Pensieve header: Solving the PR^4 relation degree by degree.

Startup

In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\People\\Frohlich"];
PP_ := Identity;
<< "../../Projects/SL2Portfolio2/Engine-Speedy.m";
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, , ]];
```

The Objects

Symmetric Algebra Objects

In[•]:=
```
sm_{i_,j_→k_} := 𝔼_{{i,j}→{k}}[b_k (β_i + β_j) + t_k (τ_i + τ_j) + a_k (α_i + α_j) + y_k (η_i + η_j) + x_k (ξ_i + ξ_j)];
sΔ_{i_→j_,k_} := 𝔼_{{i}→{j,k}}[β_i (b_j + b_k) + τ_i (t_j + t_k) + α_i (a_j + a_k) + η_i (y_j + y_k) + ξ_i (x_j + x_k)];
sS_{i_} := 𝔼_{{i}→{i}}[-β_i b_i - τ_i t_i - α_i a_i - η_i y_i - ξ_i x_i];
sη_{i_} := 𝔼_{{}→{i}}[0];
sϵ_{i_} := 𝔼_{{i}→{}}[0];
```

In[•]:=
```
sσ_{i_→j_} := 𝔼_{{i}→{j}}[β_i b_j + τ_i t_j + α_i a_j + η_i y_j + ξ_i x_j];
sϒ_{i_→j_,k_,l_,m_} := 𝔼_{{i}→{j,k,l,m}}[β_i b_k + τ_i t_k + α_i a_l + η_i y_j + ξ_i x_m];
```

The CU Definitions

In[•]:=
```
cΛ = (η_i + ℯ^(-γ α_i - ϵ β_i) η_j / (1 + γ ϵ η_j ξ_i)) y_k + (β_i + β_j + Log[1 + γ ϵ η_j ξ_i] / ϵ) b_k +
     (α_i + α_j + Log[1 + γ ϵ η_j ξ_i] / γ) a_k + (ℯ^(-γ α_j - ϵ β_j) ξ_i / (1 + γ ϵ η_j ξ_i) + ξ_j) x_k;
Define[cm_{i,j→k} = 𝔼_{{i,j}→{k}}[cΛ]]
```

In[•]:=
```
Define[cσ_{i→j} = sσ_{i,j} /. τ_i → 0, cϵ_i = sϵ_i, cη_i = sη_i, cΔ_{i→j,k} = sΔ_{i→j,k},
  cS_i = sS_i // sϒ_{i→1,2,3,4} // cm_{4,3→i} // cm_{i,2→i} // cm_{i,1→i}];
```

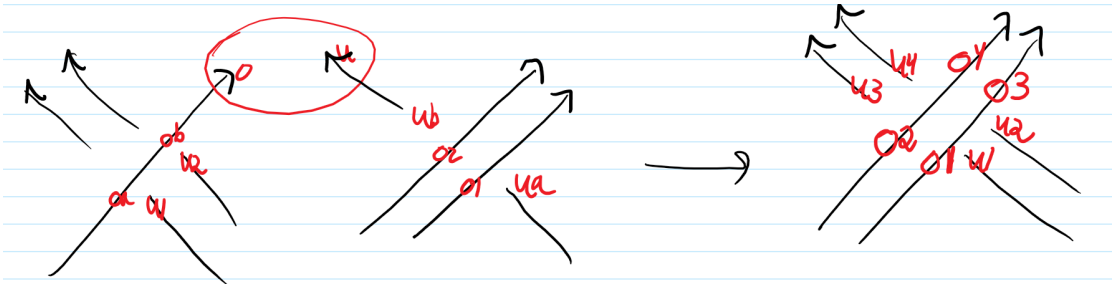
In[•]:=
```
$k = 1; (*ħ=γ=1;*)
```

Booting Up QU

In[•]:=
```
Define[aσ_{i→j} = 𝔼_{{i}→{j}}[a_j α_i + x_j ξ_i], bσ_{i→j} = 𝔼_{{i}→{j}}[b_j β_i + y_j η_i]]
```

```
In[•]:= Define[am_{i,j→k} = 𝔼_{{i,j}→{k}}[(α_i + α_j) a_k + (𝓡_j^{-1} ξ_i + ξ_j) x_k],
          bm_{i,j→k} = 𝔼_{{i,j}→{k}}[(β_i + β_j) b_k + (η_i + e^{-ϵ β_i} η_j) y_k]]
```

The PR^4 Equation




```
rhs = (R_{o1,u1} R_{o3,u2} R_{o2,u3} R_{o4,u4}) // (bm_{o1,o3→o1} bm_{o2,o4→o2} am_{u1,u3→u1} am_{u2,u4→u2})
```

```
Out[•]= 𝔼_{{}→{o1,o2,u1,u2}}[ħ a_{u1} b_{o1} + ħ a_{u2} b_{o1} + ħ a_{u1} b_{o2} + ħ a_{u2} b_{o2}, ħ B_{o2} x_{u1} y_{o1} + ħ B_{o2} x_{u2} y_{o1} + ħ x_{u1} y_{o2} + ħ x_{u2} y_{o2},
  1 + (-ħ^2 a_{u1} B_{o2} x_{u2} y_{o1} - 1/4 γ ħ^3 B_{o2}^2 x_{u1}^2 y_{o1}^2 - 1/4 γ ħ^3 B_{o2}^2 x_{u2}^2 y_{o1}^2 -
      ħ^2 a_{u1} x_{u2} y_{o2} + γ ħ^3 B_{o2} x_{u1} x_{u2} y_{o1} y_{o2} - 1/4 γ ħ^3 x_{u1}^2 y_{o2}^2 - 1/4 γ ħ^3 x_{u2}^2 y_{o2}^2) ϵ + O[ϵ]^2]
```

```
In[•]:= lhs = (R_{oa,u1} R_{ob,u2} // bm_{oa,ob→o}) (R_{o1,ua} R_{o2,ub} // am_{ua,ub→u}) // P_{o,u};
```

```
In[•]:= lhs ≡ rhs
```

```
Out[•]= True
```

Solving the PR^4 Equation

```
In[•]:= Invert[R_, k_]_{i_,j_} := Module[{n1, n2},
          𝔼_{{i,j}→{}}[β_i α_j / ħ, η_i ξ_j / ħ, 1 + If[k == 0, 0, (Invert[R, k - 1]_{i,j})_k[3] -
              (R_{n1,n2} // ((Invert[R, 0]_{n1,j})_k (Invert[R, k - 1]_{i,n2})_k))[3]]]
```

```
In[•]:= RR[0]_{i_,j_} := 𝔼_{}→{i,j}[ħ a_j b_i, ħ x_j y_i, 1];
        PPP[0]_{i_,j_} := Invert[RR[0], 0]_{i,j};
```

```
In[•]:= Eq[k_] :=
         Last[(RR[k]_{o1,u1} RR[k]_{o3,u2} RR[k]_{o2,u3} RR[k]_{o4,u4}) // (bm_{o1,o3→o1} bm_{o2,o4→o2} am_{u1,u3→u1} am_{u2,u4→u2})] -
         Last[(RR[k]_{oa,u1} RR[k]_{ob,u2} // bm_{oa,ob→o}) (RR[k]_{o1,ua} RR[k]_{o2,ub} // am_{ua,ub→u}) // PPP[k]_{o,u}]
```

```
In[•]:= (RR[0]_{o1,u1} RR[0]_{o3,u2} RR[0]_{o2,u3} RR[0]_{o4,u4}) // (bm_{o1,o3→o1})
```

```
Out[•]= 𝔼_{{}→{o1,o2,o4,u1,u2,u3,u4}}[ħ a_{u1} b_{o1} + ħ a_{u2} b_{o1} + ħ a_{u3} b_{o2} + ħ a_{u4} b_{o4},
  ħ x_{u1} y_{o1} + ħ x_{u2} y_{o1} + ħ x_{u3} y_{o2} + ħ x_{u4} y_{o4}, 1 - ħ^2 a_{u1} x_{u2} y_{o1} ϵ + O[ϵ]^2]
```

In[•]:=
```
bm_{i,k→k}
```
Out[•]=
$$\mathbb{E}_{\{i,k\}\to\{k\}}\left[b_k\,\beta_i+b_k\,\beta_k,\ y_k\,\eta_i+y_k\,\eta_k,\ 1-y_k\,\beta_i\,\eta_k\,\epsilon+O[\epsilon]^2\right]$$

In[•]:=
```
Block[{$k = 0}, Eq[0]]
```
Out[•]=
$$O[\epsilon]^1$$

In[•]:=
```
Block[{$k = 1}, Eq[0]]
```
Out[•]=
$$\gamma\,\hbar^3\,B_{o2}\,x_{u1}\,x_{u2}\,y_{o1}\,y_{o2}\,\epsilon+O[\epsilon]^2$$

In[•]:=
```
RR[1]_{i_,j_} := 𝔼_{}→{i,j}[ħ a_j b_i, ħ x_j y_i,
    1 + ϵ (f_1[b_i] + f_2[b_i] y_i x_j + f_3[b_i] y_i^2 x_j^2 + f_4[b_i] a_j + f_5[b_i] a_j y_i x_j + f_6[b_i] a_j^2) + O[ϵ]^2];
PPP[1]_{i_,j_} := Invert[RR[1], 1]_{i,j};
PPP[1]_{i,j}
```
Out[•]=
$$\mathbb{E}_{\{i,j\}\to\{\}}\left[\frac{\alpha_j\,\beta_i}{\hbar},\ \frac{\eta_i\,\xi_j}{\hbar},\ 1+\left(-f_1\left[\frac{\alpha_j}{\hbar}\right]-\frac{\eta_i\,\xi_j\,f_2\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^2}-\frac{\eta_i^2\,\xi_j^2\,f_3\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^4}-\frac{\beta_i\,f_4\left[\frac{\alpha_j}{\hbar}\right]}{\hbar}-\frac{\beta_i\,\eta_i\,\xi_j\,f_5\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^3}-\frac{\beta_i^2\,f_6\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^2}\right)\epsilon+O[\epsilon]^2\right]$$

In[•]:=
```
Block[{$k = 1}, RR[1]_{i,j} // PPP[1]_{i,k}]
```
Out[•]=
$$\mathbb{E}_{\{k\}\to\{j\}}\left[a_j\,\alpha_k,\ x_j\,\xi_k,\ 1+O[\epsilon]^2\right]$$

In[•]:=
```
Eq[1] // FullSimplify
```
Out[•]=
$$\begin{aligned}&\left(f_1[b_{o1}]+f_1[b_{o2}]-f_1[b_{o1}+b_{o2}]+\right.\\&\quad x_{u1}\,y_{o2}\left(2\,x_{u2}\,y_{o2}\left(-f_3[b_{o2}]+f_3[b_{o1}+b_{o2}]\right)+a_{u2}\left(-f_5[b_{o2}]+f_5[b_{o1}+b_{o2}]\right)\right)-\\&\quad 2\,B_{o2}^2\,x_{u1}\,x_{u2}\,y_{o1}^2\left(f_3[b_{o1}]-f_3[b_{o1}+b_{o2}]+\gamma^2\,\hbar^2\,f_6[b_{o2}]\right)+\\&\quad B_{o2}\,y_{o1}\left(a_{u1}\,x_{u2}\left(-f_5[b_{o1}]+f_5[b_{o1}+b_{o2}]+2\,\gamma\,\hbar\,f_6[b_{o2}]\right)+x_{u1}\left(x_{u2}\,y_{o2}\right.\right.\\&\qquad\left.\left.\left(\gamma\,\hbar^3+4\,f_3[b_{o1}+b_{o2}]+2\,\gamma\,\hbar\,f_5[b_{o2}]\right)+a_{u2}\left(-f_5[b_{o1}]+f_5[b_{o1}+b_{o2}]+2\,\gamma\,\hbar\,f_6[b_{o2}]\right)\right)\right)+\\&\quad a_{u1}\left(x_{u2}\,y_{o2}\left(-f_5[b_{o2}]+f_5[b_{o1}+b_{o2}]\right)-2\,a_{u2}\left(f_6[b_{o1}]+f_6[b_{o2}]-f_6[b_{o1}+b_{o2}]\right)\right)\Big)\,\epsilon+O[\epsilon]^2\end{aligned}$$

In[•]:=
```
CoefficientRules[
 Simplify@SeriesCoefficient[Block[{$k = 1}, Eq[1]], 1], {a_u1, a_u2, x_u1, x_u2, y_o1, y_o2}]
```
Out[•]=
$$\begin{aligned}&\{\{1,1,0,0,0,0\}\to-2\,f_6[b_{o1}]-2\,f_6[b_{o2}]+2\,f_6[b_{o1}+b_{o2}],\\&\{1,0,0,1,1,0\}\to-B_{o2}\,f_5[b_{o1}]+B_{o2}\,f_5[b_{o1}+b_{o2}]+2\,\gamma\,\hbar\,B_{o2}\,f_6[b_{o2}],\\&\{1,0,0,1,0,1\}\to-f_5[b_{o2}]+f_5[b_{o1}+b_{o2}],\\&\{0,1,1,0,1,0\}\to-B_{o2}\,f_5[b_{o1}]+B_{o2}\,f_5[b_{o1}+b_{o2}]+2\,\gamma\,\hbar\,B_{o2}\,f_6[b_{o2}],\\&\{0,1,1,0,0,1\}\to-f_5[b_{o2}]+f_5[b_{o1}+b_{o2}],\\&\{0,0,1,1,2,0\}\to-2\,B_{o2}^2\,f_3[b_{o1}]+2\,B_{o2}^2\,f_3[b_{o1}+b_{o2}]-2\,\gamma^2\,\hbar^2\,B_{o2}^2\,f_6[b_{o2}],\\&\{0,0,1,1,1,1\}\to\gamma\,\hbar^3\,B_{o2}+4\,B_{o2}\,f_3[b_{o1}+b_{o2}]+2\,\gamma\,\hbar\,B_{o2}\,f_5[b_{o2}],\\&\{0,0,1,1,0,2\}\to-2\,f_3[b_{o2}]+2\,f_3[b_{o1}+b_{o2}],\\&\{0,0,0,0,0,0\}\to f_1[b_{o1}]+f_1[b_{o2}]-f_1[b_{o1}+b_{o2}]\}\end{aligned}$$

In[•]:=
```
SolveAlways[Replace[x_ → x == 0] /@ Values@CoefficientRules[
    Simplify@SeriesCoefficient[Block[{$k = 1}, Eq[1]], 1], {au1, au2, xu1, xu2, yo1, yo2}],
 {bo1, Bo1, bo2, Bo2, bo4, Bo4, bu1, Bu1, bu2, Bu2, bu3, Bu3, bu4, Bu4}]
```

... InverseFunction: Inverse functions are being used. Values may be lost for multivalued inverses.

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... InverseFunction: Inverse functions are being used. Values may be lost for multivalued inverses.

... General: Further output of InverseFunction::ifun will be suppressed during this calculation.

Out[•]=
$$\left\{\left\{f_1[b_{o1}] \to -f_1[b_{o2}] + f_1[b_{o1}+b_{o2}],\right.\right.$$
$$f_3[b_{o2}] \to \frac{1}{4}\left(-\gamma\hbar^3 - 2\gamma\hbar f_5[b_{o2}]\right),\ f_5[b_{o1}] \to f_5[b_{o2}] + 2\gamma\hbar f_6[b_{o2}],$$
$$f_3[b_{o1}] \to -\frac{1}{4}\gamma\hbar\left(\hbar^2 + 2f_5[b_{o2}] + 4\gamma\hbar f_6[b_{o2}]\right),\ f_6[b_{o1}] \to -f_6[b_{o2}] + f_6[b_{o1}+b_{o2}],$$
$$\left.\left.f_5[b_{o1}+b_{o2}] \to f_5[b_{o2}],\ f_3[b_{o1}+b_{o2}] \to \frac{1}{4}\left(-\gamma\hbar^3 - 2\gamma\hbar f_5[b_{o2}]\right)\right\}\right\}$$

In[•]:=
```
Values@CoefficientRules[
    Simplify@SeriesCoefficient[Block[{$k = 1}, Eq[1]] /. f1|2|4|5|6[_] → 0, 1],
    {au1, au2, xu1, xu2, yo1, yo2}] // Column
```

$$-2B_{o2}^2 f_3[b_{o1}] + 2B_{o2}^2 f_3[b_{o1}+b_{o2}]$$

Out[•]=
$$\gamma\hbar^3 B_{o2} + 4B_{o2} f_3[b_{o1}+b_{o2}]$$
$$-2f_3[b_{o2}] + 2f_3[b_{o1}+b_{o2}]$$

In[•]:=
```
CoefficientRules[Simplify@
    SeriesCoefficient[Block[{$k = 1}, Eq[1]] /. {f1|4|5|6[_] → 0, f3[_] → -γ ħ^3/4}, 1],
  {au1, au2, xu1, xu2, yo1, yo2}] // Column
```

Out[•]=

In[•]:=
```
RR[1]i_,j_ := 𝔼{}→{i,j}[ħ aj bi, ħ xj yi,
  1 + ϵ (f1[bi] + f2[bi] yi xj + f3[bi] yi^2 xj^2 + f4[bi] aj + f5[bi] aj yi xj + f6[bi] aj^2) + O[ϵ]^2 /.
   {f1|4|5|6[_] → 0, f3[_] → -γ ħ^3/4, f2[_] → 0}];
RR[1]i,j
```

Out[•]=
$$\mathbb{E}_{\{\}\to\{i,j\}}\left[\hbar\, a_j\, b_i,\ \hbar\, x_j\, y_i,\ 1 - \frac{1}{4}\left(\gamma\hbar^3 x_j^2 y_i^2\right)\epsilon + O[\epsilon]^2\right]$$

In[•]:=
```
RR[2][i_,j_] :=
  𝔼{}→{i,j}[ħ a_j b_i, ħ x_j y_i, 1 - 1/4 (γ ħ^3 x_j^2 y_i^2) ϵ + (f0,0[b_i] + a_j f0,1[b_i] + a_j^2 f0,2[b_i] + a_j^3 f0,3[b_i] +
      a_j^4 f0,4[b_i] + a_j^5 f0,5[b_i] + a_j^6 f0,6[b_i] + x_j y_i f1,0[b_i] + a_j x_j y_i f1,1[b_i] +
      a_j^2 x_j y_i f1,2[b_i] + a_j^3 x_j y_i f1,3[b_i] + a_j^4 x_j y_i f1,4[b_i] + a_j^5 x_j y_i f1,5[b_i] + x_j^2 y_i^2 f2,0[b_i] +
      a_j x_j^2 y_i^2 f2,1[b_i] + a_j^2 x_j^2 y_i^2 f2,2[b_i] + a_j^3 x_j^2 y_i^2 f2,3[b_i] + a_j^4 x_j^2 y_i^2 f2,4[b_i] +
      x_j^3 y_i^3 f3,0[b_i] + a_j x_j^3 y_i^3 f3,1[b_i] + a_j^2 x_j^3 y_i^3 f3,2[b_i] + a_j^3 x_j^3 y_i^3 f3,3[b_i]) ϵ^2 + O[ϵ]^3];
PPP[2][i_,j_] := Invert[RR[2], 2][i,j];
PPP[2][i,j]
```

Out[•]=
$$\mathbb{E}_{\{i,j\}\to\{\}}\Big[\frac{\alpha_j\,\beta_i}{\hbar},\ \frac{\eta_i\,\xi_j}{\hbar},$$
$$1+\frac{\gamma\,\eta_i^2\,\xi_j^2\,\epsilon}{4\,\hbar}+\left(\frac{1}{8}\gamma^2\,\eta_i^2\,\xi_j^2+\frac{\gamma^2\,\eta_i^3\,\xi_j^3}{4\,\hbar}+\frac{\gamma^2\,\eta_i^4\,\xi_j^4}{16\,\hbar^2}-\mathsf{f}_{0,0}\Big[\frac{\alpha_j}{\hbar}\Big]-\frac{\beta_i\,\mathsf{f}_{0,1}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar}-\frac{\beta_i^2\,\mathsf{f}_{0,2}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^2}-\right.$$
$$\frac{\beta_i^3\,\mathsf{f}_{0,3}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^3}-\frac{\beta_i^4\,\mathsf{f}_{0,4}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^4}-\frac{\beta_i^5\,\mathsf{f}_{0,5}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^5}-\frac{\beta_i^6\,\mathsf{f}_{0,6}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^6}-\frac{\eta_i\,\xi_j\,\mathsf{f}_{1,0}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^2}-\frac{\beta_i\,\eta_i\,\xi_j\,\mathsf{f}_{1,1}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^3}-$$
$$\frac{\beta_i^2\,\eta_i\,\xi_j\,\mathsf{f}_{1,2}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^4}-\frac{\beta_i^3\,\eta_i\,\xi_j\,\mathsf{f}_{1,3}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^5}-\frac{\beta_i^4\,\eta_i\,\xi_j\,\mathsf{f}_{1,4}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^6}-\frac{\beta_i^5\,\eta_i\,\xi_j\,\mathsf{f}_{1,5}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^7}-\frac{\eta_i^2\,\xi_j^2\,\mathsf{f}_{2,0}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^4}-$$
$$\frac{\beta_i\,\eta_i^2\,\xi_j^2\,\mathsf{f}_{2,1}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^5}-\frac{\beta_i^2\,\eta_i^2\,\xi_j^2\,\mathsf{f}_{2,2}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^6}-\frac{\beta_i^3\,\eta_i^2\,\xi_j^2\,\mathsf{f}_{2,3}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^7}-\frac{\beta_i^4\,\eta_i^2\,\xi_j^2\,\mathsf{f}_{2,4}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^8}-$$
$$\left.\left.\frac{\eta_i^3\,\xi_j^3\,\mathsf{f}_{3,0}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^6}-\frac{\beta_i\,\eta_i^3\,\xi_j^3\,\mathsf{f}_{3,1}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^7}-\frac{\beta_i^2\,\eta_i^3\,\xi_j^3\,\mathsf{f}_{3,2}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^8}-\frac{\beta_i^3\,\eta_i^3\,\xi_j^3\,\mathsf{f}_{3,3}\left[\frac{\alpha_j}{\hbar}\right]}{\hbar^9}\right)\epsilon^2+O[\epsilon]^3\right]$$

In[•]:=
```
Eq[2]
```

Out[•]=
$$\left(1-\mathbb{E}_{\{\}\to\{o,o1,o2,u,u1,u2\}}\left[\hbar\,a_{u1}\,b_o+\hbar\,a_{u2}\,b_o+\hbar\,a_u\,b_{o1}+\hbar\,a_u\,b_{o2},\ \hbar\,x_{u1}\,y_o+\hbar\,x_{u2}\,y_o+\hbar\,B_{o2}\,x_u\,y_{o1}+\hbar\,x_u\,y_{o2},\ 1\right]\right)+$$
$$\left(-\hbar^2\,a_{u1}\,B_{o2}\,x_{u2}\,y_{o1}-\frac{1}{4}\gamma\,\hbar^3\,B_{o2}^2\,x_{u1}^2\,y_{o1}^2-\frac{1}{4}\gamma\,\hbar^3\,B_{o2}^2\,x_{u2}^2\,y_{o1}^2-\hbar^2\,a_{u1}\,x_{u2}\,y_{o2}+\right.$$
$$\gamma\,\hbar^3\,B_{o2}\,x_{u1}\,x_{u2}\,y_{o1}\,y_{o2}-\frac{1}{4}\gamma\,\hbar^3\,x_{u1}^2\,y_{o2}^2-\frac{1}{4}\gamma\,\hbar^3\,x_{u2}^2\,y_{o2}^2+$$
$$\frac{1}{4}\left(4\,\hbar^2\,a_{u1}\,x_{u2}\,y_o+\gamma\,\hbar^3\,x_{u1}^2\,y_o^2+\gamma\,\hbar^3\,x_{u2}^2\,y_o^2+\gamma\,\hbar^3\,B_{o2}^2\,x_u^2\,y_{o1}^2+\gamma\,\hbar^3\,x_u^2\,y_{o2}^2\right)\mathbb{E}_{\{\}\to\{o,o1,o2,u,u1,u2\}}^{(0,0,1)}\left[\hbar\,a_{u1}\,b_o+\hbar\,a_{u2}\,b_o+\hbar\,a_u\,b_{o1}+\hbar\,a_u\,b_{o2},\ \hbar\,x_{u1}\,y_o+\hbar\,x_{u2}\,y_o+\hbar\,B_{o2}\,x_u\,y_{o1}+\hbar\,x_u\,y_{o2},\ 1\right]\Big)\epsilon+O[\epsilon]^2$$

In[•]:=
```
Simplify@Block[{$k = 2}, Eq[2]]
```