

Pensieve header: Comparing  $SG$  and  $SG^T$ .

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\APAI"];
```

```
In[ ]:= Once[<< KnotTheory` ; << Rot.m];
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

Loading Rot.m from <http://drorbn.net/APAI> to compute rotation numbers.

```
In[ ]:= DepthList[K_] := Module[{Cs, n, dd, k, s, i, j},
  Cs = Rot[K][[1]]; n = Length[Cs];
  dd = Table[0, {2 n + 1}];
  For[k = 1, k ≤ n, k++, {s, i, j} = Cs[[k]]; dd[[i + 1, j + 1]] = {s, -s}];
  FoldList[Plus, dd]
]
```

```
In[ ]:= DepthList /@ AllKnots[{3, 7}]
```

Out[ ]:=

```
{ {0, 1, 0, 1, 0, 1, 0}, {0, 1, 2, 1, 0, 1, 2, 1, 0},
  {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0}, {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0},
  {0, 1, 2, 1, 0, 1, 0, 1, 2, 1, 0, 1, 0}, {0, 1, 2, 1, 0, 1, 0, 1, 2, 1, 0, 1, 0},
  {0, 1, 0, 1, 0, -1, 0, 1, 0, -1, 0, -1, 0}, {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0},
  {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0}, {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0},
  {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0}, {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0},
  {0, 1, 0, 1, 0, 1, 0, 1, 0, -1, 0, 1, 0, -1, 0}, {0, 1, 2, 1, 0, 1, 2, 1, 2, 1, 0, 1, 2, 1, 0} }
```

```

In[*]:= K = Knot[4, 1];
A := Module[{Cs, phi, n, A, k, s, i, j},
  {Cs, phi} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  For[k = 1, k <= n, k++, {s, i, j} = Cs[[k]];
    A[[{i, j}, {i + 1, j + 1}]] +=  $\begin{pmatrix} -T^5 & T^5 - 1 \\ 0 & -1 \end{pmatrix}$ ];
  A
];
G := Inverse[A];
DM := DiagonalMatrix[T^DepthList[K]]; DM
A // MatrixForm
G // MatrixForm

```

Out[\*]=

- {1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, T, 0, 0, 0, 0, 0, 0, 0}, {0, 0, T<sup>2</sup>, 0, 0, 0, 0, 0, 0},
- {0, 0, 0, T, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, T, 0, 0, 0},
- {0, 0, 0, 0, 0, 0, T<sup>2</sup>, 0, 0}, {0, 0, 0, 0, 0, 0, 0, T, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1}

Out[\*]//MatrixForm=

$$\begin{pmatrix} 1 & -T & 0 & 0 & -1+T & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{T} & 0 & 0 & -1+\frac{1}{T} & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -T & 0 & 0 & -1+T \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1+\frac{1}{T} & 0 & 0 & 0 & 1 & -\frac{1}{T} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[\*]//MatrixForm=

$$\begin{pmatrix} 1 & \frac{T-3T^2+T^3}{1-3T+T^2} & \frac{T(T-3T^2+T^3)}{1-3T+T^2} & \frac{T-3T^2+T^3}{1-3T+T^2} & 1 & \frac{T-3T^2+T^3}{1-3T+T^2} & \frac{T(T-3T^2+T^3)}{1-3T+T^2} & \frac{T-3T^2+T^3}{1-3T+T^2} & 1 \\ 0 & 1 & -\frac{T^2}{1-3T+T^2} & -\frac{T}{1-3T+T^2} & -\frac{T}{1-3T+T^2} & -\frac{T^2}{1-3T+T^2} & \frac{(1-2T)T}{1-3T+T^2} & \frac{1-2T}{1-3T+T^2} & 1 \\ 0 & 0 & -\frac{T^2}{1-3T+T^2} & -\frac{T}{1-3T+T^2} & -\frac{T}{1-3T+T^2} & -\frac{T^2}{1-3T+T^2} & \frac{(1-2T)T}{1-3T+T^2} & \frac{1-2T}{1-3T+T^2} & 1 \\ 0 & 0 & \frac{T(T-T^2)}{1-3T+T^2} & \frac{1-2T}{1-3T+T^2} & \frac{1-2T}{1-3T+T^2} & \frac{T-2T^2}{1-3T+T^2} & -\frac{T^3}{1-3T+T^2} & -\frac{T^2}{1-3T+T^2} & 1 \\ 0 & 0 & \frac{T(T-T^2)}{1-3T+T^2} & \frac{T-T^2}{1-3T+T^2} & \frac{1-2T}{1-3T+T^2} & \frac{T-2T^2}{1-3T+T^2} & -\frac{T^3}{1-3T+T^2} & -\frac{T^2}{1-3T+T^2} & 1 \\ 0 & 0 & \frac{(1-T)T}{1-3T+T^2} & \frac{1-T}{1-3T+T^2} & \frac{1-T}{1-3T+T^2} & \frac{1-2T}{1-3T+T^2} & -\frac{T^2}{1-3T+T^2} & -\frac{T}{1-3T+T^2} & 1 \\ 0 & 0 & \frac{(1-T)T}{1-3T+T^2} & \frac{1-T}{1-3T+T^2} & \frac{1-T}{1-3T+T^2} & \frac{T-T^2}{1-3T+T^2} & -\frac{T^2}{1-3T+T^2} & -\frac{T}{1-3T+T^2} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[\*]:= Factor[A.G] // MatrixForm

Out[\*]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[\*]:= Factor[G.(DM /. T -> T^1).(A^T /. T -> T^1)] // MatrixForm

Out[\*]//MatrixForm=

$$\begin{pmatrix} -((-1+T)T(1+T)) & -((-1+T)T^2(1+T)) & (-1+T)T(1+T) & (-1+T)(1+T) & -((-1+T)T(1+T)) \\ -\frac{T(-1+2T-3T^2+T^3)}{1-3T+T^2} & \frac{(-1+T)T(-1+2T+T^2)}{1-3T+T^2} & \frac{(-1+T)T(-1-2T+T^2)}{1-3T+T^2} & -\frac{(-1+T)T}{1-3T+T^2} & \frac{(-1+T)(1-2T+2T^2+T^3)}{1-3T+T^2} \\ -\frac{(-1+T)T}{1-3T+T^2} & \frac{T^4}{1-3T+T^2} & \frac{(-1+T)T(-1-2T+T^2)}{1-3T+T^2} & -\frac{(-1+T)T}{1-3T+T^2} & \frac{(-1+T)(1-2T+2T^2+T^3)}{1-3T+T^2} \\ -\frac{(-1+T)(-1+2T)}{1-3T+T^2} & \frac{(-1+T)T^4}{1-3T+T^2} & \frac{-1+2T}{1-3T+T^2} & -\frac{(-1+T)(-1+2T)}{1-3T+T^2} & \frac{2(-1+T)T(-1+T+T^2)}{1-3T+T^2} \\ -\frac{(-1+T)(-1+2T)}{1-3T+T^2} & \frac{(-1+T)T^4}{1-3T+T^2} & \frac{(-1+T)T}{1-3T+T^2} & -\frac{1-2T-T^2+T^3}{1-3T+T^2} & \frac{2(-1+T)T(-1+T+T^2)}{1-3T+T^2} \\ -\frac{(-1+T)^2}{1-3T+T^2} & \frac{(-1+T)T^3}{1-3T+T^2} & \frac{-1+T}{1-3T+T^2} & -\frac{(-1+T)^2}{1-3T+T^2} & \frac{T(3-5T+3T^2)}{1-3T+T^2} \\ -\frac{(-1+T)^2}{1-3T+T^2} & \frac{(-1+T)T^3}{1-3T+T^2} & \frac{-1+T}{1-3T+T^2} & -\frac{(-1+T)^2}{1-3T+T^2} & \frac{(-1+T)T(-3+T+T^2)}{1-3T+T^2} \\ 0 & 0 & 0 & 0 & -1+T \\ 0 & 0 & 0 & 0 & -1+T \end{pmatrix}$$