Pensieve header: Mathematica notebook for A Perturbed Alexander Invariant.

```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\APAI"];
```

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```
In[•]:= Once[<< KnotTheory`; << Rot.m];
```

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```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

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```
Loading Rot.m from http://drorbn.net/APAI to compute rotation numbers.
```

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```
In[•]:= R1[s_, i_, j_] := s (g_{ji} (g_{j⁺,j} + g_{j,j⁺} - g_{ij}) - g_{ii} (g_{j,j⁺} - 1) - 1 / 2);
ρ[K_] := ρ[K] = Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ρ1},
    {Cs, φ} = Rot[K]; n = Length[Cs];
    A = IdentityMatrix[2 n + 1];
    Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ((-T^s  T^s - 1)
                                                               (  0     -1  )))];
    Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
    G = Inverse[A];
    ρ1 = Sum_{k=1}^{n} R1 @@ Cs[[k]] - Sum_{k=1}^{2 n} φ[[k]] (g_{kk} - 1 / 2);
    Factor@{Δ, Δ^2 ρ1 /. α_⁺ :> α + 1 /. g_{α_,β_} :> G[[α, β]]}];
```

The g-Rules

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```
In[•]:= δ_{i_,j_} := If[i === j, 1, 0];
gRules_{s_,i_,j_} := {g_{iβ_} :> δ_{iβ} + T_j^s g_{i⁺,β} + (1 - T_j^s) g_{j⁺,β}, g_{jβ_} :> δ_{jβ} + g_{j⁺,β},
   g_{α_,i} :> T_j^-s (g_{α,i⁺} - δ_{α,i⁺}), g_{α_,j} :> g_{α,j⁺} - (1 - T_j^s) g_{αi} - δ_{α,j⁺}}
(α_⁺)⁺ := α^"++"; (* this is for cosmetic reasons only *)
```

Invariance Under R3

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In[•]:=
```
lhs = Simplify[R1[1, j, k] + R1[1, i, k+] + R1[1, i+, j+] //.
    gRules1,j,k ⋃ gRules1,i,k+ ⋃ gRules1,i+,j+ /. Tα_+ → Tα]
rhs = Simplify[R1[1, i, j] + R1[1, i+, k] + R1[1, j+, k+] //.
    gRules1,i,j ⋃ gRules1,i+,k ⋃ gRules1,j+,k+ /. Tα_+ → Tα]
Simplify[lhs == rhs]
```

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$$\frac{1}{2\,T_j\,T_k}\Big(2\,(g_{i^{++},i^{++}}-g_{j^{++},i^{++}})\,(T_k\,(g_{j^{++},i^{++}}-g_{k^{++},i^{++}})+g_{k^{++},i^{++}})-T_j\Big(-2\,(g_{j^{++},i^{++}}+g_{j^{++},j^{++}}-g_{k^{++},i^{++}}-g_{k^{++},j^{++}})\,(g_{k^{++},i^{++}}+g_{k^{++},j^{++}})+T_k\Big(3-2\,g^2_{j^{++},i^{++}}-2\,g_{j^{++},j^{++}}+2\,g_{k^{++},i^{++}}+2\,g_{i^{++},k^{++}}\,g_{k^{++},i^{++}}+2\,g_{j^{++},j^{++}}\,g_{k^{++},i^{++}}-2\,g^2_{k^{++},i^{++}}+2\,g_{k^{++},j^{++}}+2\,g_{j^{++},j^{++}}\,g_{k^{++},j^{++}}+2\,g_{j^{++},k^{++}}\,g_{k^{++},j^{++}}-4\,g_{k^{++},i^{++}}\,g_{k^{++},j^{++}}-2\,g^2_{k^{++},j^{++}}+2\,g_{j^{++},i^{++}}\,(1+g_{i^{++},j^{++}}-2\,g_{j^{++},j^{++}}+g_{k^{++},i^{++}}+g_{k^{++},j^{++}})+2\,g_{j^{++},j^{++}}\,g_{k^{++},k^{++}}-4\,g_{k^{++},i^{++}}\,g_{k^{++},k^{++}}-4\,g_{k^{++},j^{++}}\,g_{k^{++},k^{++}}+2\,g_{i^{++},i^{++}}\,(-2+g_{j^{++},i^{++}}+g_{j^{++},j^{++}}+g_{k^{++},k^{++}})\Big)\Big)\Big)$$

Out[•]=
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$$\frac{1}{2\,T_j\,T_k}\Big(2\,(g_{i^{++},i^{++}}-g_{j^{++},i^{++}})\,(T_k\,(g_{j^{++},i^{++}}-g_{k^{++},i^{++}})+g_{k^{++},i^{++}})-T_j\Big(-2\,(g_{j^{++},i^{++}}+g_{j^{++},j^{++}}-g_{k^{++},i^{++}}-g_{k^{++},j^{++}})\,(g_{k^{++},i^{++}}+g_{k^{++},j^{++}})+T_k\Big(3-2\,g^2_{j^{++},i^{++}}-2\,g_{j^{++},j^{++}}+2\,g_{k^{++},i^{++}}+2\,g_{i^{++},k^{++}}\,g_{k^{++},i^{++}}+2\,g_{j^{++},j^{++}}\,g_{k^{++},i^{++}}-2\,g^2_{k^{++},i^{++}}+2\,g_{k^{++},j^{++}}+2\,g_{j^{++},j^{++}}\,g_{k^{++},j^{++}}+2\,g_{j^{++},k^{++}}\,g_{k^{++},j^{++}}-4\,g_{k^{++},i^{++}}\,g_{k^{++},j^{++}}-2\,g^2_{k^{++},j^{++}}+2\,g_{j^{++},i^{++}}\,(1+g_{i^{++},j^{++}}-2\,g_{j^{++},j^{++}}+g_{k^{++},i^{++}}+g_{k^{++},j^{++}})+2\,g_{j^{++},j^{++}}\,g_{k^{++},k^{++}}-4\,g_{k^{++},i^{++}}\,g_{k^{++},k^{++}}-4\,g_{k^{++},j^{++}}\,g_{k^{++},k^{++}}+2\,g_{i^{++},i^{++}}\,(-2+g_{j^{++},i^{++}}+g_{j^{++},j^{++}}+g_{k^{++},k^{++}})\Big)\Big)\Big)$$

Out[•]=
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True

Invariance Under R2c

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In[•]:=
```
Simplify[R1[-1, i, j+] + R1[1, i+, j] - (gj+,j+ - 1 / 2)]
lhs = Simplify[
  R1[-1, i, j+] + R1[1, i+, j] - (gj+,j+ - 1 / 2) //. gRules-1,i,j+ ⋃ gRules1,i+,j /. Tα_+ → Tα]
```

Out[•]=
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$$\frac{1}{2}-(-1+g_{j,j^+})\,g_{i^+,i^+}+g_{j,i^+}\,(g_{j,j^+}-g_{i^+,j}+g_{j^+,j})+g_{i,i}\,(-1+g_{j^+,j^{++}})-g_{j^+,i}\,(-g_{i,j^+}+g_{j^{++},j^+}+g_{j^+,j^{++}})-g_{j^+,j^+}$$

Out[•]=
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$$\frac{1}{2}-g_{j^{++},j^{++}}$$

Invariance Under R1l

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In[•]:=
```
lhs1 = R1[1, i+, i] - (gi+,i+ - 1 / 2)
lhs2 = lhs1 //. {gi+,β_ :> T^-1 δi+,β + gi++,β, gi,β_ :> δi,β + gi+,β}
Simplify[lhs2]
```

Out[•]=
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$$g_{i,i^+}^2 - g_{i^+,i^+} - (-1 + g_{i,i^+})\, g_{i^+,i^+}$$

Out[•]=
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$$-\frac{1}{T} - g_{i^{++},i^+} - \left(-1 + \frac{1}{T} + g_{i^{++},i^+}\right)\left(\frac{1}{T} + g_{i^{++},i^+}\right) + \left(\frac{1}{T} + g_{i^{++},i^+}\right)^2$$

Out[•]=
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$$0$$

R1r, R2b, and Sw$^+$.

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In[•]:=
```
Simplify[R1[1, i, i+] + (gi+,i+ - 1 / 2) //. {   (* R1r *)
    gi,β_ :> δi,β + T gi+,β + (1 - T) gi++,β, gi+,β_ :> δi+,β + gi++,β,
    gα_,i :> T^-1 (gα,i+ - δα,i+), gα_,i+ :> T gα,i++ - (1 - T) δα,i+ - T δα,i++ }]
```

Out[•]=
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$$0$$

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In[•]:=
```
Simplify[R1[1, i, j] + R1[-1, i+, j+] //. gRules1,i,j ⋃ gRules-1,i+,j+ /. Tα_+ → Tα]   (* R2b *)
```

Out[•]=
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$$0$$

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In[•]:=
```
(gi,i - 1 / 2) + (gj,j - 1 / 2) - (gi+,i+ - 1 / 2) - (gj+,j+ - 1 / 2) //. gRules1,i,j   (* Sw+ *)
```

Out[•]=
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$$0$$