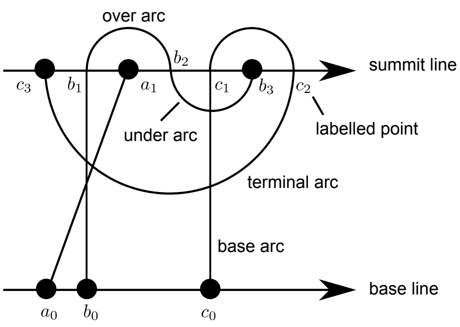
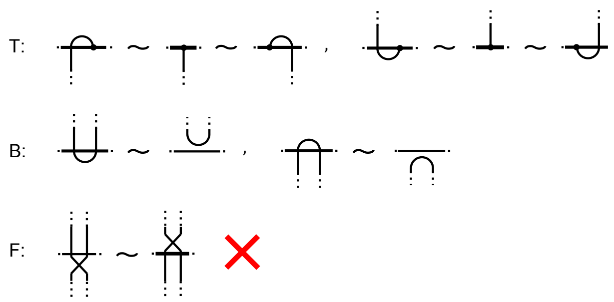
A virtual curve diagram

No arcs crossing above the summit line



Equivalence of virtual curve diagrams and a forbidden move

Curves interacting with the summit line



Using T and B movies, a virtual curve diagram can be put into a unique normal form with a minimal amount of points. This gives a way to distinguish two virtual curve diagrams.

The virtual free group

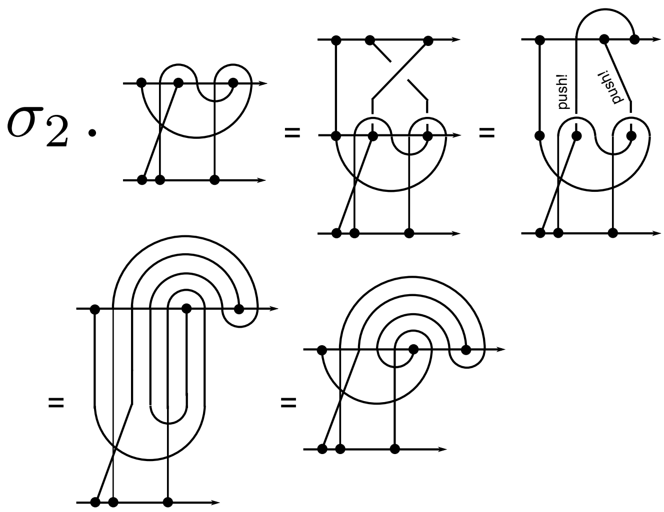
A virtual curve diagram can also be encoded using the "virtual free group" $VF_n.$ The virtual free group VF_n consists of "valid" words with "letters" of the form $(x_i^{\pm 1}, j),$ where $1 \leq i \leq n$ and $j \in \mathbb{N}.$ A valid word is one in which the second coordinates are all distinct. If all the letters of two valid words agree on the first coordinate, and the second coordinates induce the same order, then the words are considered equivalent. For example, the words $(x_1, 3)(x_2^{-1}, 1)(x_1, 2)$ and $(x_1, 51)(x_2^{-1}, 7)(x_1, 23)$ are equivalent. Additionally there is the following relation: $(x_i, j)(x_i^{-1}, j \pm 1) = (x_i^{-1}, j)(x_i, j \pm 1) = \epsilon.$ Any valid word has a unique normal form of minimal length.

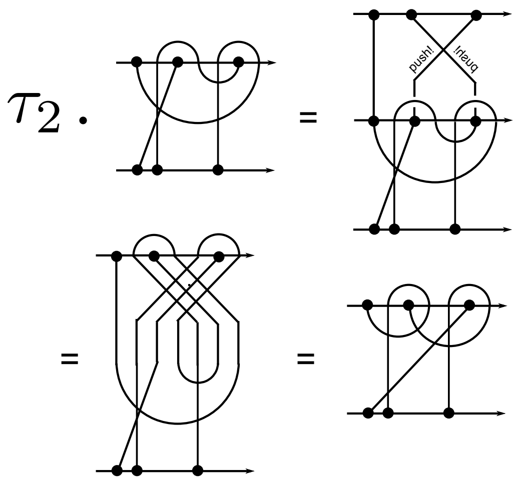
A virtual curve diagram can be encoded by a valid n-tuple of words. An n-tuple is valid when the concatenation of the n words is valid. The same equivalence applies on the words, except the order induced by the second coordinates of the concatenation must be preserved.

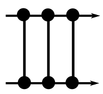
There is a 1-1 correspondence between valid n-tuples and virtual curve diagrams. For example the virtual curve diagram in the top left corner corresponds to the 3-tuple $\big((x_2, 1), (x_2, 3)(x_3, 1)(x_2^{-1}, 2), (x_3, 3)(x_1, 1)(x_3^{-1}, 2)\big)$.

The left action of braid and permutation generators

Just the familiar mapping class group action of a braid on a punctured disk





Claim: The left action on the trivial diagram  , is faithful.