Solving linearized 5-gon in emergent $\mathcal{P}$ and relation to KV

```
SetDirectory["C:/Users/kunoy/Dropbox/MyNotes/Research/Mathematica/WithDBN"];
(* SetDirectory["/Users/kunohome/Dropbox/MyNotes/Research/Mathematica/WithDBN"]; *)
<< FreeLie.m
```

```
FreeLie` implements / extends
 {*, +, **, $SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,
  AllWords, Arbitrator, AS, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop,
  cw, CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE,
  Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,
  Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support,
  t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ι, Λ, σ, τ, ħ, ⎵, ⏜}.

FreeLie` is in the public domain. Dror Bar-Natan is committed
  to support it within reason until July 15, 2022. This is version 240218.
```

Preparation from algebra

The map R

KV equations

(EM) GRT Equations

Experimental Computations

1. Solving linearized pentagon in emergent $\mathcal{P}$

```
SolEMPent[k_] := BasisKer[AllLyndonWords[k, {LW[x], LW[y]}], P3, AssBasis_{x,y}[k - 1]]
```

```
Do[Print[k, " and ", Timing[Length@SolEMPent[k]], L[k]], {k, 1, 16}]
```

1 and {0., 1}{}

2 and {0., 1}{}

3 and {0., 1}{$\overline{\sigma_3}$}

4 and {0., 0}{}

5 and {0., 1}{$\overline{\sigma_5}$}

6 and {0., 0}{}

7 and {0., 1}{$\overline{\sigma_7}$}

8 and {0., 1}{$\overline{\sigma_3\,\sigma_5}$}

9 and {0.015625, 1}{$\overline{\sigma_9}$}

10 and {0.09375, 1}{$\overline{\sigma_3\,\sigma_7}$}

11 and {0.28125, 2}{$\overline{\sigma_{11}}$, $\overline{\sigma_3\ \overline{\sigma_3\,\sigma_5}}$}

12 and {0.671875, 2}{$\overline{\sigma_3\,\sigma_9}$, $\overline{\sigma_5\,\sigma_7}$}

13 and {2.65625, 3}{$\overline{\sigma_{13}}$, $\overline{\sigma_3\ \overline{\sigma_3\,\sigma_7}}$, $\overline{\overline{\sigma_3\,\sigma_5}\,\sigma_5}$}

14 and {138.547, 3}{$\overline{\sigma_3\,\sigma_{11}}$, $\overline{\sigma_5\,\sigma_9}$, $\overline{\sigma_3\ \overline{\sigma_3\ \overline{\sigma_3\,\sigma_5}}}$}

15 and {826.938, 4}{$\overline{\sigma_{15}}$, $\overline{\sigma_3\ \overline{\sigma_3\,\sigma_9}}$, $\overline{\sigma_3\ \overline{\sigma_5\,\sigma_7}}$, $\overline{\overline{\sigma_3\,\sigma_7}\,\sigma_5}$}

16 and {4052.47, 5}{$\overline{\sigma_3\,\sigma_{13}}$, $\overline{\sigma_5\,\sigma_{11}}$, $\overline{\sigma_7\,\sigma_9}$, $\overline{\sigma_3\ \overline{\sigma_3\ \overline{\sigma_3\,\sigma_7}}}$, $\overline{\sigma_3\ \overline{\overline{\sigma_3\,\sigma_5}\,\sigma_5}}$}

2. Checking EM 5-gon implies |R(y,x)x + R(x,y)y| = Duflo

```
RightComposition[R_{x,y}, 𝒮]@SolEMPent[3]
```

$\{-\widehat{xxy}-\widehat{xyy}\}$

```
Timing[RightComposition[R_{x,y}, 𝒮, δ_2]@SolEMPent[8]]
```

{0.1875, {0}}

3. |R(y,x)x + R(x,y)y| is trivial for commutators in Lie($\sigma_{2\,k+1}$)?

```
Timing[diver[ν_8[Part[SolEMPent[8], 1]]]]
```

{0.015625, 0}

```
Timing[RightComposition[R_{x,y}, 𝒮]@SolEMPent[8]]
```

{0.015625, {0}}

4. Checking EM 5-gon implies KV1 and KV2

```
k = 11;
Timing[Table[{CheckSpecial[ν_k[ψ]], δ_2@diver[ν_k[ψ]]}, {ψ, SolEMPent[k]}]]
```

{24.2969, {{0, 0}, {0, 0}}}

5. Checking KV1 + 2-cycle + 3-cycle implies |R(y,x)x + R(x,y)y| = Duflo (the answer is NO.)

```
k = 7;
RightComposition[R_{x,y}, 𝒮, δ_2]@
 BasisKer[BasisKer[SolKV1[k], threecycle_k, AllLyndonWords[k, {LW[x], LW[y]}]],
  RightComposition[Change_k, twist_k], AllLyndonWords[k, {LW[x], LW[y]}]]
```

$$\{45\,\widehat{xxyxxyz}+45\,\widehat{xxyxxzy}+45\,\widehat{xxyxxzz}-45\,\widehat{xxyxyxz}-45\,\widehat{xxyxzxy}-45\,\widehat{xxyxzxz}+45\,\widehat{xxyyxxz}+45\,\widehat{xxyyxyz}+45\,\widehat{xxyyxzy}+45\,\widehat{xxyyxzz}+45\,\widehat{xxyzxxz}+45\,\widehat{xxyzxyy}+45\,\widehat{xxyzxyz}+45\,\widehat{xxyzxzy}+45\,\widehat{xxyzxzz}+45\,\widehat{xxzxxzy}-45\,\widehat{xxzxyxy}-45\,\widehat{xxzxyxz}-45\,\widehat{xxzxyzz}-45\,\widehat{xxzxzxy}+45\,\widehat{xxzxzyz}+45\,\widehat{xxzyxyy}+45\,\widehat{xxzyxyz}+45\,\widehat{xxzyxzy}-45\,\widehat{xxzyyzz}+45\,\widehat{xxzyzxz}+45\,\widehat{xxzyzyz}+45\,\widehat{xxzzxyy}+45\,\widehat{xxzzxzy}-45\,\widehat{xxzzyxz}-45\,\widehat{xxzzyyz}-45\,\widehat{xxzzyzz}-45\,\widehat{xyxyxyz}-45\,\widehat{xyxyxzy}-45\,\widehat{xyxyxzz}-45\,\widehat{xyxyyxz}-45\,\widehat{xyxyzxz}-45\,\widehat{xyxzxyy}-45\,\widehat{xyxzxyz}-45\,\widehat{xyxzxzy}-45\,\widehat{xyxzxzz}-45\,\widehat{xyxzyxz}-45\,\widehat{xyxzzxz}-45\,\widehat{xyyxzxz}-45\,\widehat{xyzxyzz}+45\,\widehat{xyzxzyz}-45\,\widehat{xyzyxzz}-45\,\widehat{xyzyyzz}+45\,\widehat{xyzyzxz}+45\,\widehat{xyzyzyz}-45\,\widehat{xyzzxzz}-45\,\widehat{xyzzyxz}-45\,\widehat{xyzzyyz}-45\,\widehat{xyzzyzz}+45\,\widehat{xzxzyyz}+45\,\widehat{xzxzyzy}+45\,\widehat{xzxzyzz}+45\,\widehat{xzxzzyz}+45\,\widehat{xzyxzyz}-45\,\widehat{xzyxzzy}+45\,\widehat{xzyyzyz}-45\,\widehat{xzyyzzy}+45\,\widehat{xzyzxzz}+45\,\widehat{xzyzyyz}+45\,\widehat{xzyzyzy}+45\,\widehat{xzyzyzz}+45\,\widehat{xzyzzyz}-45\,\widehat{xzzxzzy}-45\,\widehat{xzzyyzy}-45\,\widehat{xzzyyzz}+45\,\widehat{xzzyzyz}-45\,\widehat{xzzyzzy},\ 48\,\widehat{xxyxxyz}+48\,\widehat{xxyxxzy}+48\,\widehat{xxyxxzz}-48\,\widehat{xxyxyxz}-48\,\widehat{xxyxzxy}-48\,\widehat{xxyxzxz}+48\,\widehat{xxyyxxz}+48\,\widehat{xxyyxyz}+48\,\widehat{xxyyxzy}+48\,\widehat{xxyyxzz}+48\,\widehat{xxyzxxz}+48\,\widehat{xxyzxyy}+48\,\widehat{xxyzxyz}+48\,\widehat{xxyzxzy}+48\,\widehat{xxyzxzz}+48\,\widehat{xxzxxzy}-48\,\widehat{xxzxyxy}-48\,\widehat{xxzxyxz}-48\,\widehat{xxzxyzz}-48\,\widehat{xxzxzxy}+48\,\widehat{xxzxzyz}+48\,\widehat{xxzyxyy}+48\,\widehat{xxzyxyz}+48\,\widehat{xxzyxzy}-48\,\widehat{xxzyyzz}+48\,\widehat{xxzyzxz}+48\,\widehat{xxzyzyz}+48\,\widehat{xxzzxyy}+48\,\widehat{xxzzxzy}-48\,\widehat{xxzzyxz}-48\,\widehat{xxzzyyz}-48\,\widehat{xxzzyzz}-48\,\widehat{xyxyxyz}-48\,\widehat{xyxyxzy}-48\,\widehat{xyxyxzz}-48\,\widehat{xyxyyxz}-48\,\widehat{xyxyzxz}-48\,\widehat{xyxzxyy}-48\,\widehat{xyxzxyz}-48\,\widehat{xyxzxzy}-48\,\widehat{xyxzxzz}-48\,\widehat{xyxzyxz}-48\,\widehat{xyxzzxz}-48\,\widehat{xyyxzxz}-48\,\widehat{xyzxyzz}+48\,\widehat{xyzxzyz}-48\,\widehat{xyzyxzz}-48\,\widehat{xyzyyzz}+48\,\widehat{xyzyzxz}+48\,\widehat{xyzyzyz}-48\,\widehat{xyzzxzz}-48\,\widehat{xyzzyxz}-48\,\widehat{xyzzyyz}-48\,\widehat{xyzzyzz}+48\,\widehat{xzxzyyz}+48\,\widehat{xzxzyzy}+48\,\widehat{xzxzyzz}+48\,\widehat{xzxzzyz}+48\,\widehat{xzyxzyz}-48\,\widehat{xzyxzzy}+48\,\widehat{xzyyzyz}-48\,\widehat{xzyyzzy}+48\,\widehat{xzyzxzz}+48\,\widehat{xzyzyyz}+48\,\widehat{xzyzyzy}+48\,\widehat{xzyzyzz}+48\,\widehat{xzyzzyz}-48\,\widehat{xzzxzzy}-48\,\widehat{xzzyyzy}-48\,\widehat{xzzyyzz}+48\,\widehat{xzzyzyz}-48\,\widehat{xzzyzzy}\}$$

```
k = 7;
threecycle_k@SolEMPent[k]
```

$$\{0\}$$