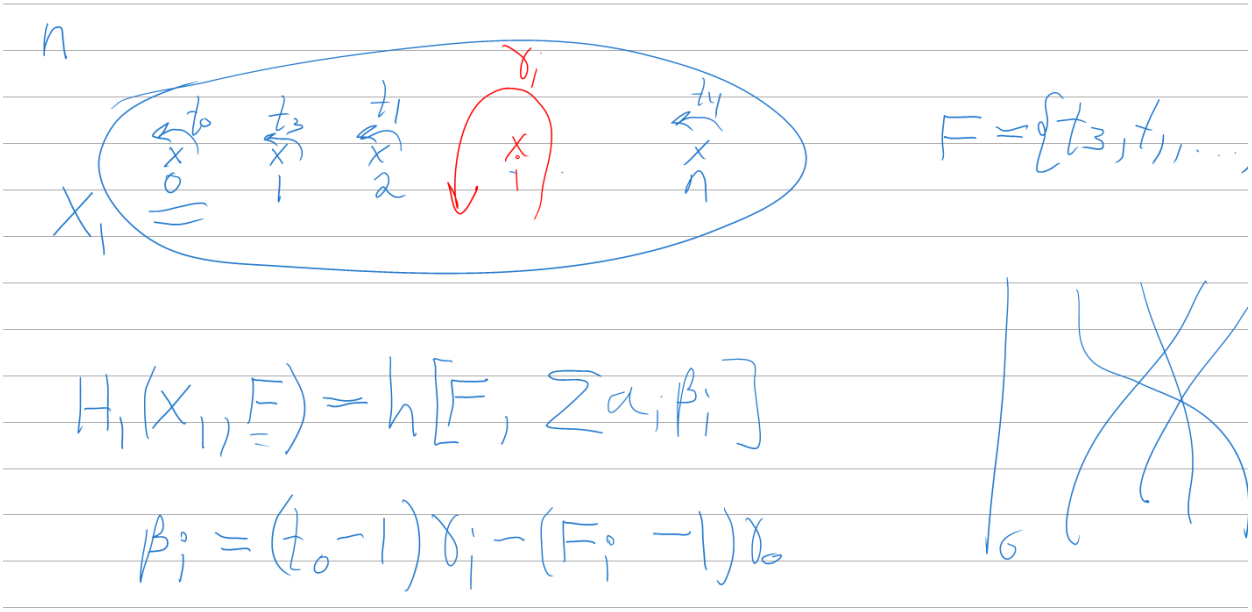

```
In[•]:= σ_i_[h[F_, L_]] := h[
          Permute[F, Cycles[{{i, i + 1}}]],
          Expand[L /. {β_i → β_{i+1}, β_{i+1} → F[[i]] β_i + (1 - F[[i + 1]]) β_{i+1}}]
         ]

In[•]:= {h[{t1, t2, t3}, β1] // σ1 // σ2 // σ1, h[{t1, t2, t3}, β1] // σ2 // σ1 // σ2}
Out[•]=
        {h[{t3, t2, t1}, β3], h[{t3, t2, t1}, β3]}

In[•]:= {h[{t1, t2, t3}, β2] // σ1 // σ2 // σ1, h[{t1, t2, t3}, β2] // σ2 // σ1 // σ2}
Out[•]=
        {h[{t3, t2, t1}, t1 β2 + β3 - t2 β3], h[{t3, t2, t1}, t1 β2 + β3 - t2 β3]}

In[•]:= {h[{t1, t2, t3}, β3] // σ1 // σ2 // σ1, h[{t1, t2, t3}, β3] // σ2 // σ1 // σ2}
Out[•]=
        {h[{t3, t2, t1}, t1 t2 β1 + t1 β2 - t1 t3 β2 + β3 - t3 β3],
         h[{t3, t2, t1}, t1 t2 β1 + t1 β2 - t1 t3 β2 + β3 - t3 β3]}

In[•]:= μ[h[F_, L1_], h[F_, L2_]] := Factor[Expand[L1 (L2 /. {t_i_ :> t_i^-1, β_i_ :> β̄_i})] /.
          {β_i_ β̄_j_ :> Piecewise[{{(t0-1) (F[[i]]-1) (1-t0 F[[i]]) / (t0 F[[i]]), i == j},
                                   {-(t0-1) (F[[i]]-1) (F[[j]]-1) / F[[j]], i < j},
                                   {-(t0-1) (F[[i]]-1) (F[[j]]-1) / (t0 F[[j]]), i > j}}]}]
```

```
In[•]:= {μ[h[{t1, t2, t3}, β1], h[{t1, t2, t3}, β2]],
          μ[h[{t1, t2, t3}, β2], h[{t1, t2, t3}, β1]] /. ti_ :> ti^-1} // Simplify
```

Out[•]=

$$\left\{-\frac{(-1+t_0)\ (-1+t_1)\ (-1+t_2)}{t_2},\ \frac{(-1+t_0)\ (-1+t_1)\ (-1+t_2)}{t_2}\right\}$$

```
In[•]:= h[{t1, t2, t3}, β2] // σ1
```

Out[•]=

$$h[\{t_2, t_1, t_3\},\ t_1\,\beta_1+\beta_2-t_2\,\beta_2]$$

```
In[•]:= {μ[h[{t1, t2, t3}, β1], h[{t1, t2, t3}, β1]],
         μ[h[{t1, t2, t3}, β1] // σ1, h[{t1, t2, t3}, β1] // σ1]}
```

Out[•]=

$$\left\{-\frac{(-1+t_0)\ (-1+t_1)\ (-1+t_0\,t_1)}{t_0\,t_1},\ -\frac{(-1+t_0)\ (-1+t_1)\ (-1+t_0\,t_1)}{t_0\,t_1}\right\}$$

```
In[•]:= {μ[h[{t1, t2, t3}, β1], h[{t1, t2, t3}, β2]],
         μ[h[{t1, t2, t3}, β1] // σ1, h[{t1, t2, t3}, β2] // σ1]}
```

Out[•]=

$$\left\{-\frac{(-1+t_0)\ (-1+t_1)\ (-1+t_2)}{t_2},\ -\frac{(-1+t_0)\ (-1+t_1)\ (-1+t_2)}{t_2}\right\}$$

```
In[•]:= {μ[h[{t1, t2, t3}, β2], h[{t1, t2, t3}, β2]],
         μ[h[{t1, t2, t3}, β2] // σ1, h[{t1, t2, t3}, β2] // σ1]}
```

Out[•]=

$$\left\{-\frac{(-1+t_0)\ (-1+t_2)\ (-1+t_0\,t_2)}{t_0\,t_2},\ -\frac{(-1+t_0)\ (-1+t_2)\ (-1+t_0\,t_2)}{t_0\,t_2}\right\}$$