}{T^2\left(-1+T-T^2+3T^3-T^4+T^5\right)} & \frac{1-2T+3T^2-3T^3+4T^4-2T^5+T^6}{T\left(1-T^2+T^3\right)} & -\frac{(-1+T)T^2\left(2-T+T^2\right)}{1-2T+4T^2-T^3+T^4} & 1 & -\frac{2-} \\ 0 & -\frac{T^2}{-1+3T-4T^2+2T^3-T^4+2T^5-T^6+T^7} & -\frac{T^2\left(1-2T+5T^2-3T^3+T^4\right)}{1-3T+5T^2-2T^3+T^4} & \frac{(-1+T)T^2}{1+T+T^3} & -\frac{T^2\left(1-T^2+T^3\right)}{2-4T+4T^2-4T^3+4T^4-2T^5+T^6} & \\ 0 & \frac{(-1+T)\left(1-T+T^2\right)}{T\left(-1+2T-T^2+T^3\right)} & \frac{1+T^2+T^3}{-1+T} & -T & \frac{-1+2T-2T^2+2T^3+T^4}{T\left(-1+3T-2T^2+T^3\right)} & \\ 0 & -\frac{T\left(1-T+T^2\right)}{-1+2T-T^2+T^3} & -\frac{T^2\left(1+T^2+T^3\right)}{(-1+T)^2} & \frac{T^3}{-1+T} & -\frac{T\left(-1+2T-2T^2+2T^3+T^4\right)}{(-1+T)\left(-1+3T-2T^2+T^3\right)} & \end{pmatrix}$$