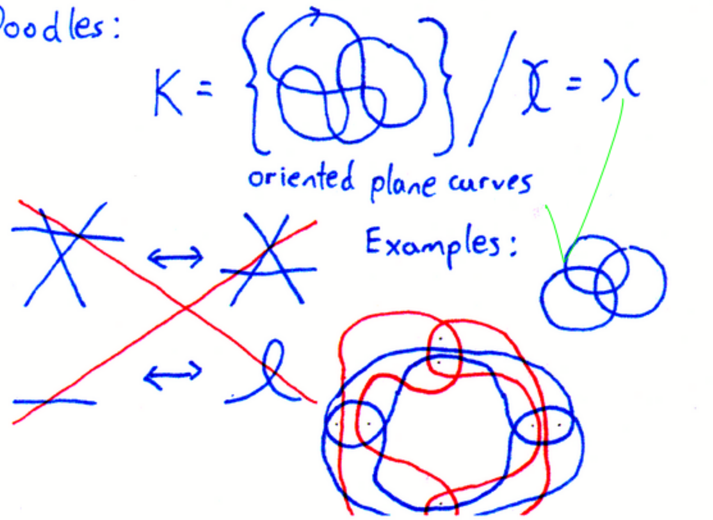
Abstract:

We will study finite-type invariants of doodles (plane curves modulo certain moves). We will construct finite-type invariants using configuration space integrals. While the topology is not difficult, the finite-type theory parallels the rich finite-type theory of knots in 3D.

Doodles:



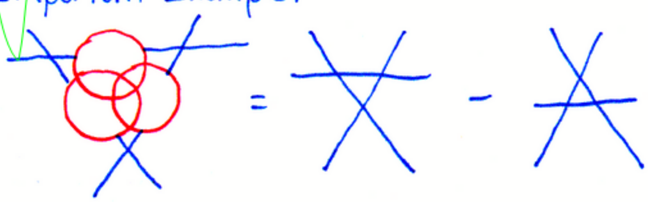
Goussarov Finite Type

n-bracelets

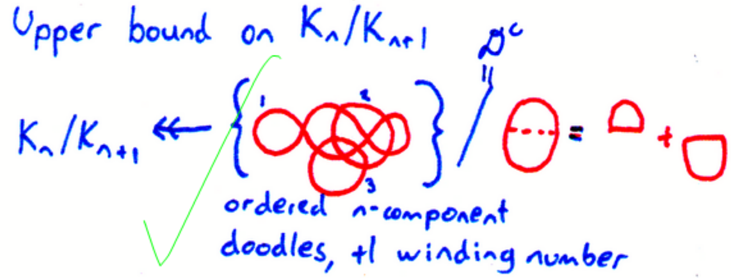
"doodles with detours" (dnd: doodles n' detours)

v is an invariant of type ≤ n if v vanishes on K_{n+1}

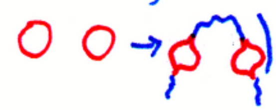
Important Example:

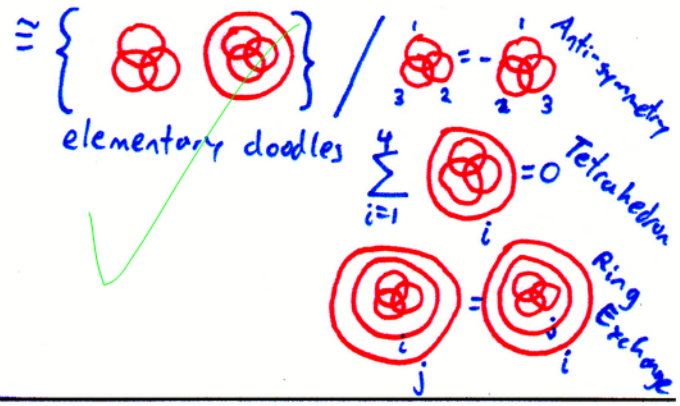


Upper bound on K_n/K_{n+1}

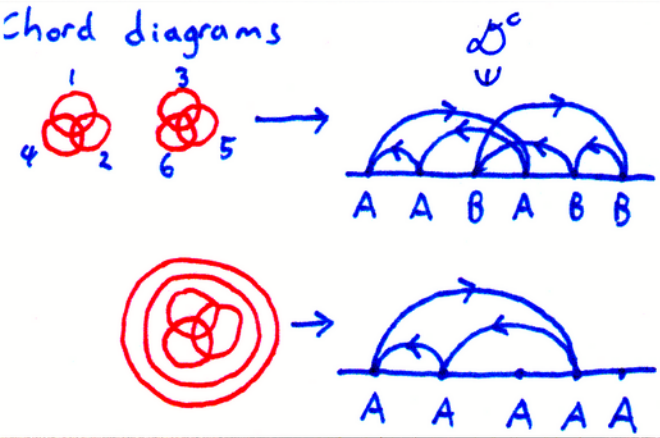


(Modulo K_{n+1}, all ways of connecting bubbles are equivalent.

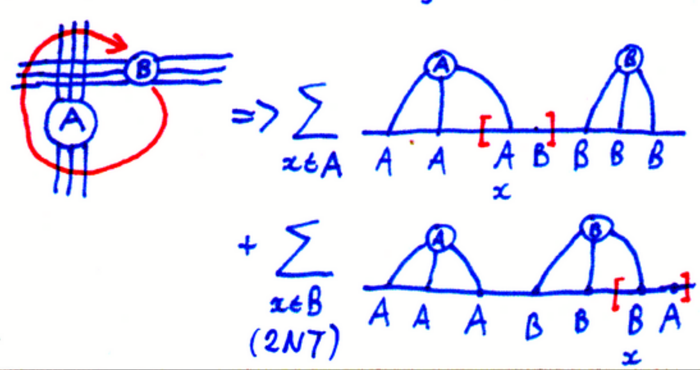




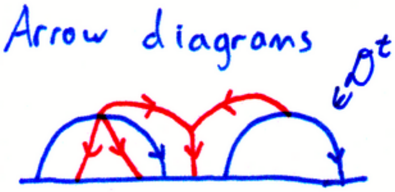
Chord diagrams



Relations on Chord Diagrams



Arrow diagrams



Gauss diagram skeleton.

2-in-1-out internal vertices

$$\begin{tikzcd} \mathcal{D}^c/\text{2NT} \\ \mathcal{A}^c \arrow[u, phantom, "\shortparallel"] \arrow[r, two heads] \arrow[rr, bend right, "T"'] & grK \arrow[r, "gr Z"] & \mathcal{A}^t \arrow[d, "?\ \text{projection map}"] \\ & & \mathcal{A}^c \end{tikzcd}$$

Configuration space integrals

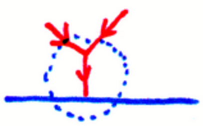
$$Z_{guess} = \sum_{\substack{\text{arrow} \\ \text{diagrams} \\ D}} \oint_{Conf(D)} \Phi^*(\omega')^{\#\text{arrows}} [D]$$

$$\downarrow \Phi$$

$$(\rightarrow, \nearrow, \searrow) \in (S^1)^3$$

$$\mathcal{A}^t = \mathcal{D}^t / \begin{matrix} \text{STU} \\ \text{IHX} \\ \text{foot swap} \\ \text{arrow exchange} \\ \text{R2 invariance} \end{matrix}$$

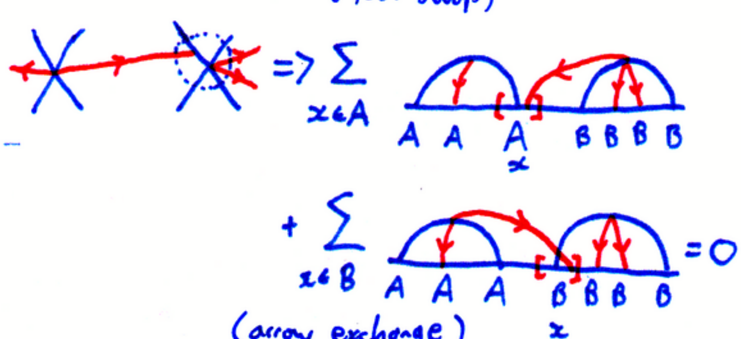
Primary Faces => Relations on Arrow Diagrams



=> STU

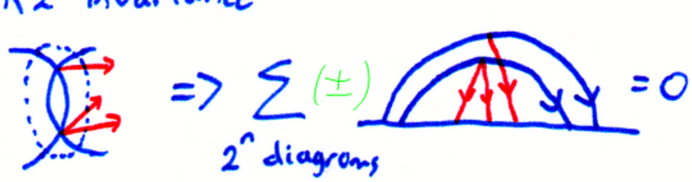
=> IHX

(foot swap)



(arrow exchange)

R2 invariance

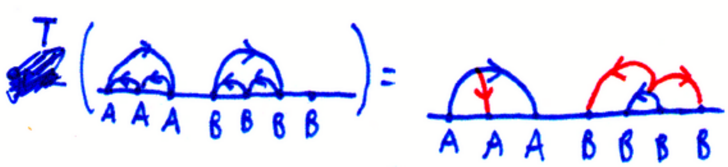


2^n diagrams

Hidden faces vanish

Usual trick: orientation reversing automorphism of Conf(D)

Evaluating Z on n-bracelets



+ 8 other diagrams

Jonathan: It seems that the configuration space integrals we defined are more naturally invariants of virtual doodles. Virtual doodles are doodles with some ordinary crossings and some virtual crossings, with the relation that triple points having three virtual crossings are allowed. (Caution: virtual doodles are not Gauss diagrams modulo Reidemeister 2.)

The integrals we defined are invariants for virtual doodles, if we use the rule that the Gauss diagram skeleton is not allowed to use virtual crossings.

What kinds of chords do our integrals detect? They detect "semi-virtuals with outer rings".

(Conjectured) Punchline: Relations on Feynman diagrams correspond with relations on chord diagrams.

This is just a matter of carefully checking the analogues of the relations we already knew. What makes this work here and not in the original theory is that we have degree 2 chords, the semi-virtuals.

What would be nice is a clean formulation of finite type for virtual doodles yielding chords which are "semi-virtuals with outer rings".