

$m \diamond n := \text{Min}[m, n];$

$S[pd\_PD, a\_List] := \text{Times} @@ \left( \{ \text{List} @@ pd, a \}^T /. \{ \right.$   
     $\{X[i_, j_, k_, l_], 0\} \Rightarrow p[i, j]_{i \diamond j} p[k, l]_{k \diamond l},$   
     $\{X[i_, j_, k_, l_], 1\} \Rightarrow p[i, l]_{i \diamond l} p[j, k]_{j \diamond k},$   
     $\{x\_X, \star\} \Rightarrow x \}$   
 $\left. \right) // . \{$   
     $p[i_, j_]_{m_} p[j_, k_]_{n_} \Rightarrow p[i, k]_{m \diamond n}$   
 $\} // . \{$   
 $X[i_, j_, k_, l_] p[i_, j_]_{m_} p[k_, l_]_{n_} \Rightarrow$   
     $(c_m c_n \rightarrow c_{m \diamond n}),$   
 $X[i_, j_, k_, l_] p[i_, l_]_{m_} p[j_, k_]_{n_} \Rightarrow$   
     $(c_{m \diamond n} \rightarrow c_m c_n)$   
 $\} // . p[[_]]_{\bar{m}} \Rightarrow c_m$