ctrl-J allows you to add/remoce tags from cells.
Cell -> cell tags -> show cell tags.

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```
In[•]:= g1[h[F_, L_]] := h[
          Permute[F, Cycles[{{1, 2}}]],
          Expand[L /. Join[{x2 → -tF⟦1⟧ x2},
              Table[xj → (1 - tF⟦j⟧)/(tF⟦2⟧ - 1) x2 + (tF⟦1⟧ - 1)/(tF⟦2⟧ - 1) xj, {j, 3, Length[F]}]]] // Simplify
        ]
```

pdf

```
In[•]:= g̅1[h[F_, L_]] := h[
          Permute[F, Cycles[{{1, 2}}]],
          Expand[L /. Join[{x2 → -1/tF⟦2⟧ x2},
              Table[xj → (1 - tF⟦j⟧)/(tF⟦2⟧ (tF⟦2⟧ - 1)) x2 + (tF⟦1⟧ - 1)/(tF⟦2⟧ - 1) xj, {j, 3, Length[F]}]]] // Simplify
        ]
```

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```
In[•]:= gi_[h[F_, L_]] /; i > 1 := h[
          Permute[F, Cycles[{{i, i + 1}}]],
          Expand[L /. {xi → xi+1, xi+1 → tF⟦i⟧ xi + (1 - tF⟦i+1⟧) xi+1}] // Simplify
        ]
```

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```
In[•]:= g̅i_[h[F_, L_]] /; i > 1 := h[
          Permute[F, Cycles[{{i, i + 1}}]],
          Expand[L /. {xi → 1/tF⟦i+1⟧ xi+1 + (tF⟦i⟧ - 1)/tF⟦i+1⟧ xi, xi+1 → xi}] // Simplify
        ]
```

```
In[•]:= {h[{1, 2, 3, 4}, α2 x2 + α3 x3 + α4 x4] // g1 // g̅1, h[{1, 2, 3, 4}, α2 x2 + α3 x3 + α4 x4] // g̅1 // g1}
Out[•]=
{h[{1, 2, 3, 4}, x2 α2 + x3 α3 + x4 α4], h[{1, 2, 3, 4}, x2 α2 + x3 α3 + x4 α4]}

In[•]:= {h[{1, 2, 3, 4}, α2 x2 + α3 x3 + α4 x4] // g2 // g̅2, h[{1, 2, 3, 4}, α2 x2 + α3 x3 + α4 x4] // g̅2 // g2}
Out[•]=
{h[{1, 2, 3, 4}, x2 α2 + x3 α3 + x4 α4], h[{1, 2, 3, 4}, x2 α2 + x3 α3 + x4 α4]}

In[•]:= {h[{1, 2, 3, 4}, α2 x2 + α3 x3 + α4 x4] // g3 // g̅3, h[{1, 2, 3, 4}, α2 x2 + α3 x3 + α4 x4] // g̅3 // g3}
Out[•]=
{h[{1, 2, 3, 4}, x2 α2 + x3 α3 + x4 α4], h[{1, 2, 3, 4}, x2 α2 + x3 α3 + x4 α4]}
```

```
In[•]:= {h[{1, 2, 3}, x2] // g2 // g1 // g2, h[{1, 2, 3}, x2] // g1 // g2 // g1}
```

```
Out[•]=
```

$$\left\{h\left[\{3,\,2,\,1\},\ \frac{t_1\,((-1+t_1)\,x_2-(-1+t_2)\,x_3)}{-1+t_3}\right],\ h\left[\{3,\,2,\,1\},\ \frac{t_1\,((-1+t_1)\,x_2-(-1+t_2)\,x_3)}{-1+t_3}\right]\right\}$$

```
In[•]:= {h[{1, 2, 3}, x3] // g2 // g1 // g2, h[{1, 2, 3}, x3] // g1 // g2 // g1}
```

```
Out[•]=
```

$$\{h[\{3,\,2,\,1\},\ -t_1\,((-1+t_1)\,x_2+x_3)],\ h[\{3,\,2,\,1\},\ -t_1\,((-1+t_1)\,x_2+x_3)]\}$$

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```
In[•]:= ι[h[F_, L1_], h[F_, L2_]] := Simplify[Expand[L1 (L2 /. {ti_ :> ti^-1, xi_ :> x̄i})]] /.
```

$$\left\{x_{i\_}\ \bar{x}_{j\_} :> \begin{cases} \dfrac{(t_{F[[1]]}-1)\,(t_{F[[i]]}-1)\,(1-t_{F[[1]]}\,t_{F[[i]]})}{t_{F[[1]]}\,t_{F[[i]]}} & i == j \\ \dfrac{-(t_{F[[1]]}-1)\,(t_{F[[i]]}-1)\,(t_{F[[j]]}-1)}{t_{F[[j]]}} & i < j \\ \dfrac{-(t_{F[[1]]}-1)\,(t_{F[[i]]}-1)\,(t_{F[[j]]}-1)}{t_{F[[1]]}\,t_{F[[j]]}} & i > j \end{cases}\right\}$$

tex

Above was the program. Let's test it:

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```
In[•]:= Table[ι[h[{1, 2, 3, 4}, xi], h[{1, 2, 3, 4}, xj]], {i, 2, 4}, {j, 2, 4}] // MatrixForm
```

```
Out[•]//MatrixForm=
```

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$$\begin{pmatrix} \frac{(-1+t_1)\,(-1+t_2)\,(1-t_1\,t_2)}{t_1\,t_2} & -\frac{(-1+t_1)\,(-1+t_2)\,(-1+t_3)}{t_3} & -\frac{(-1+t_1)\,(-1+t_2)\,(-1+t_4)}{t_4} \\ -\frac{(-1+t_1)\,(-1+t_2)\,(-1+t_3)}{t_1\,t_2} & \frac{(-1+t_1)\,(-1+t_3)\,(1-t_1\,t_3)}{t_1\,t_3} & -\frac{(-1+t_1)\,(-1+t_3)\,(-1+t_4)}{t_4} \\ -\frac{(-1+t_1)\,(-1+t_2)\,(-1+t_4)}{t_1\,t_2} & -\frac{(-1+t_1)\,(-1+t_3)\,(-1+t_4)}{t_1\,t_3} & \frac{(-1+t_1)\,(-1+t_4)\,(1-t_1\,t_4)}{t_1\,t_4} \end{pmatrix}$$

```
In[•]:= Table[
 Table[ι[h[{1, 2, 3, 4}, x_i] // g_r, h[{1, 2, 3, 4}, x_j] // g_r], {i, 2, 4}, {j, 2, 4}] //
   Simplify // MatrixForm,
 {r, 3}
]
```

Out[•]=

$$\left\{\begin{pmatrix} \frac{(-1+t_1)(-1+t_2)(1-t_1 t_2)}{t_1 t_2} & -\frac{(-1+t_1)(-1+t_2)(-1+t_3)}{t_3} & -\frac{(-1+t_1)(-1+t_2)(-1+t_4)}{t_4} \\ -\frac{(-1+t_1)(-1+t_2)(-1+t_3)}{t_1 t_2} & -\frac{(-1+t_1)(-1+t_3)(-1+t_1 t_3)}{t_1 t_3} & -\frac{(-1+t_1)(-1+t_3)(-1+t_4)}{t_4} \\ -\frac{(-1+t_1)(-1+t_2)(-1+t_4)}{t_1 t_2} & -\frac{(-1+t_1)(-1+t_3)(-1+t_4)}{t_1 t_3} & -\frac{(-1+t_1)(-1+t_4)(-1+t_1 t_4)}{t_1 t_4} \end{pmatrix},\right.$$

$$\begin{pmatrix} \frac{(-1+t_1)(-1+t_2)(1-t_1 t_2)}{t_1 t_2} & -\frac{(-1+t_1)(-1+t_2)(-1+t_3)}{t_3} & -\frac{(-1+t_1)(-1+t_2)(-1+t_4)}{t_4} \\ -\frac{(-1+t_1)(-1+t_2)(-1+t_3)}{t_1 t_2} & -\frac{(-1+t_1)(-1+t_3)(-1+t_1 t_3)}{t_1 t_3} & -\frac{(-1+t_1)(-1+t_3)(-1+t_4)}{t_4} \\ -\frac{(-1+t_1)(-1+t_2)(-1+t_4)}{t_1 t_2} & -\frac{(-1+t_1)(-1+t_3)(-1+t_4)}{t_1 t_3} & \frac{(-1+t_1)(-1+t_4)(1-t_1 t_4)}{t_1 t_4} \end{pmatrix},$$

$$\left.\begin{pmatrix} \frac{(-1+t_1)(-1+t_2)(1-t_1 t_2)}{t_1 t_2} & -\frac{(-1+t_1)(-1+t_2)(-1+t_3)}{t_3} & -\frac{(-1+t_1)(-1+t_2)(-1+t_4)}{t_4} \\ -\frac{(-1+t_1)(-1+t_2)(-1+t_3)}{t_1 t_2} & \frac{(-1+t_1)(-1+t_3)(1-t_1 t_3)}{t_1 t_3} & -\frac{(-1+t_1)(-1+t_3)(-1+t_4)}{t_4} \\ -\frac{(-1+t_1)(-1+t_2)(-1+t_4)}{t_1 t_2} & -\frac{(-1+t_1)(-1+t_3)(-1+t_4)}{t_1 t_3} & -\frac{(-1+t_1)(-1+t_4)(-1+t_1 t_4)}{t_1 t_4} \end{pmatrix}\right\}$$

```
In[•]:= Table[
 Table[ι[h[{1, 2, 3, 4}, x_i] // g_r, h[{1, 2, 3, 4}, x_j] // g_r] == ι[h[{1, 2, 3, 4}, x_i],
      h[{1, 2, 3, 4}, x_j]], {i, 2, 4}, {j, 2, 4}] // Simplify // MatrixForm,
 {r, 3}
]
```

Out[•]=

$$\left\{\begin{pmatrix} \text{True} & \text{True} & \text{True} \\ \text{True} & \text{True} & \text{True} \\ \text{True} & \text{True} & \text{True} \end{pmatrix}, \begin{pmatrix} \text{True} & \text{True} & \text{True} \\ \text{True} & \text{True} & \text{True} \\ \text{True} & \text{True} & \text{True} \end{pmatrix}, \begin{pmatrix} \text{True} & \text{True} & \text{True} \\ \text{True} & \text{True} & \text{True} \\ \text{True} & \text{True} & \text{True} \end{pmatrix}\right\}$$