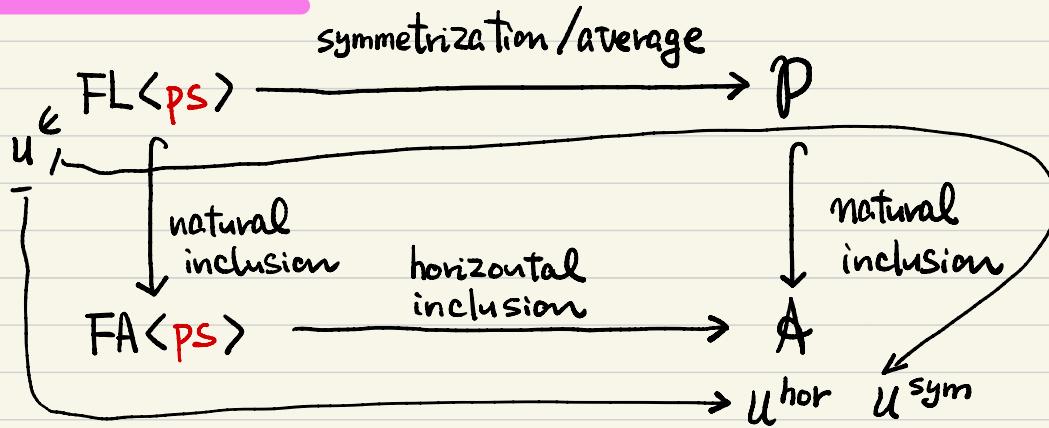


## Question on P



This diagram does not commute. What is  $u^{\text{sym}} - u^{\text{hor}}$ ?

In what follows, I'll do some sample computations, and give a formula to compute it recursively ( see the last two pages ).

$$\textcircled{1} \quad \mathcal{U} = [x, [x, y]]$$

$$\mathcal{U}^{\text{sym}} = \frac{1}{2} \left( \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} + \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right)$$

$$= \frac{1}{2} \left( \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} - \begin{array}{c} \text{Diagram 7} \\ \text{Diagram 8} \end{array} + \begin{array}{c} \text{Diagram 9} \\ \text{Diagram 10} \end{array} - \begin{array}{c} \text{Diagram 11} \\ \text{Diagram 12} \end{array} \right)$$

$$= \frac{1}{2} \left( \begin{array}{c} \text{Diagram 13} \\ \text{Diagram 14} \\ \text{Diagram 15} \\ \text{Diagram 16} \end{array} - \begin{array}{c} \text{Diagram 17} \\ \text{Diagram 18} \\ \text{Diagram 19} \\ \text{Diagram 20} \end{array} - \begin{array}{c} \text{Diagram 21} \\ \text{Diagram 22} \\ \text{Diagram 23} \\ \text{Diagram 24} \end{array} + \begin{array}{c} \text{Diagram 25} \\ \text{Diagram 26} \\ \text{Diagram 27} \\ \text{Diagram 28} \end{array} \right) \\ + \left( \begin{array}{c} \text{Diagram 29} \\ \text{Diagram 30} \\ \text{Diagram 31} \\ \text{Diagram 32} \end{array} - \begin{array}{c} \text{Diagram 33} \\ \text{Diagram 34} \\ \text{Diagram 35} \\ \text{Diagram 36} \end{array} - \begin{array}{c} \text{Diagram 37} \\ \text{Diagram 38} \\ \text{Diagram 39} \\ \text{Diagram 40} \end{array} + \begin{array}{c} \text{Diagram 41} \\ \text{Diagram 42} \\ \text{Diagram 43} \\ \text{Diagram 44} \end{array} \right)$$

$$= \frac{1}{2} \left( \text{Diagram 1} - \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} - \text{Diagram 5} + \text{Diagram 6} \right)$$

$$\text{Diagram 1} - \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} - \text{Diagram 5} + \text{Diagram 6}$$

$$= u^{\text{hor}} + \frac{1}{2} \left( \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} - \text{Diagram 4} \right)$$

$$\text{Diagram 1} + \text{Diagram 2}$$

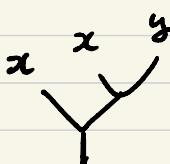
$$= yx + xy \in \text{FA}\langle \text{ps} \rangle \subset P$$

Conclusion:

$$u = [x, [x, y]]$$

$$u^{\text{sym}} - u^{\text{hor}}$$

$$= \frac{1}{2}(yx + xy)$$



$$\textcircled{2} \quad u = [x, y], [x, z]$$

$$2u^{\text{sym}} = \text{Diagram 1} + \text{Diagram 2} = \text{Diagram 3} - \text{Diagram 4} + \text{Diagram 5} - \text{Diagram 6}$$

$$= \text{Diagram 7} - \text{Diagram 8} - \text{Diagram 9} + \text{Diagram 10} + \text{Diagram 11} - \text{Diagram 12} - \text{Diagram 13} + \text{Diagram 14}$$

$$= \text{Diagram 15} - \text{Diagram 16} - \text{Diagram 17} + \text{Diagram 18} - \text{Diagram 19} + \text{Diagram 20} + \text{Diagram 21} - \text{Diagram 22}$$

$$+ \text{Diagram 23} - \text{Diagram 24} - \text{Diagram 25} + \text{Diagram 26} - \text{Diagram 27} + \text{Diagram 28} + \text{Diagram 29} - \text{Diagram 30}$$

$$= \text{Diagram 31} - \text{Diagram 32} - \text{Diagram 33} + \text{Diagram 34} - \text{Diagram 35} + \text{Diagram 36} + \text{Diagram 37} - \text{Diagram 38}$$

$$+ \text{Diagram 39} - \text{Diagram 40} - \text{Diagram 41} + \text{Diagram 42} - \text{Diagram 43} + \text{Diagram 44} + \text{Diagram 45} - \text{Diagram 46}$$

$$= 2\mathcal{U}^{\text{hor}} + \left( - \begin{array}{c} (1) \\ | \\ \text{Diagram} \\ | \\ (2) \end{array} + \begin{array}{c} (2) \\ | \\ \text{Diagram} \\ | \\ (3) \end{array} + \begin{array}{c} (3) \\ | \\ \text{Diagram} \\ | \\ (4) \end{array} - \begin{array}{c} (4) \\ | \\ \text{Diagram} \\ | \\ (1) \end{array} \right. \\ \left. + \begin{array}{c} (5) \\ | \\ \text{Diagram} \\ | \\ (6) \end{array} - \begin{array}{c} (6) \\ | \\ \text{Diagram} \\ | \\ (7) \end{array} - \begin{array}{c} (7) \\ | \\ \text{Diagram} \\ | \\ (8) \end{array} + \begin{array}{c} (8) \\ | \\ \text{Diagram} \\ | \\ (5) \end{array} \right)$$

$$\begin{array}{cccc} (1)(2) & (3)(4) & (5)(6) & (7)(8) \end{array}$$

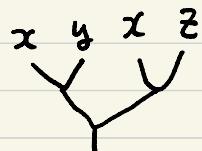
$$= \begin{array}{cccc} \text{Diagram} & - & \text{Diagram} & + \end{array} \begin{array}{cccc} \text{Diagram} & - & \text{Diagram} & \end{array}$$

$$= \begin{array}{ccccc} \text{Diagram} & - & \text{Diagram} & = & zxy - yxz \end{array}$$

Conclusion:

$$\mathcal{U} = [[x, y], [x, z]]$$

$$\mathcal{U}^{\text{sym}} - \mathcal{U}^{\text{hor}} = \frac{1}{2}(zxy - yxz)$$



$$③ \mathcal{U} = [x, [y, [x, z]]]$$

$$2\mathcal{U}^{\text{sym}} = \text{Diagram 1} + \text{Diagram 2} = \text{Diagram 3} - \text{Diagram 4} + \text{Diagram 5} - \text{Diagram 6}$$

$$= \text{Diagram 7} - \text{Diagram 8} - \text{Diagram 9} + \text{Diagram 10} + \text{Diagram 11} - \text{Diagram 12} - \text{Diagram 13} + \text{Diagram 14}$$

$$= \text{Diagram 15} - \text{Diagram 16} - \text{Diagram 17} + \text{Diagram 18} - \text{Diagram 19} + \text{Diagram 20} + \text{Diagram 21} - \text{Diagram 22}$$

$$+ \text{Diagram 23} - \text{Diagram 24} - \text{Diagram 25} + \text{Diagram 26} - \text{Diagram 27} + \text{Diagram 28} + \text{Diagram 29} - \text{Diagram 30}$$

$$= \text{Diagram 31} - \text{Diagram 32} - \text{Diagram 33} + \text{Diagram 34} - \text{Diagram 35} + \text{Diagram 36} + \text{Diagram 37} - \text{Diagram 38}$$

$$+ \text{Diagram 39} - \text{Diagram 40} - \text{Diagram 41} + \text{Diagram 42} - \text{Diagram 43} + \text{Diagram 44} + \text{Diagram 45} - \text{Diagram 46}$$

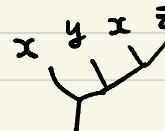
$$= 2\mathcal{U}^{\text{hor}} + \left( -\begin{array}{c} (1) \\ | \\ \text{---} \end{array} + \begin{array}{c} (2) \\ | \\ \text{---} \end{array} + \begin{array}{c} (3) \\ | \\ \text{---} \end{array} - \begin{array}{c} (4) \\ | \\ \text{---} \end{array} \right. \\ \left. + \begin{array}{c} (5) \\ | \\ \text{---} \end{array} - \begin{array}{c} (6) \\ | \\ \text{---} \end{array} - \begin{array}{c} (7) \\ | \\ \text{---} \end{array} + \begin{array}{c} (8) \\ | \\ \text{---} \end{array} \right)$$



$$\begin{array}{c} (1)(2) \\ | \\ \text{---} \end{array} + \begin{array}{c} (3)(4) \\ | \\ \text{---} \end{array} + \begin{array}{c} (5)(6) \\ | \\ \text{---} \end{array} + \begin{array}{c} (7)(8) \\ | \\ \text{---} \end{array} = \begin{array}{c} (1) \\ | \\ \text{---} \end{array} - \begin{array}{c} (2) \\ | \\ \text{---} \end{array} + \begin{array}{c} (3) \\ | \\ \text{---} \end{array} - \begin{array}{c} (4) \\ | \\ \text{---} \end{array}$$

Conclusion:

$$\mathcal{U}^{\text{sym}} - \mathcal{U}^{\text{hor}} = \frac{1}{2} (yzx - xzy)$$



$$\begin{aligned} &= \begin{array}{c} (1) \\ | \\ \text{---} \end{array} + \begin{array}{c} (2) \\ | \\ \text{---} \end{array} \\ &= \begin{array}{c} (1) \\ | \\ \text{---} \end{array} - \begin{array}{c} (2) \\ | \\ \text{---} \end{array} = yzx - xzy \end{aligned}$$

(changed sign!)

Let  $R(u) := u^{\text{hor}} - u^{\text{sym}}$ . We claim that  $R(u) \in \text{FA}(\text{ps})[1] \subset \mathcal{P}$  and

Prop  $R([u, v]) = [u, R(v)] + [R(u), v]$

$$+ \frac{1}{2} \sum_i ((\partial_i v) x_i (\partial_i u)^* - (\partial_i u) x_i (\partial_i v)^*)$$

$R(x) = R(y) = 0$

proof

$$[u, v]^{\text{sym}} = \begin{array}{c} \text{Diagram showing } u \text{ and } v \text{ as parallel wires with a crossing} \\ \text{between them.} \end{array} = \begin{array}{c} \text{Diagram showing } u \text{ and } v \text{ as parallel wires with a crossing} \\ \text{between them, plus a green arrow pointing up indicating a } \\ \text{contribution from the diagram above.} \end{array} + \frac{1}{2} \sum_i ((\partial_i u) x_i (\partial_i v)^* + (\partial_i v) x_i (\partial_i u)^*)$$

See 240117

$$\begin{array}{c} \text{Diagram showing } u \text{ and } v \text{ as parallel wires with a crossing} \\ \text{between them, minus the diagram above it.} \end{array} = \begin{array}{c} \text{Diagram showing } u \text{ and } v \text{ as parallel wires with a crossing} \\ \text{between them, minus the diagram above it, plus a green } \\ \text{brace indicating the cancellation of the terms from the } \\ \text{diagram above.} \end{array} - \sum_i (\partial_i v) x_i (\partial_i u)^*$$

$$\xrightarrow{\sim} [u, v]^{\text{sym}} = \left| \begin{array}{c} \cancel{\#}(v) \\ \cancel{\#}(u) \end{array} \right| - \left| \begin{array}{c} \cancel{\#}(u) \\ \cancel{\#}(v) \end{array} \right| + \frac{1}{2} \sum_i \left( (\partial_i u) x_i (\partial_i v)^* - (\partial_i v) x_i (\partial_i u)^* \right)$$

$$= [u^{\text{sym}}, v^{\text{sym}}] + \frac{1}{2} \sum_i \left( (\partial_i u) x_i (\partial_i v)^* - (\partial_i v) x_i (\partial_i u)^* \right)$$

$$\left[ u^{\text{hor}} - R(u), v^{\text{hor}} - R(v) \right] = [u^{\text{hor}}, v^{\text{hor}}] - [u, R(v)] - [R(u), v]$$

$$= [u, v]^{\text{hor}} - [u, R(v)] - [R(u), v]$$

$\rightsquigarrow$  get the formula for  $R([u, v]) = [u, v]^{\text{hor}} - [u, v]^{\text{sym}}$

//

I still don't know a closed formula....