quantum mechanics or quantum theory, branch of mathematical physics that deals with the emission and absorption of energy by matter and with the motion of material particles. Because it holds that energy and matter exist in tiny, discrete amounts, quantum mechanics is particularly applicable to ELEMENTARY PARTICLES and the interactions between them. According to the older theories of classical physics, energy is treated solely as a continuous phenomenon (i.e., WAVES), and matter is assumed to occupy a very specific region of space and to move in a continuous manner. According to the quantum theory, energy is emitted and absorbed in a small packet, called a quantum (pl. quanta), which in some situations behaves as particles of matter do; particles exhibit certain wave-like properties when in motion and are no longer viewed as localized in a given region but spread out to some degree. The quantum theory thus proposes a dual nature for both waves and particles, with one aspect predominating in some situations and the other in others. Quantum mechanics is needed to explain many properties of matter, such as the temperature dependence of the SPECIFIC HEAT of solids, as well as when very small quantities of matter or energy are involved, as in the interaction of elementary particles and fields, but the theory of RELATIVITY assumes importance in the special situation where very large speeds are involved. Together they form the theoretical basis of modern physics. The results of classical physics were approximations of those quantum mechanics for large scale events and those of RELATIVITY when ordinary speeds are involved. Quantum theory was developed principally over a period of thirty years. The first contribution was the explanation of BLACKBODY RADIATION in 1900 by MAX PLANCK, who proposed that the energies of any harmonic oscillators, such as the atoms of a blackbody radiator, are restricted to certain values, each of which is an integral multiple of a basic minimum value. In 1905 ALBERT EINSTEIN proposed that the radiation itself is also quantized, and he used the new theory to explain the PHOTON IS EFFECT. Niels BOHR used the quantum theory in 1913 to explain both atomic structure and atomic spectra, showing the connection between the energy levels of an atom's electrons and the frequencies of light emitted and absorbed by the atom. Quantum mechanics, the final mathematical formulation of the quantum theory, was developed during the 1920s. In 1924 Louis de BROGLIE proposed that particles exhibit wave-like properties. This hypothesis was confirmed experimentally in 1927 by CLAYTON J. DAVISON and Lester H. Germer, who observed DIFFRACTION of a beam of electrons. Two different formulations of quantum mechanics are presented in 1926 by Erwin SCHROEDINGER and Werner HEISENBERG. Schrödinger's work function, which is related to the probability of finding a particle at a given point in space. The matrix mechanics of Werner HEISENBERG (1925) makes no mention of wave function.
שאולו לתلامידים يستנוולו קולסיות והוגיות לתמשיקאים

שם: _____________________________

חות, שנות לימוד, תואר: _____________________________

טלפון: _____________________________

דאר אלקטורוני: _____________________________

ozilla: _____________________________

ה отношения להישרדות: _____________________________

(כוש, קלי, מורה, לאו, מסעמה, מעניין,

שאולו לתلامידים يستנוולו קולסיות והוגיות לתמשיקאים

שם: _____________________________

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דאר אלקטורוני: _____________________________

ozilla: _____________________________

ה יחסים להישרדות: _____________________________

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ozilla: _____________________________

ה יחסים להישרדות: _____________________________

(כוש, קלי, מורה, לאו, מסעמה, מעניין,

(כוש, קלי, מורה, לאו, מסעמה, מעניין,)
COURSE INTRODUCTION:
WHAT HAPPENS TO A QUANTUM PARTICLE ON A PENDULUM \( \frac{\pi}{2} \) SECONDS AFTER IT IS TOSSED IN?

DROR BAR-NATAN

Follows a lecture given by the author in the “trivial notions” seminar in Harvard on April 29, 1989.

ABSTRACT. This subject is the best one-hour introduction I know for the mathematical techniques that appear in quantum mechanics — in one short lecture we start with a meaningful question, visit Schrödinger’s equation, operators and exponentiation of operators, Fourier analysis, path integrals, the least action principle, and Gaussian integration, and at the end we land with a meaningful and interesting answer.

CONTENTS

1. The Question 1
2. The Solution 2
3. The Lemmas 3
4. The Morals 5

1. THE QUESTION

Let the complex valued function \( \psi = \psi(t, x) \) be a solution of the Schrödinger equation

\[
\frac{\partial \psi}{\partial t} = -i \left( -\frac{1}{2} \Delta_x + \frac{1}{2} x^2 \right) \psi \quad \text{with} \quad \psi|_{t=0} = \psi_0.
\]

What is \( \psi|_{t=T=\frac{\pi}{2}} \)?

In fact, the major part of our discussion will work just as well for the general Schrödinger equation,

\[
\frac{\partial \psi}{\partial t} = -iH \psi, \quad H = -\frac{1}{2} \Delta_x + V(x), \quad \psi|_{t=0} = \psi_0, \quad \text{arbitrary } T,
\]

where:

- \( \psi \) is the “wave function”, with \( |\psi(t, x)|^2 \) representing the probability of finding our particle at time \( t \) in position \( x \).
- \( H \) is the “energy”, or the “Hamiltonian”.
- \( -\frac{1}{2} \Delta_x \) is the “kinetic energy”.
- \( V(x) \) is the “potential energy at \( x \)”. 

Date: This edition: March 27, 1999;  First edition: April 29, 1989.
WASHINGTON -- Though dead for nearly two centuries, Joseph Louis Lagrange, the greatest French mathematician, is about to make news in the developing Chinese espionage scandal.

A book frequently checked out of the library at Los Alamos National Laboratory is a seminal 1970s' work by David Pierre entitled "Mathematical Programming via Augmented Lagrangians: an Introduction to Computer Programs."

Reached in Montana, Dr. Pierre explained that Lagrange multipliers, weighted with algebraic constraint equations, form composite functions.

That didn't help, so I read off a question that investigators of China's penetration of our laboratories are now asking: "How can the Lagrangian codes be applied to the history of our nuclear tests to develop, with supercomputers, three-dimensional modeling that obviates the need for explosive tests?"

"You'd better check with a physicist," said Dr. Pierre, a mathematician unconnected with secrecy. "They can model dynamic systems -- like airplanes or missiles -- based on Lagrangian functions. If they have a big enough computer, and a good enough program -- and the benchmarks of previous tests -- they can mathematically simulate what you're after."

In so checking, I was able to get an idea of the New Nuclear Espionage, a spy system combining open dual-use purchasing with clandestine collection, both protected by politico-diplomatic enticements. This is a far cry from the simple stealing of specs for the Soviets by the Rosenbergs.

Thanks to the lax, sell-'em-anything decisions of the Clinton White House and Commerce Department, China bought advanced computers or their key components. That gave them the "big enough computer."

Then China's mathematicians and physicists were able to learn from their friendly associates in the U.S. what types of augmented Lagrangians were used in nuclear as well as missile programs, or "codes." That gave them the "good enough program."

Then into the Lagrangian-coded supercomputers went the "benchmarks" -- the experience of our tests as well as their own -- to give them the information on which to base their simulations. Feed the benchmarks into the codes on the supercomputer, and --
presto! -- the People's Liberation Army leaps a decade ahead in its the race to nuclear-weapon parity.

That's my theory, based not on leaks but on common sense. When the thousand-page Cox Committee investigative report is finally cleared by a nervous National Security Council; when the C.I.A.'s belated jeremiad assesses the damage caused by the derelict guardians of our security; when Senate and House intelligence committees issue reports, and when Piffiab conducts its usual internal whitewash absolving the Clinton White House of creating a culture of permissiveness to China's political, trade and scientific penetrations -- we'll see if my theory holds heavy water.

In the meantime, we can expect the President to continue his familiar legal obfuscations, where "to the best of my knowledge" he says he cannot be "sure" of any espionage at all. National political-security adviser Samuel Berger will tell different stories in public and in secret.

Secretary of Energy Bill Richardson will lock E-mail barn doors only after press pressure and will rely on misleading "lie detector" tests (that the spy Aldrich Ames showed lead to false security).

But little by little, the interrelated truth will out. Logic suggests that the theft of our W-88 MIRV'ed missile will be followed, as the night the day, by news of the loss of the secrets of our W-87 or W-89, whatever warhead technology that may be.

A word about the much-maligned media. A column here remembering the C.I.A. mole Larry Wu-tai Chin on Jan. 2, 1997, was a howl in the night, but shoe-leather reporting by Jeff Gerth in The Times on May 15 and 17, 1998, of Lieut. Col. Liu Chaoying's penetrations pushed the House into appointing the bipartisan Cox committee.

Cox's still-secret findings then energized the Reno-restrained F.B.I. and the moribund D.O.E.

Though The Washington Post is doing a fine job on local coverage, NBC's "Meet the Press" and ABC's "Nightline" -- with The Times's Gerth, James Risen and editor Stephen Engelberg in the lead -- have advanced this global story and help protect your safety. 

Vive Lagrange!
\[
\int_{a}^{b} F(x, y, y') \, dx = y' |_{a}^{b} - \int_{a}^{b} F(x, y, y') \, dx
\]

\[
F(x, y, y') = \frac{\lambda y^{2} + y'^{2}}{\sqrt{y^{2} + y'^{2}}}
\]

\[
y(x) = \frac{x}{\lambda} + C
\]

\[
y(x) = \frac{x}{\lambda} + C, \quad y(0) = 0
\]

\[
y(x) = \frac{x}{\lambda} + C, \quad g(x, y) = 0
\]

\[
J = \int_{a}^{b} y \, dx
\]

\[
G = \int_{a}^{b} y^2 \, dx
\]

\[
F_{y} = \frac{\partial F}{\partial x} = 0
\]

\[
G_{y} = \frac{\partial G}{\partial x} = 0
\]

\[
\frac{\partial}{\partial x} \left( F - y' F_{y} \right) = y' F_{y} - F y' - F y'' y' = 0
\]

\[
y' = \frac{\lambda y^{2} + y'^{2}}{\sqrt{y^{2} + y'^{2}}}
\]

\[
y = C \cosh \frac{x-C}{\lambda}
\]

\[
f(x, y) = 0, \quad g(x, y) = 0
\]

\[
\nabla h = 0
\]

\[
h_{x} = f + xg
\]

\[
F_{y} = \frac{\partial F}{\partial x} = 0
\]

\[
f = y + x \cosh \frac{x-C}{\lambda}
\]

\[
g = \frac{x-C}{\lambda}
\]

\[
J = \int_{a}^{b} y \, dx
\]

\[
G = \int_{a}^{b} y^2 \, dx
\]
\[ 0 = 1 - \lambda \frac{dy}{dx} \frac{y'}{\sqrt{1+y'^2}} \]

\[ \frac{\lambda y'}{\sqrt{1+y'^2}} = x - C_1 \]

\[ y' = \frac{x - C_1}{\sqrt{x^2 - (x-C_1)^2}} \]

\[ y = C_2 - \sqrt{x^2 - (x-C_1)^2} \]

\[ x = (x-C_1)^2 + (y-C_2)^2 \]
\[ F_{1} : L^{2}(R^{n}) \rightarrow L^{2}(R^{n}) \text{ such that } \|F\|_{L^{2}(R^{n})} \leq 1/\sqrt{3n} \]

\[ p_{1} \quad F_{0} = I \quad F_{1} \circ F_{a} = F_{1} \circ F_{a} \]

\[ F_{\frac{a}{a_{0}}} = \ldots \]

\[ (\alpha \in R^{n}, \beta) \quad \text{for some } \alpha \in R^{n}, \beta \in R \]

\[ y(0) = 0 \quad y(1) = 1 \quad y \rightarrow \int_{0}^{1} y'y' dx \]

\[ y(0) = 0 \quad y(1) = 1 \quad y \rightarrow \int_{0}^{1} y y' dx \]

\[ y(0) = 0 \quad y(1) = 1 \quad y \rightarrow \int_{0}^{1} x y y' dx \]

\[ y \rightarrow \int_{a}^{b} \frac{y'^2}{x^3} dx \]

\[ y \rightarrow \int_{a}^{b} (y^2 + y'^2 + 2ye^x) dx \]

\[ y(0) = 0 \quad y(1) = 1 \quad \int_{0}^{1} (y'^2 + x^2) dx \]

\[ y_{0} \in C_{0}^{2} \quad \text{for } x \in (0, 1) \]

\[ y \in D^{2} \quad y' \in D^{1} \quad y'' \in L^{2} \]

\[ y_{0} = x \]

\[ \int_{0}^{1} y_{0} y_{0} = 0 \]

\[ y_{0}^{2} \rightarrow y_{0} \rightarrow 0 \quad y_{0}^{3} \rightarrow 0 \quad y_{0}^{4} \rightarrow 0 \]
EULER-LAGRANGE ACCORDING TO ITZYKSON-ZUBER

\[ \frac{\partial L(x)}{\partial \varphi_i(x)} - \partial_\mu \frac{\partial L(x)}{\partial [\partial_\mu \varphi_i(x)]} = 0 \]

\[ 0 = \frac{[(x)^2 \phi \eta e]}{(x)Je} e_\eta e - \frac{(x)^2 \phi e}{(x)Je} \]
$L(q) = \int_a^b L(t, q, \dot{q}) dt$  \quad F(x + \Delta x) = F(x) + \langle DF \rangle \dot{x} \Delta x$

$\delta L = \langle EL_q(q), \delta q \rangle  \quad EL_q L = \frac{\partial L}{\partial q} - \frac{\partial}{\partial t} \left( \frac{\partial L}{\partial \dot{q}} \right)$

$0 = L(q) = \int_c^b C(t, q, \dot{q}) dt \quad \text{or} \quad \dot{q} \text{ is a constant}\ (C = \text{cons.}) \quad L(q) \neq 0 \quad \Rightarrow \quad \frac{\partial}{\partial q} \neq 0$

$EL_q L + \lambda EL_q (c) = 0$

$L = \int \sqrt{1 + y'^2} dx \quad L = \int_a^b y \, dx$

$\frac{xy'}{\sqrt{1+y'^2}} = x - C_1 \quad \Rightarrow \quad 1 - \lambda \frac{y'}{\sqrt{1+y'^2}} = 0$

$\sqrt{x^2 - (x-C_1)^2} \leq y \leq -\sqrt{x^2 - (x-C_1)^2}$

$(x-C_1)^2 + (y-C_1)^2 = x^2$

$\frac{1}{\sqrt{1+y'^2}} = \frac{1}{\sqrt{4+12\lambda^2}} \Rightarrow y = \sqrt{x^2 - (x-C_1)^2}$

\( L = -\frac{1}{2} \frac{\partial}{\partial q} \frac{1}{2} \frac{\partial}{\partial \dot{q}} \)  \quad Itignson-Zuber  \quad 1996
Special relativity

Arise from attempt to unify Newtonian mechanics with Maxwell's theory of electromagnetism.

**Newtonian mechanics**

1. Every body continues in its state of constant velocity motion unless acted upon by an external force.

\[ \frac{d}{dt} (\text{momentum}) = \text{force} \]

2. \( F = ma \)

3. Every action has an equal and opposite reaction.

**Galilean relativity**

Experiments don't (can't) distinguish between different frames moving with constant velocity relative to each other.

\[ \begin{cases} t' = t \\ x' = x - vt \end{cases} \]

**Classical electromagnetism**

\[ \begin{align*}
\mathbf{E}, \mathbf{B} & \text{ electric, magnetic fields} \\
\mathbf{E} \times \mathbf{B} & = -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \cdot \mathbf{E} & = \frac{\rho}{\varepsilon_0} \\
\nabla \times \mathbf{B} & = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{j} \\
\n\nabla \cdot \mathbf{B} & = 0
\end{align*} \]

Without local charge & current densities,

\[ \nabla \times (\nabla \times \mathbf{E}) = -\frac{\partial^2 \mathbf{E}}{\partial t^2} = -\frac{\varepsilon_0}{\mu_0} \nabla^2 \mathbf{E} \]

or

\[ \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) \mathbf{E} = 0 \]

**Light:** instance of electromagnetic wave

\[ \begin{align*}
\mathbf{E} & = E_0 \cos \left( \frac{2\pi}{\lambda} - \frac{2\pi}{c} t \right) \\
\mathbf{B} & = \mathbf{B}_0 \cos \left( \frac{2\pi}{\lambda} - \frac{2\pi}{c} t \right)
\end{align*} \]

Speed in vacuum, \( c \approx 3 \times 10^8 \text{ms}^{-1} \)
### Table 18-1 Classical Physics

Maxwell's equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$</td>
<td>(Flux of $E$ through a closed surface) = (Charge inside)/$\varepsilon_0$</td>
</tr>
<tr>
<td>II. $\nabla \times E = -\frac{\partial B}{\partial t}$</td>
<td>(Line integral of $E$ around a loop) = $-\frac{d}{dt}$ (Flux of $B$ through the loop)</td>
</tr>
<tr>
<td>III. $\nabla \cdot B = 0$</td>
<td>(Flux of $B$ through a closed surface) = 0</td>
</tr>
<tr>
<td>IV. $c^2 \nabla \times B = \frac{j}{\varepsilon_0} + \frac{\partial E}{\partial t}$</td>
<td>$c^2$ (Integral of $B$ around a loop) = (Current through the loop)/$\varepsilon_0$ + $\frac{\partial}{\partial t}$ (Flux of $E$ through the loop)</td>
</tr>
</tbody>
</table>

Conservation of charge

$\nabla \cdot j = -\frac{\partial \rho}{\partial t}$

(Flux of current through a closed surface) = $-\frac{\partial}{\partial t}$ (Charge inside)

Force law

$F = q(E + v \times B)$

Law of motion

$\frac{d}{dt}(p) = F$, where $p = \frac{mv}{\sqrt{1 - v^2/c^2}}$ (Newton’s law, with Einstein’s modification)

Gravitation

$F = -G \frac{m_1 m_2}{r^2} e_r$
\[ S(A) = \sqrt{\text{tr}(A^2) + J^A} \]

\[ \oint_C \vec{F} \cdot d\vec{r} = 0 \quad \text{for} \quad \oint_C \vec{F} = 0 \]

\[ \text{div} \vec{B} = 0 \quad \text{curl} \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

\[ \vec{F} = -\nabla E \quad \text{curl} \vec{B} = -\frac{\partial E}{\partial t} \]

\[ \text{Stokes' theorem} \quad \oint_C \vec{F} \cdot d\vec{r} = \iint_S \nabla \times \vec{F} \cdot d\vec{S} \]

\[ \vec{E} = \frac{\vec{p}}{4\pi \varepsilon_0} - \vec{B} \times \vec{v} \quad \vec{B} = \frac{\vec{J}}{4\pi} \times \vec{r} \]

\[ \vec{E} = \vec{E}_0 + \vec{E}_1 + \vec{E}_2 \quad \vec{B} = \vec{B}_0 + \vec{B}_1 + \vec{B}_2 \]

\[ \vec{F} = \vec{E} \cdot \vec{v} \quad \vec{F} = \frac{m \vec{v}}{\tau} \]

\[ \vec{v} = \vec{E} \times (\vec{B} \times \vec{v}) + \frac{1}{m} \nabla \times \vec{E} \]

\[ \vec{E} = \frac{Q}{4\pi \varepsilon_0 \rho} \quad \vec{B} = \frac{\mu_0 I}{4\pi} \]

\[ \text{MioKeik} \]
$1996 7 \text{ פלניר} \left\{ \begin{array}{l}
    q_i = (q_i, 1, \ldots, n) \\
    F = F_{q_i} \left( q_i, \frac{1}{2} \right)
\end{array} \right.$

$F = R_{xy} \left( u(x,y) \right) = \frac{
\left[ \begin{array}{c}
1 \\
-1
\end{array} \right] \cdot\left[ \begin{array}{c}
\frac{\partial u(x,y)}{\partial x} \\
\frac{\partial u(x,y)}{\partial y}
\end{array} \right] = \frac{
\partial^2 u(x,y)}{\partial x^2} - \frac{\partial^2 u(x,y)}{\partial y^2}
\right. \left. \right.$

$\begin{aligned}
    \frac{\partial u}{\partial x} &= \frac{1}{2} \\
    \frac{\partial u}{\partial y} &= \frac{1}{2}
\end{aligned}$

$H = \sum q_i P_i - F$

$[q_i, p_i] \leq H - \frac{1}{2} (p_i^2) \leq H = 0 \Rightarrow \sum P_i = 0 \Rightarrow F = \frac{1}{2} p_i^2 - H$

$[\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}] = \left[ \begin{array}{c}
q_i \\
p_i
\end{array} \right]$

$\sum q_i p_i + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} + \frac{1}{2} \frac{\partial^2 u}{\partial y^2} = H = \sum q_i p_i + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} + \frac{1}{2} \frac{\partial^2 u}{\partial y^2}$

$\left\{ f, g \right\} = \sum \frac{\partial f}{\partial x} \frac{\partial g}{\partial x} - \sum \frac{\partial f}{\partial y} \frac{\partial g}{\partial y}$

$\left\{ F, H \right\} = \left\{ F, H \right\}$

\[ 2.5 \]

$\left\{ F, H \right\} = \left\{ F, H \right\}$

$\left\{ F, H \right\} = \left\{ F, H \right\}$

$\frac{H}{F} = \left[ \begin{array}{c}
F \\
H
\end{array} \right] \Rightarrow \left[ \begin{array}{c}
q_i \\
p_i
\end{array} \right] = \left[ \begin{array}{c}
F \\
H
\end{array} \right] = \left[ \begin{array}{c}
1 \\
0
\end{array} \right]$

$\left[ \begin{array}{c}
q_i \\
p_i
\end{array} \right] = \left[ \begin{array}{c}
F \\
H
\end{array} \right] = \left[ \begin{array}{c}
1 \\
0
\end{array} \right]$

$\left[ \begin{array}{c}
q_i \\
p_i
\end{array} \right] = \left[ \begin{array}{c}
F \\
H
\end{array} \right] = \left[ \begin{array}{c}
1 \\
0
\end{array} \right]$

$\left[ \begin{array}{c}
q_i \\
p_i
\end{array} \right] = \left[ \begin{array}{c}
F \\
H
\end{array} \right] = \left[ \begin{array}{c}
1 \\
0
\end{array} \right]$

$\frac{d^2 u}{\partial x^2} - \frac{d^2 u}{\partial y^2} = 0$
Non-Commutative (Quantum) Probability
Classical and Quantum Mechanics for Mathematicians, HUJI 1999
Dror Bar-Natan

Claim: In the quantum probability space \((\mathbb{R}^4, \nu)\) where \(\nu\) is the unit vector \(\nu = \frac{\sqrt{3}}{2}(0 \ 1 \ -1 \ 0)^T\), one has \(p(A = B) = p(B = C) = p(C = D) = \frac{3}{4}\) and \(p(D = A) = 0\), where \(A, B, C,\) and \(D\) are the random variables corresponding to the matrices:

\[
A = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix} \quad ; \quad B = \begin{pmatrix}
-\frac{\sqrt{3}}{2} & 0 & 0 & 0 \\
\frac{\sqrt{3}}{2} & 0 & 0 & 0 \\
0 & \frac{\sqrt{3}}{2} & 0 & -\frac{\sqrt{3}}{2} \\
0 & 0 & \frac{\sqrt{3}}{2} & \frac{1}{2}
\end{pmatrix}
\]

\[
C = \begin{pmatrix}
-\frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} & 0 \\
0 & -\frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\
-\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} & 0 \\
0 & -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2}
\end{pmatrix} \quad ; \quad D = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]

Mathematica 2.0 for SPARC
Copyright 1988-91 Wolfram Research, Inc.
-- Terminal graphics initialized --

\[
\text{In}[1] := \text{v} = 1/2 \text{Sqrt}[2] \{0, 1, -1, 0\}; q = 1/2 \text{Sqrt}[3];
\]

\[
\text{In}[2] := \text{A1 = DiagonalMatrix}[\{1, 1, -1, -1\}] ; \text{A4 = DiagonalMatrix}[\{1, -1, 1, -1\}] ;
\]

\[
\text{In}[3] := \text{A2 = \{-1/2, q, 0, 0\}, \{q, 1/2, 0, 0\}, \{0, 0, -1/2, q\}, \{0, 0, q, 1/2\}};
\]

\[
\text{In}[4] := \text{A3 = \{-1/2, 0, -q, 0\}, \{0, -1/2, 0, -q\}, \{-q, 0, 1/2, 0\}, \{0, q, 1/2\}};
\]

\[
\text{In}[5] := \{\text{Eigenvalues}[\text{A1}], \text{Eigenvalues}[\text{A2}], \text{Eigenvalues}[\text{A3}], \text{Eigenvalues}[\text{A4}]\}
\]

\[
\text{Out}[5] := \{\{1, -1, 1, -1\}, \{1, -1, 1, -1\}, \{1, -1, 1, -1\}, \{1, -1, 1, -1\}\}
\]

\[
\]

\[
\text{Out}[6] := \{\text{True, True, True, True}\}
\]

\[
\text{In}[7] := \text{pequal[M1_, M2_]} := 1 - \nu.(\text{M1-M2}).(\text{M1-M2}).\nu / 4
\]

\[
\text{In}[8] := \{\text{pequal[A1,A2], pequal[A2,A3], pequal[A3,A4], pequal[A4,A1]}\}
\]

\[
3 \ 3 \ 3
\]

\[
\text{Out}[8] := \{\text{-, -, -}, 0\}
\]

\[
4 \ 4 \ 4
\]

Quantum Behavior

1-1 Atomic mechanics

"Quantum mechanics" is the description of the behavior of matter and light in all its details, in particular, of the happenings on an atomic scale. Things on a very small scale behave like nothing that you have any direct experience about. They do not behave like waves, they do not behave like particles, they do not behave like clouds, or billiard balls, or weights on springs, or like anything that you have ever seen.

Newton thought that light was made up of particles, but then it was discovered that it behaves like a wave. Later, however (in the beginning of the twentieth century), it was found that light indeed sometimes behaves like a particle. Historically, the electron, for example, was thought to behave like a particle, and then it was found that in many respects it behaves like a wave. So it really behaves like neither. Now we have given up. We say: "It is like neither."

There is one lucky break, however—electrons behave just like light. The quantum behavior of atomic objects (electrons, proton, neutron, photon, and so on) are all the same for all of them; they are all "particulate waves" or whatever you want to call them. So what we learn about the properties of electrons (which we shall use or our examples) will apply also to all "particles," including photons of light.

The gradual accumulation of information about atomic and small-scale behavior during the first quarter of this century, which gave some indication about how small things do behave, produced an increasing confusion which was finally solved in 1926 and 1927 by Schrödinger, Heisenberg, and Born. They finally obtained a consistent description of the behavior of matter on a small scale. We follow the main features of that description in this chapter.

Because atomic behavior is so unlike ordinary experience, it is very difficult to get used to, and it appears peculiar and mysterious to everyone—both to the novice and to the experienced physicist. Even the experts do not understand it the way they would like to, and it is perfectly reasonable that they should not, because it is different, direct, human experience and of human intuition applies to large objects. We know how large objects will act, but things on a small scale just do not act that way. So we have to learn about them in a sort of abstract or imaginative fashion and not by connection with our direct experience.

In this chapter we shall tackle immediately the basic element of the mysterious behavior in its most strange form. We choose to examine a phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has made it the heart of quantum mechanics. In reality, it contains the only mystery. We cannot make the mystery go away by "explaining" how it works. We will just describe how it works. In telling you how it works, we will have told you about the basic peculiarities of all quantum mechanics.

1-2 An experiment with bullets

To try to understand the quantum behavior of electrons, we shall compare and contrast their behavior in a particular experimental setup, with the more familiar behavior of particles like bullets, and with the behavior of waves like light waves. We consider first the behavior of bullets in the experimental setup shown diagrammatically in Fig. 1-1. We have a machine gun that shoots a stream of bullets. It is not a very good gun, in that it sprays the bullets (randomly) over a fairly large angular spread, as indicated in the figure. In front of the gun we have

![Fig. 1-1. Interference experiment with bullets.](image-url)
Is the moon there when nobody looks? Reality and the quantum theory

Einstein maintained that quantum metaphysics entails spooky actions at a distance; experiments have now shown that what bothered Einstein is not a debatable point but the observed behavior of the real world.

N. David Mermin

Quantum mechanics is magic!

In May 1935, Albert Einstein, Boris Podolsky, and Nathan Rosen published an argument that quantum mechanics fails to provide a complete description of physical reality. Today, 60 years later, the EPR paper and the theoretical and experimental work it inspired remain remarkable for both their flaws and their illustration they provide of one of the most bizarre aspects of the world revealed to us by the quantum theory.

Einstein's talent for saying memorable things did him a disservice when he declared "God does not play dice," for it has been held ever since that the basis for his opposition to quantum mechanics was the claim that a fundamental understanding of the world can only be statistical. But the EPR paper, his most powerful attack on the quantum theory, focused on a quite different aspect: the doctrine that physical properties have in general no objective reality independent of the act of observation. As Pascual Jordan put it, "Observations not only disturb what has to be measured, they produce it... We compel [the electrons] to assume a definite position... We ourselves produce the results of measurement."

Jordan's statement is something of a trauma for contemporary physicists. Underlying it, we have all been taught, is the disruption of what is being measured by the act of measurement, made unavoidable by the existence of the quantum of action, which generally makes it impossible even in principle to construct probes that can yield the information classical intuition expects to be there.

Einstein didn't like this. He wanted things out there to have properties, whether or not they were measured. We often discussed his notions on objective reality. I recall that during one walk Einstein suddenly stopped, turned to me, and asked whether I really believed that the EPR paper describes a situation ingeniously contrived to test the quantum theory in an apparent space-time region A. So far from B that there is no possibility of the measurement in A exerting an influence on region B by any known dynamical mechanism. Under these conditions, Einstein maintained that the properties in A must have existed all along.

Spooky actions at a distance

Many of his simplest and most explicit statements of this position can be found in Einstein's correspondence with Max Born. Throughout the book (which sometimes reads like a Nabokov novel), Born, pained by Einstein's disdain for the statistical character of the quantum theory, repeatedly falls, both in his letters and in his later commentary on the correspondence, into the error of what is really bothering Einstein. Einstein tries to answer over and over again, without success, to make himself clear. In March 1948, for example, he writes:

That which really exists in A should not depend on what kind of measurement is carried out in part of space A; it should also be independent of whether or not any measurement at all is carried out in space A. If one adheres to this program, one can hardly consider the (quanta-thetrical) description as a complete representation of the physically real. If one tries to do so in spite of this, one has to assume that the physically real in B suffers a sudden change as a result of measurement in A. My instinct for physics bristles at this.

Or, in March 1947:

I cannot seriously believe in (the quantum theory) because it cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance.

The "spooky actions at a distance" (spukhafte Fernwirkungen) are the quintessence of a definite value of a property by the system in region B by virtue of the measurement carried out in region A. The EPR paper presents a way function that describes two correlated particles localized in regions A and B far apart. In this particular two-particle state one can learn (in the sense of being able to predict with certainty...

David Mermin is director of the Laboratory for Atomic and Solid State Physics at Cornell. He is currently completing a book about quantum mechanics for today's readers as the person who "boops" an internationally accepted scientific term. With N. W. Ashcroft, he is starting updating the world's funniest solid-state physics text. He says he is bothered by the theorem, but may have rocks in his

\[ S(x/B)S(a) = e^{\frac{1}{2} (x-B)^2} S(x+B) \]

\[ s(a) = \int s(x) S(x) \, dx \]

\[ S(a) = S(a) \]

\[ s(a) = s(a) + s(a+b) = S(a)S(b) \]

\[ S(a)S(b) = \exp \left( -\frac{(a-b)^2}{2} \right) \]

\[ s(a)S(b) = S(c) \]

\[ (d\sigma) = \int d^3p \, \frac{1}{\sqrt{2}} \left[ e^{i(k-p fight)S(x)} \right] \]

\[ \int_{-\infty}^{\infty} a(x, \beta) S(x, \beta) = 0 \]

\[ \int_{-\infty}^{\infty} a(-\beta) S(x, \beta) = 0 \]

\[ a = 0 \text{ for } S(x) = 0 \]

\[ s(x, \beta) = 0 \]

\[ E = E^* \]

\[ E S(x, \beta) E = e^{-\frac{1}{2} (x + \beta)^2} E \]

\[ E^2 = E \]
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מרית חיות

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תורת לתומי צלילים על התחנה.

המוכן "כָּלַ֖ד שָׁאַלְתָּ עַל הַמַּמְשָׁלָה" מַאֵֽת גְּנוֹנֶֽגֶל לְיַוְּלֵֽנֶגֶל מִן:

בִּימֵ֣ים הַֽמַּמְשָׁלָה קָֽצֶֽת נַאֲנֵֽי. לָא הַבָּנָ֖י נָאוּךְ הַבָּהֲרִים כִּלְלָֽיָה לְחֹיָֽה וּנְעֵֽי גְּוָֽיִֽס הַגְּוָֽיִֽס וּנְעֵֽי גְּוָֽיִֽס.

גֹּדוֹ. הָאָמְפוֹרָו הַחֲבִיב הָרֵאָה לְאָדָא口头 שֶׁלָּשָּם.

סֶדֶדָה שֶל שָׁאַלְתָּ, מְסְבַּתֶּה דֵּבָרִים גַּאֲלָה בְּרָאוֹחַ, רֹזְחָה לֹעָשׂוּת בָּרָבָא, נַעֲנָה רֹזְחִי לֹעָשׂוּת הַגְּוָֽיִֽס הַגְּוָֽיִֽס.

כּוֹסְטֵה. חֶקֶר הָשָׁוֵיָה מְזוֹּנָת לְוַֽלְפֵּכָת שָׁאַלְתָּ הַמְּמֻנָּה הָלֵֽפִיק הַשָּׁוֵיָה עַקְּבִּית.

הָאָמְפוֹרָו הַדְּבָרִים אֶת הַשָּׁוֵיָה עַל. מִרְוּל, הָאָם שָׁאֵל מַדָּאָה מְמֻנָּה בְּפָשֵׁיָה בְּכָרָב בְּכָרָב. נִיּוּר, אָמְרִית.

יָאָם שָׁאֵל סֶבֶר שְׁמֵעִיָּה חוֹרָב בְּמַשְׁמַעְתָּ בְּבָהֲרִים תָּכְנִי, בְּהַפֶּר הַסָּפָר מְלַכְּפִּים שָׁלֵנָת.

יָאָם שָׁאֵל סֶבֶר שְׁמֵעִיָּה זֹהיִּים לְמַשְׁמַעְתָּ הַמַּמְשָׁלָה מְלַכְּפִּים שָׁלֵנָת.

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אֶזְלוֹל שָׁאֵל בּוֹדְךָ חִדָּוָה הַשָּׁוֵיָה הָלֵֽפִיק?

לָאָם שָׁאֵל סֶבֶר שְׁמֵעִיָּה לֶעָזְרֵי רוֹבִיק וְלֶעָזְרֵי רוֹבִיק לָאָם שָׁאֵל.

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\[ V = \text{span}(V_+ \oplus \mathcal{W}_+)
\]
\[ W = \text{span}(W_+) \]
\[ \alpha_\mathcal{J} = V \otimes W = [V \otimes W] \]
\[ \mathcal{G}(V_+ \otimes W) = \pm V_+ \otimes W
\]
\[ M(V_+ \otimes W_+) = \pm \alpha_\mathcal{U} V \]
\[ \alpha : V \rightarrow \mathcal{W}_\mathcal{J}
\]
\[ \alpha V = V \otimes W
\]
\[ \mathcal{U} : \mathcal{W}_\mathcal{J} \otimes \mathcal{W}_\mathcal{J} = U(V_+ \otimes W_-) = V_+ \otimes V_-
\]
\[ \begin{align*}
\text{U: } & (1, 0) \rightarrow (0, 0) \rightarrow (1, 0) \\
& (0, 1) \rightarrow (1, 1) \rightarrow (0, 1)
\end{align*} \]
מכינה קלאסית קוראנית למתמטיקאים
מונד א', סמסר ב', 1999
דרור בר-ניט

משק הביהנה: סעותים.
הונורטרון מוקר בשמורת: איני.

cono כבננה של א奮 משנני הנושאים הבאות:
1. משואות מקטורים.
2. השטבות קוראניות.