

Pensieve header: Just colour-coded $SA\phi$ -matrices.

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\APAI"];
```

```
In[ ]:= Once[<< KnotTheory` ; << Rot.m];
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

Loading Rot.m from <http://drorbn.net/APAI> to compute rotation numbers.

```
In[ ]:= R1[s_, i_, j_] := s (g_{j^*,j} + g_{j,j^*} - g_{ij}) - g_{ii} (g_{j,j^*} - 1) - 1 / 2);
rho[K_] := rho[K] = Module[{Cs, phi, n, A, s, i, j, k, Delta, G, rho1},
  {Cs, phi} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} -> (A[[{i, j}, {i + 1, j + 1}]] += ( -T^s T^s - 1 ))];
  Delta = T^(-Total[phi] - Total[Cs[[All, 1]]) / 2) Det[A];
  G = Inverse[A];
  rho1 = Sum_{k=1}^n R1 @@ Cs[[k]] - Sum_{k=1}^{2^n} phi[[k]] (g_{kk} - 1 / 2);
  Factor@{Delta, Delta^2 rho1 /. alpha_+ -> alpha + 1 /. g_{alpha, beta_} -> G[[alpha, beta]]};
```

```
In[ ]:= Table[Hue[k / 16], {k, 0, 16}]
```

Out[]:=



```
In[ ]:= ColouredA[K_] := Module[{Cs, phi, n, A, alpha, beta, k, s, i, j},
  {Cs, phi} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  For[k = 1, k <= n, k++,
    {s, i, j} = Cs[[k]];
    A[[{i, j}, {i + 1, j + 1}]] += Map[Style[#, Background -> Hue[k / 2 n]] &, ( -T^s T^s - 1 ), {2}];
  ];
  MatrixForm[A]
]
```

In[]:= ColouredA[Knot[3, 1]]

Out[]//MatrixForm=

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{T} & 0 & 0 & -1 + \frac{1}{T} & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 + \frac{1}{T} & 0 & 1 & -\frac{1}{T} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 + \frac{1}{T} & 0 & 1 & -\frac{1}{T} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[]:= ColouredA[Knot[4, 1]]

Out[]//MatrixForm=

$$\begin{pmatrix} 1 & -1 & 0 & 0 & -1 + T & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{T} & 0 & 0 & -1 + \frac{1}{T} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 + T \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 + \frac{1}{T} & 0 & 0 & 0 & 1 & -\frac{1}{T} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[]:= ColouredA[Knot[6, 3]]

⌘ KnotTheory: Loading precomputed data in PD4Knots`

Out[]//MatrixForm=

$$\begin{pmatrix} 1 & -1 & 0 & 0 & -1 + T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & -1 + T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{1}{T} & 0 & 0 & 0 & 0 & -1 + \frac{1}{T} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 + T & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{1}{T} & 0 & 0 & -1 + \frac{1}{T} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 + \frac{1}{T} & 0 & 0 & 0 & 1 & -\frac{1}{T} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[]:= Do[Echo[K → ColouredA[K]], {K, AllKnots[{3, 7}]}];

» Knot[3, 1] →

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{T} & 0 & 0 & -1 + \frac{1}{T} & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 + \frac{1}{T} & 0 & 1 & -\frac{1}{T} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 + \frac{1}{T} & 0 & 1 & -\frac{1}{T} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

» Knot [4, 1] →

$$\begin{pmatrix} 1 & -T & 0 & 0 & -1+T & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{T} & 0 & 0 & -1+\frac{1}{T} & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -T & 0 & 0 & -1+T \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1+\frac{1}{T} & 0 & 0 & 0 & 1 & -\frac{1}{T} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

» Knot [5, 1] →

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{T} & 0 & 0 & 0 & 0 & -1+\frac{1}{T} & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{T} & 0 & 0 & 0 & 0 & -1+\frac{1}{T} & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1+\frac{1}{T} & 0 & 0 & 0 & 1 & -\frac{1}{T} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1+\frac{1}{T} & 0 & 0 & 0 & 1 & -\frac{1}{T} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1+\frac{1}{T} & 0 & 0 & 0 & 1 & -\frac{1}{T} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

» Knot [5, 2] →

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{T} & 0 & 0 & 0 & 0 & -1+\frac{1}{T} & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1+\frac{1}{T} & 0 & 1 & -\frac{1}{T} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{1}{T} & 0 & 0 & -1+\frac{1}{T} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1+\frac{1}{T} & 0 & 0 & 0 & 1 & -\frac{1}{T} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1+\frac{1}{T} & 0 & 0 & 0 & 1 & -\frac{1}{T} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

» Knot [6, 1] →

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1+T & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1+\frac{1}{T} & 0 & 1 & -\frac{1}{T} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{1}{T} & 0 & 0 & 0 & -1+\frac{1}{T} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1+T & 0 & 0 & 0 & 1 & -T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1+\frac{1}{T} & 0 & 1 & -\frac{1}{T} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1+\frac{1}{T} & 0 & 0 & 0 & 0 & 1 & -\frac{1}{T} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

