Pensieve header: Hour 29: Perturbing the Heisenberg-algebra knot invariant.

Recall,

$$m_k^{ij} = e^{(\xi_i+\xi_j)\,x_k+(\pi_i+\pi_j)\,p_k-\xi_i\,\pi_j},$$

$$R_\epsilon = e^{(T-1)\,(p_i-p_j)\,x_j+R'};\ \ R' = \sum_{k=1}^{\$k} \epsilon^k\, R^{(k)},$$

$$_A\mathcal{L}_B\,//\,_B\mathcal{M}_C = e^{\sum_{i\in B}\partial_{z_i}\partial_{\zeta_i}}(\mathcal{L}\cdot\mathcal{M}),$$

$$\langle F:\mathcal{E}\rangle_B = e^{\frac{1}{2}\sum_{u,v\in B}F_{uv}\,\partial^u\partial^v}\,\mathcal{E}\,|_{z_B=0} \text{ and } [F:\mathcal{E}]_B = e^{\frac{1}{2}\sum_{u,v\in B}F_{uv}\,\partial^u\partial^v}\,\mathcal{E}$$

(Note, the two are equi-computable: clearly if we know how to compute $[F:\mathcal{E}]$ we also know how to compute $\langle F:\mathcal{E}\rangle$, and also $[F:\mathcal{E}]\,|_{z_B\to\bar{z}_B} = \langle F:\mathcal{E}\,|_{z_B\to z_B+\bar{z}_B}\rangle$).

$Z_\lambda := \log\left[\lambda F:e^E\right]$ satisfies $Z_0 = E$ and the "synthesis equation",

$$\partial_\lambda Z_\lambda = \frac{1}{2}\,F_{uv}(\partial_u\partial_v Z_\lambda + (\partial_u Z_\lambda)\,(\partial_v Z_\lambda)).$$

Lemma 1. $\left\langle F:\mathcal{E}\,e^{\frac{1}{2}\sum_{i,j\in B}G_{ij}\,z_i\,z_j}\right\rangle_B = \det(I-GF)^{-1/2}\,\langle F(I-GF)^{-1}:\mathcal{E}\rangle_B.$

Lemma 2. $\langle F:\mathcal{E}\,e^{\sum_{i\in B}y_i\,z_i}\rangle_B = e^{\frac{1}{2}\sum_{i,j\in B}F_{ij}\,z_i\,z_j}\,\langle F:\mathcal{E}\,|_{z_B\to z_B+Fy_B}\rangle_B.$

To solve the synthesis equation in general, we write $Z_\lambda = \sum Z[m]\,\lambda^m$ and then solve iteratively $Z[0] = Z_0 = E$ and

$$(m+1)\,Z[m+1] = \frac{1}{2}\,F_{uv}\big(\partial_u\partial_v Z[m] + \textstyle\sum_j(\partial_u Z[j])\cdot(\partial_v Z[m-j])\big).$$

Definition. A power series f in an auxiliary variable ϵ and in the z_i's, including $i\notin B$, is called *docile* if every monomial μ in it satisfies $\deg_z\mu \le 2\deg_\epsilon\mu + 2$; we will short that to $\deg_z f \le 2\deg_\epsilon f + 2$.

Claim 1. The synthesis equation preserves docility: if E is docile then so is Z_λ, and in particular, so is $\log\langle F:e^E\rangle$.

(And so it makes sense to restrict our attention to docile perturbations!)

Claim 2. Restricting attention to $\{z_i : i\in B\}$, if $\deg_{z_B} E \le 4\deg_\epsilon E$ then $\deg_{z_B} Z_\lambda \le 4\deg_\epsilon Z_\lambda - 2\deg_\lambda Z_\lambda$ and thus $\deg_\lambda Z_\lambda \le 2\deg_\epsilon Z_\lambda$.

Claim 2 implies that if $E\,|_{\epsilon=0}$ is independent of z_B and if we only care about Z_λ up to ϵ^k, then the iterative process for finding Z_λ terminates at $Z[2k]$.

Conclusion. We can compute efficiently (in polynomial time!) if all of our generating functions are of the form $\omega\,e^{Q+P}$, where ω is a scalar, Q is an ϵ-free quadratic, and $P = \sum_{k=1}^{\$k} P^{(k)}\,\epsilon^k$, where $\deg P^{(k)} \le 2k+2$.

On to the implementation...

𝔼[ω, Q, P_ϵSeries] represents $\omega\,e^{Q+P}$, where ω is a scalar, Q is an ϵ-free quadratic, and $P = \sum_{k=0}^{\$k} P[\![k]\!]\,\epsilon^k$ is a docile perturbation (it is ill-advised to include ω in P because then it will have log terms, so always, $P[\![0]\!] = 0$).

Initialization and minor utilities

```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Classes\\21-1350-KnotTheory"];
        Once[<< "Common.m"];
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
In[•]:= $k=1;
```

```
In[•]:= CCF[ℰ_] := ExpandDenominator@ExpandNumerator@Together[ℰ];
        CF[ℰ_List] := CF /@ ℰ;
        CF[ℰ_ϵSeries] := CF /@ ℰ;
        CF[ℰ_] := Module[
          {vs = Cases[ℰ, (p | x | π | ξ)_, ∞] ⋃ {p | x | π | ξ}},
          Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) :> CCF[c] (Times @@ vs^ps)]
        ];
        CF[ℰ_𝔼] := CF /@ ℰ;
        CF[𝔼_sp___[ℰs___]] := CF /@ 𝔼_sp[ℰs];
```

```
In[•]:= ϵSeries /: S1_ϵSeries ≡ S2_ϵSeries :=
          Length[S1] == Length[S2] ∧ Inner[CF[#1] == CF[#2] &, S1, S2, And];
        ϵSeries[0] := ϵSeries @@ Table[0, $k + 1];
        ϵSeries /: S1_ϵSeries + S2_ϵSeries :=
          ϵSeries @@ Table[S1[[k]] + S2[[k]], {k, Min[Length@S1, Length@S2]}];
        ϵSeries /: S1_ϵSeries * S2_ϵSeries := ϵSeries @@
            Table[Sum[S1[[j + 1]] * S2[[k - j + 1]], {j, 0, k}], {k, 0, Min[Length@S1, Length@S2] - 1}];
        ϵSeries /: c_ * S_ϵSeries := (c #) & /@ S;
        ϵSeries /: ∂vs___ S_ϵSeries := (s ↦ ∂vs s) /@ S;
```

The Main Program

Variables and their duals:

```
In[•]:= {p*, x*, π*, ξ*} = {π, ξ, p, x};
        (vs_List)* := (v ↦ v*) /@ vs;
        (u_i_)* := (u*)_i;
```

𝔼 operations:

```
In[•]:= 𝔼 /: 𝔼[ω1_, Q1_, P1_] ≡ 𝔼[ω2_, Q2_, P2_] := CF[ω1 == ω2] ∧ CF[Q1 == Q2] ∧ (P1 ≡ P2);
        𝔼 /: 𝔼[ω1_, Q1_, P1_] 𝔼[ω2_, Q2_, P2_] := 𝔼[ω1 ω2, Q1 + Q2, P1 + P2];
        𝔼_d1_→r1_[ℰ1s___] ≡ 𝔼_d2_→r2_[ℰ2s___] ^:= (d1 == d2) ∧ (r1 == r2) ∧ (𝔼[ℰ1s] ≡ 𝔼[ℰ2s]);
        𝔼_d1_→r1_[ℰ1s___] 𝔼_d2_→r2_[ℰ2s___] ^:= 𝔼_(d1⋃d2)→(r1⋃r2) @@ (𝔼[ℰ1s] 𝔼[ℰ2s]);
```

```
In[•]:= 𝔼d1_→r1_[ℰ1s___] // 𝔼d2_→r2_[ℰ2s___] := Module[{is = r1 ⋂ d2, lvs},
  lvs = Flatten@Table[{x$@i, p$@i}, {i, is}];
  𝔼(d1⋃Complement[d2,is])→(r2⋃Complement[r1,is]) @@ (Zip_{lvs⋃lvs*}[lvs*.lvs, Times[
      𝔼[ℰ1s] /. Table[(v : x | p)_i → v$@i, {i, is}],
      𝔼[ℰ2s] /. Table[(v : ξ | π)_i → v$@i, {i, is}]
    ]])
 ]
```

```
In[•]:= Zip_vs_[ℱ_, ℰ_] := ⟨ℱ, ℰ⟩ // Zip1_vs // Zip2_vs // Zip3_vs
```

Getting rid of the quadratic using lemma 1.

Lemma 1. $\left\langle F : \mathcal{E}\, e^{\frac{1}{2}\sum_{i,j\in B} G_{ij} z_i z_j} \right\rangle_B = \det(I - \mathrm{GF})^{-1/2} \left\langle F(I - \mathrm{GF})^{-1} : \mathcal{E} \right\rangle_B$.

```
In[•]:= Zip1_{} = Identity;
Zip1_vs_@⟨ℱ_, 𝔼[ω_, Q_, P_]⟩ := Module[{ℐ, F, G, u, v},
  ℐ = IdentityMatrix@Length@vs;
  F = Table[∂u,v ℱ, {u, vs*}, {v, vs*}];
  G = Table[∂u,v Q, {u, vs}, {v, vs}];
  CF /@ ⟨vs*.F.Inverse[ℐ - G.F].vs* / 2,
    𝔼[PowerExpand@Factor[ω Det[ℐ - G.F]^-1/2], Q - vs.G.vs / 2, P]⟩
 ]
```

Getting rid of linear terms using Lemma 2.

Lemma 2. $\langle F : \mathcal{E}\, e^{\sum_{i\in B} y_i z_i} \rangle_B = e^{\frac{1}{2}\sum_{i,j\in B} F_{ij} z_i z_j} \left\langle F : \mathcal{E} \mid_{Z_B \to Z_B + \mathrm{F} y_B} \right\rangle_B$.

```
In[•]:= Zip2_{} = Identity;
Zip2_vs_@⟨ℱ_, 𝔼[ω_, Q_, P_]⟩ := Module[{F, Y, u, v},
  F = Table[∂u,v ℱ, {u, vs*}, {v, vs*}];
  Y = Table[∂v Q, {v, vs}];
  CF /@ ⟨ℱ, 𝔼[ω, Q - Y.vs + Y.F.Y / 2, P /. Thread[vs → vs + F.Y]]⟩
 ]
```

Dealing with Feynman diagrams without ever seeing them, using the synthesis equation and iteration. Write $Z_\lambda = \sum Z[m]\,\lambda^m$ and then $Z[0] = Z_0 = E$ and

$$Z[m+1] = \frac{1}{2\,(m+1)}\, F_{uv}\big(\partial_u \partial_v Z[m] + \textstyle\sum_j (\partial_u Z[j]) \cdot (\partial_v Z[m-j])\big),$$

and we only care to compute up to $Z[[2\ \$k]]$:

```
In[•]:= Zip3vs_@⟨ℱ_, 𝔼[ω_, Q_, P_]⟩ := Module[{Z, u, v, m, j},
  Z[0] = P;
  For[m = 0, m < 2 $k, ++m,
   Z[m + 1] = CF[1/(2 (m + 1))
      Sum[∂u*,v* ℱ (∂u,v Z[m] + Sum[(∂u Z[j]) (∂v Z[m - j]), {j, 0, m}]), {u, vs}, {v, vs}]]
  ];
  𝔼[ω, Q, CF[Sum[Z[m], {m, 0, 2 $k}] /. Table[v → 0, {v, vs}]]]
 ]
```

The Basic Tensors

```
In[•]:= η_ := 𝔼{}→{i}[1, 0, ϵSeries[0]];
m_,_→_ := 𝔼{i,j}→{k}[1, -ξi πj + (πi + πj) pk + (ξi + ξj) xk, ϵSeries[0]]
```

```
In[•]:= AllMonomials[{}, 0] = {1};
AllMonomials[{}, d_Integer] /; d > 0 := {};
AllMonomials[{v_, vs___}, d_Integer] :=
  Join @@ Table[v^(d-k) AllMonomials[{vs}, k], {k, 0, d}];
AllMonomials[vs_List, {d_}] := Join @@ Table[AllMonomials[vs, k], {k, 0, d}];
```

```
In[•]:= Basis[js_List, m_] := Flatten@Outer[Times,
    AllMonomials[Table[pj, {j, js}], m], AllMonomials[Table[xj, {j, js}], m]];
Basis[js_List, {m_}] := Flatten@Table[Basis[js, k], {k, 0, m}]
```

```
In[•]:= Basis[{i, j}, {2}]
```

$$\text{Out[•]=}\ \{1,\ p_i\,x_i,\ p_i\,x_j,\ p_j\,x_i,\ p_j\,x_j,\ p_i^2\,x_i^2,\ p_i^2\,x_i\,x_j,\ p_i^2\,x_j^2,\ p_i\,p_j\,x_i^2,\ p_i\,p_j\,x_i\,x_j,\ p_i\,p_j\,x_j^2,\ p_j^2\,x_i^2,\ p_j^2\,x_i\,x_j,\ p_j^2\,x_j^2\}$$

```
In[•]:= GenericCombination[bas_, c_] := bas.Table[cj, {j, Length@bas}];
GenericCombination[bas_, c_k_] := bas.Table[ck,j, {j, Length@bas}];
```

```
In[•]:= GenericCombination[Basis[{i, j}, {2}], c1]
```

$$\text{Out[•]=}\ c_{1,1} + p_i\,x_i\,c_{1,2} + p_i\,x_j\,c_{1,3} + p_j\,x_i\,c_{1,4} + p_j\,x_j\,c_{1,5} + p_i^2\,x_i^2\,c_{1,6} + p_i^2\,x_i\,x_j\,c_{1,7} + p_i^2\,x_j^2\,c_{1,8} +$$
$$p_i\,p_j\,x_i^2\,c_{1,9} + p_i\,p_j\,x_i\,x_j\,c_{1,10} + p_i\,p_j\,x_j^2\,c_{1,11} + p_j^2\,x_i^2\,c_{1,12} + p_j^2\,x_i\,x_j\,c_{1,13} + p_j^2\,x_j^2\,c_{1,14}$$

In[•]:=
```
R_{i_,j_} := 𝔼_{}→{i,j}[T^{1/2}, (T - 1) (p_i - p_j) x_j ,
    ϵSeries @@ Prepend[0] @ Table[GenericCombination[Basis[{i, j}, {k + 1}], c_k], {k, $k}]];
R̄_{i_,j_} := 𝔼_{}→{i,j}[T^{-1/2}, (T^{-1} - 1) (p_i - p_j) x_j,
    ϵSeries @@ Prepend[0] @ Table[GenericCombination[Basis[{i, j}, {k + 1}], d_k], {k, $k}]];
C_{i_} := 𝔼_{}→{i}[T^{1/2}, 0, ϵSeries @@ Prepend[0] @
      Table[GenericCombination[Basis[{i}, {k + 1}], e_k], {k, $k}]];
C̄_{i_} := 𝔼_{}→{i}[T^{-1/2}, 0, ϵSeries @@ Prepend[0] @
      Table[GenericCombination[Basis[{i}, {k + 1}], f_k], {k, $k}]];
```

In[•]:=
```
{R_{1,2}, R̄_{1,2}, C_1, C̄_1}
```

Out[•]=
$$\Big\{\mathbb{E}_{\{\}\to\{1,2\}}\Big[\sqrt{T},\ (-1+T)\ (p_1-p_2)\ x_2,$$
$$\epsilon\text{Series}\Big[0,\ c_{1,1}+p_1 x_1 c_{1,2}+p_1 x_2 c_{1,3}+p_2 x_1 c_{1,4}+p_2 x_2 c_{1,5}+p_1^2 x_1^2 c_{1,6}+p_1^2 x_1 x_2 c_{1,7}+p_1^2 x_2^2 c_{1,8}+$$
$$p_1 p_2 x_1^2 c_{1,9}+p_1 p_2 x_1 x_2 c_{1,10}+p_1 p_2 x_2^2 c_{1,11}+p_2^2 x_1^2 c_{1,12}+p_2^2 x_1 x_2 c_{1,13}+p_2^2 x_2^2 c_{1,14}\Big]\Big],$$
$$\mathbb{E}_{\{\}\to\{1,2\}}\Big[\frac{1}{\sqrt{T}},\ \Big(-1+\frac{1}{T}\Big)\ (p_1-p_2)\ x_2,\ \epsilon\text{Series}\Big[0,$$
$$d_{1,1}+p_1 x_1 d_{1,2}+p_1 x_2 d_{1,3}+p_2 x_1 d_{1,4}+p_2 x_2 d_{1,5}+p_1^2 x_1^2 d_{1,6}+p_1^2 x_1 x_2 d_{1,7}+p_1^2 x_2^2 d_{1,8}+$$
$$p_1 p_2 x_1^2 d_{1,9}+p_1 p_2 x_1 x_2 d_{1,10}+p_1 p_2 x_2^2 d_{1,11}+p_2^2 x_1^2 d_{1,12}+p_2^2 x_1 x_2 d_{1,13}+p_2^2 x_2^2 d_{1,14}\Big]\Big],$$
$$\mathbb{E}_{\{\}\to\{1\}}\Big[\sqrt{T},\ 0,\ \epsilon\text{Series}\Big[0,\ e_{1,1}+p_1 x_1 e_{1,2}+p_1^2 x_1^2 e_{1,3}\Big]\Big],$$
$$\mathbb{E}_{\{\}\to\{1\}}\Big[\frac{1}{\sqrt{T}},\ 0,\ \epsilon\text{Series}\Big[0,\ f_{1,1}+p_1 x_1 f_{1,2}+p_1^2 x_1^2 f_{1,3}\Big]\Big]\Big\}$$

In[•]:=
```
RMoves := {
  (R_{1,2} R_{4,3} R_{5,6} // m_{1,4→1} // m_{2,5→2} // m_{3,6→3}) ≡ (R_{2,3} R_{4,5} R_{1,6} // m_{1,4→1} // m_{2,5→2} // m_{3,6→3}),
  (R_{1,2} R̄_{3,4} // m_{1,3→1} // m_{2,4→2}) ≡ (η_1 η_2),
  (C_1 C̄_2 // m_{1,2→1}) ≡ η_1,
  (R_{1,4} R̄_{5,2} C̄_3 // m_{2,4→2} // m_{1,3→1} // m_{1,5→1}) ≡ C̄_1 η_2,
  (C_3 R_{1,2} // m_{2,3→2} // m_{2,1→1}) ≡ (C̄_3 R_{1,2} // m_{1,3→1} // m_{1,2→1}),
  (C̄_2 R_{1,3} // m_{1,2→1} // m_{1,3→1}) ≡ η_1,  (C̄_2 R̄_{3,1} // m_{1,2→1} // m_{1,3→1}) ≡ η_1,
  (C_2 R̄_{1,3} // m_{1,2→1} // m_{1,3→1}) ≡ η_1, (C_2 R_{3,1} // m_{1,2→1} // m_{1,3→1}) ≡ η_1
 }
```

Solving for R, C, \$k = 1

In[•]:=
```
$k = 1;
{R1,2, C1}
unknowns = Cases[{R1,2, R̄1,2, C1, C̄1}, (c | d | e | f)$k,_, ∞] // Union
```

Out[•]=
$$\Big\{\mathbb{E}_{\{\}\to\{1,2\}}\Big[\sqrt{T},\ (-1+T)\ (p_1-p_2)\ x_2,$$
$$\in\text{Series}\Big[0,\ c_{1,1}+p_1\,x_1\,c_{1,2}+p_1\,x_2\,c_{1,3}+p_2\,x_1\,c_{1,4}+p_2\,x_2\,c_{1,5}+p_1^2\,x_1^2\,c_{1,6}+p_1^2\,x_1\,x_2\,c_{1,7}+p_1^2\,x_2^2\,c_{1,8}+$$
$$p_1\,p_2\,x_1^2\,c_{1,9}+p_1\,p_2\,x_1\,x_2\,c_{1,10}+p_1\,p_2\,x_2^2\,c_{1,11}+p_2^2\,x_1^2\,c_{1,12}+p_2^2\,x_1\,x_2\,c_{1,13}+p_2^2\,x_2^2\,c_{1,14}\Big]\Big],$$
$$\mathbb{E}_{\{\}\to\{1\}}\Big[\sqrt{T},\ 0,\ \in\text{Series}\Big[0,\ e_{1,1}+p_1\,x_1\,e_{1,2}+p_1^2\,x_1^2\,e_{1,3}\Big]\Big]\Big\}$$

Out[•]=
$$\{c_{1,1},\ c_{1,2},\ c_{1,3},\ c_{1,4},\ c_{1,5},\ c_{1,6},\ c_{1,7},\ c_{1,8},\ c_{1,9},\ c_{1,10},\ c_{1,11},\ c_{1,12},\ c_{1,13},\ c_{1,14},\ d_{1,1},\ d_{1,2},\ d_{1,3},$$
$$d_{1,4},\ d_{1,5},\ d_{1,6},\ d_{1,7},\ d_{1,8},\ d_{1,9},\ d_{1,10},\ d_{1,11},\ d_{1,12},\ d_{1,13},\ d_{1,14},\ e_{1,1},\ e_{1,2},\ e_{1,3},\ f_{1,1},\ f_{1,2},\ f_{1,3}\}$$

In[•]:=
```
Short[errors = CCF /@ Cases[RMoves, a_ == b_ :> a - b], 25]
```

Out[•]//Short=
$$\Big\{T\,p_1\,x_3\,c_{1,2}-T^2\,p_1\,x_3\,c_{1,2}+p_1\,x_3\,c_{1,3}-T\,p_1\,x_3\,c_{1,3}-p_2\,x_1\,c_{1,4}+T\,p_2\,x_1\,c_{1,4}+p_3\,x_1\,c_{1,4}-T\,p_3\,x_1\,c_{1,4}+$$
$$p_1\,x_2\,c_{1,4}-T\,p_1\,x_2\,c_{1,4}-p_2\,x_2\,c_{1,4}+2\,T\,p_2\,x_2\,c_{1,4}-T^2\,p_2\,x_2\,c_{1,4}-T\,p_3\,x_2\,c_{1,4}+T^2\,p_3\,x_2\,c_{1,4}+$$
$$T\,p_2\,x_3\,c_{1,4}-T^2\,p_2\,x_3\,c_{1,4}+p_1\,x_3\,c_{1,5}-T\,p_1\,x_3\,c_{1,5}-2\,p_1^2\,x_1\,x_2\,c_{1,6}+2\,T\,p_1^2\,x_1\,x_2\,c_{1,6}+2\,T\,p_1\,p_2\,x_2^2\,c_{1,6}-$$
$$2\,T^2\,p_1\,p_2\,x_2^2\,c_{1,6}+2\,T\,p_1^2\,x_1\,x_3\,c_{1,6}-2\,T^2\,p_1^2\,x_1\,x_3\,c_{1,6}+T^2\,p_1^2\,x_3^2\,c_{1,6}-2\,T^3\,p_1^2\,x_3^2\,c_{1,6}+T^4\,p_1^2\,x_3^2\,c_{1,6}+$$
$$2\,T\,p_1\,p_2\,x_2\,x_3\,c_{1,7}-2\,T^2\,p_1\,p_2\,x_2\,x_3\,c_{1,7}+T\,p_1^2\,x_3^2\,c_{1,7}-2\,T^2\,p_1^2\,x_3^2\,c_{1,7}+T^3\,p_1^2\,x_3^2\,c_{1,7}+2\,p_1^2\,x_2\,x_3\,c_{1,8}-$$
$$2\,T\,p_1^2\,x_2\,x_3\,c_{1,8}+p_1^2\,x_3^2\,c_{1,8}-4\,T\,p_1^2\,x_3^2\,c_{1,8}+3\,T^2\,p_1^2\,x_3^2\,c_{1,8}+2\,T\,p_1\,p_2\,x_3^2\,c_{1,8}-2\,T^2\,p_1\,p_2\,x_3^2\,c_{1,8}-$$
$$p_1\,p_2\,x_1^2\,c_{1,9}+T\,p_1\,p_2\,x_1^2\,c_{1,9}+p_1\,p_3\,x_1^2\,c_{1,9}-T\,p_1\,p_3\,x_1^2\,c_{1,9}-2\,p_1\,p_2\,x_1\,x_2\,c_{1,9}+4\,T\,p_1\,p_2\,x_1\,x_2\,c_{1,9}-$$
$$2\,T^2\,p_1\,p_2\,x_1\,x_2\,c_{1,9}-2\,T\,p_1\,p_3\,x_1\,x_2\,c_{1,9}+2\,T^2\,p_1\,p_3\,x_1\,x_2\,c_{1,9}+p_1^2\,x_2^2\,c_{1,9}-2\,T\,p_1^2\,x_2^2\,c_{1,9}+$$
$$T^2\,p_1^2\,x_2^2\,c_{1,9}-p_1\,p_2\,x_2^2\,c_{1,9}+\text{<<62>>}+T^2\,p_1^2\,x_2^2\,c_{1,12}-p_2^2\,x_2^2\,c_{1,12}+4\,T\,p_2^2\,x_2^2\,c_{1,12}-6\,T^2\,p_2^2\,x_2^2\,c_{1,12}+$$
$$4\,T^3\,p_2^2\,x_2^2\,c_{1,12}-T^4\,p_2^2\,x_2^2\,c_{1,12}+2\,T\,p_1\,p_3\,x_2^2\,c_{1,12}-2\,T^2\,p_1\,p_3\,x_2^2\,c_{1,12}-2\,T\,p_2\,p_3\,x_2^2\,c_{1,12}+$$
$$6\,T^2\,p_2\,p_3\,x_2^2\,c_{1,12}-6\,T^3\,p_2\,p_3\,x_2^2\,c_{1,12}+2\,T^4\,p_2\,p_3\,x_2^2\,c_{1,12}-T^2\,p_3^2\,x_2^2\,c_{1,12}+2\,T^3\,p_3^2\,x_2^2\,c_{1,12}-$$
$$T^4\,p_3^2\,x_2^2\,c_{1,12}+2\,T\,p_2^2\,x_1\,x_3\,c_{1,12}-2\,T^2\,p_2^2\,x_1\,x_3\,c_{1,12}+T^2\,p_2^2\,x_3^2\,c_{1,12}-2\,T^3\,p_2^2\,x_3^2\,c_{1,12}+T^4\,p_2^2\,x_3^2\,c_{1,12}+$$
$$T\,p_2^2\,x_1\,x_3\,c_{1,13}-T^2\,p_2^2\,x_1\,x_3\,c_{1,13}-2\,T\,p_2\,p_3\,x_1\,x_3\,c_{1,13}+2\,T^2\,p_2\,p_3\,x_1\,x_3\,c_{1,13}+T\,p_3^2\,x_1\,x_3\,c_{1,13}-$$
$$T^2\,p_3^2\,x_1\,x_3\,c_{1,13}+p_1^2\,x_2\,x_3\,c_{1,13}-2\,T\,p_1^2\,x_2\,x_3\,c_{1,13}+T^2\,p_1^2\,x_2\,x_3\,c_{1,13}-p_2^2\,x_2\,x_3\,c_{1,13}+4\,T\,p_2^2\,x_2\,x_3\,c_{1,13}-$$
$$4\,T^2\,p_2^2\,x_2\,x_3\,c_{1,13}+T^3\,p_2^2\,x_2\,x_3\,c_{1,13}+2\,T\,p_1\,p_3\,x_2\,x_3\,c_{1,13}-2\,T^2\,p_1\,p_3\,x_2\,x_3\,c_{1,13}-2\,T\,p_2\,p_3\,x_2\,x_3\,c_{1,13}+$$
$$4\,T^2\,p_2\,p_3\,x_2\,x_3\,c_{1,13}-2\,T^3\,p_2\,p_3\,x_2\,x_3\,c_{1,13}-T^2\,p_3^2\,x_2\,x_3\,c_{1,13}+T^3\,p_3^2\,x_2\,x_3\,c_{1,13}+T\,p_2^2\,x_3^2\,c_{1,13}-$$
$$2\,T^2\,p_2^2\,x_3^2\,c_{1,13}+T^3\,p_2^2\,x_3^2\,c_{1,13}+2\,p_2^2\,x_2\,x_3\,c_{1,14}-2\,T\,p_2^2\,x_2\,x_3\,c_{1,14}+p_1^2\,x_3^2\,c_{1,14}-2\,T\,p_1^2\,x_3^2\,c_{1,14}+$$
$$T^2\,p_1^2\,x_3^2\,c_{1,14}+2\,T\,p_1\,p_3\,x_3^2\,c_{1,14}-2\,T^2\,p_1\,p_3\,x_3^2\,c_{1,14}-2\,T\,p_2\,p_3\,x_3^2\,c_{1,14}+2\,T^2\,p_2\,p_3\,x_3^2\,c_{1,14},$$
$$\frac{\text{<<1>>}}{T^2},\ \text{<<5>>},\ \frac{\text{<<1>>}}{\text{<<1>>}},\ \frac{T^2\,c_{\text{<<1>>}}+\text{<<36>>}+\text{<<1>>}}{T^2}\Big\}$$

In[•]:=
```
eqns = Thread[0 == Union @@ (CoefficientRules[#, {x1, x2, x3, p1, p2, p3}][[ ;; , 2]] & /@ errors)]
```

Out[•]=
$$\Big\{0==c_{1,4}-T\,c_{1,4},\ 0==-c_{1,4}+T\,c_{1,4},\ 0==T\,c_{1,4}-T^2\,c_{1,4},\ 0==-c_{1,4}+2\,T\,c_{1,4}-T^2\,c_{1,4},$$
$$0==-T\,c_{1,4}+T^2\,c_{1,4},\ 0==T\,c_{1,2}-T^2\,c_{1,2}+c_{1,3}-T\,c_{1,3}+c_{1,5}-T\,c_{1,5},$$
$$0==-2\,c_{1,6}+2\,T\,c_{1,6},\ 0==2\,T\,c_{1,6}-2\,T^2\,c_{1,6},\ 0==c_{1,9}-T\,c_{1,9},$$
$$0==-c_{1,9}+T\,c_{1,9},\ 0==2\,T\,c_{1,9}-2\,T^2\,c_{1,9},\ 0==-2\,c_{1,9}+4\,T\,c_{1,9}-2\,T^2\,c_{1,9},$$
$$0==-2\,T\,c_{1,9}+2\,T^2\,c_{1,9},\ 0==2\,T\,c_{1,6}-2\,T^2\,c_{1,6}-c_{1,9}+4\,T\,c_{1,9}-4\,T^2\,c_{1,9}+T^3\,c_{1,9},$$
$$0==2\,T\,c_{1,8}-2\,T^2\,c_{1,8}+T^2\,c_{1,9}-2\,T^3\,c_{1,9}+T^4\,c_{1,9}+T\,c_{1,10}-2\,T^2\,c_{1,10}+T^3\,c_{1,10},$$
$$0==2\,T\,c_{1,7}-2\,T^2\,c_{1,7}-c_{1,10}+4\,T\,c_{1,10}-3\,T^2\,c_{1,10}+2\,c_{1,11}-2\,T\,c_{1,11},$$

$$0 == T^2 c_{1,9} - T^3 c_{1,9} + 2 T c_{1,12} - 2 T^2 c_{1,12},\ 0 == c_{1,12} - T^2 c_{1,12},\ 0 == -c_{1,12} + 2 T c_{1,12} - T^2 c_{1,12},$$

$$0 == c_{1,9} - 2 T c_{1,9} + T^2 c_{1,9} + c_{1,12} - 2 T c_{1,12} + T^2 c_{1,12},\ 0 == -2 T c_{1,12} + 2 T^2 c_{1,12},$$

$$0 == -4 T c_{1,12} + 8 T^2 c_{1,12} - 4 T^3 c_{1,12},\ 0 == -2 c_{1,12} + 6 T c_{1,12} - 6 T^2 c_{1,12} + 2 T^3 c_{1,12},$$

$$0 == -2 T^2 c_{1,12} + 2 T^3 c_{1,12},\ 0 == -T^2 c_{1,12} + 2 T^3 c_{1,12} - T^4 c_{1,12},$$

$$0 == -c_{1,12} + 4 T c_{1,12} - 6 T^2 c_{1,12} + 4 T^3 c_{1,12} - T^4 c_{1,12},\ 0 == -2 T c_{1,12} + 6 T^2 c_{1,12} - 6 T^3 c_{1,12} + 2 T^4 c_{1,12},$$

$$0 == 2 T c_{1,13} - 2 T^2 c_{1,13},\ 0 == T c_{1,13} - T^2 c_{1,13},\ 0 == 2 T c_{1,12} - 2 T^2 c_{1,12} + T c_{1,13} - T^2 c_{1,13},$$

$$0 == 2 c_{1,8} - 2 T c_{1,8} + c_{1,10} - 2 T c_{1,10} + T^2 c_{1,10} + c_{1,13} - 2 T c_{1,13} + T^2 c_{1,13},\ 0 == -2 T c_{1,13} + 2 T^2 c_{1,13},$$

$$0 == -2 T c_{1,13} + 4 T^2 c_{1,13} - 2 T^3 c_{1,13},\ 0 == T^2 c_{1,12} - 2 T^3 c_{1,12} + T^4 c_{1,12} + T c_{1,13} - 2 T^2 c_{1,13} + T^3 c_{1,13},$$

$$0 == -T^2 c_{1,13} + T^3 c_{1,13},\ 0 == -c_{1,13} + 4 T c_{1,13} - 4 T^2 c_{1,13} + T^3 c_{1,13} + 2 c_{1,14} - 2 T c_{1,14},$$

$$0 == 2 T c_{1,14} - 2 T^2 c_{1,14},\ 0 == T^2 c_{1,6} - 2 T^3 c_{1,6} + T^4 c_{1,6} + T c_{1,7} - 2 T^2 c_{1,7} + T^3 c_{1,7} +$$
$$c_{1,8} - 4 T c_{1,8} + 3 T^2 c_{1,8} + c_{1,11} - 2 T c_{1,11} + T^2 c_{1,11} + c_{1,14} - 2 T c_{1,14} + T^2 c_{1,14},$$

$$0 == -2 T c_{1,14} + 2 T^2 c_{1,14},\ 0 == c_{1,1} + d_{1,1},\ 0 == c_{1,1} - c_{1,4} + \frac{c_{1,4}}{T} + 2 c_{1,12} + \frac{2 c_{1,12}}{T^2} - \frac{4 c_{1,12}}{T} + d_{1,1},$$

$$0 == c_{1,2} + c_{1,4} - \frac{c_{1,4}}{T} - 2 c_{1,9} + \frac{2 c_{1,9}}{T} - 4 c_{1,12} - \frac{4 c_{1,12}}{T^2} + \frac{8 c_{1,12}}{T} + d_{1,2},$$

$$0 == \frac{c_{1,4}}{T} + \frac{4 c_{1,12}}{T^2} - \frac{4 c_{1,12}}{T} + d_{1,4},\ 0 == c_{1,2} + d_{1,2} + d_{1,4} - T d_{1,4},\ 0 == c_{1,4} + T d_{1,4},$$

$$0 == c_{1,2} - \frac{c_{1,2}}{T} + \frac{c_{1,3}}{T} + d_{1,3} + d_{1,5} - T d_{1,5},\ 0 == c_{1,4} - \frac{c_{1,4}}{T} + \frac{c_{1,5}}{T} + T d_{1,5},$$

$$0 == c_{1,4} - \frac{c_{1,4}}{T} + \frac{c_{1,5}}{T} - 4 c_{1,12} - \frac{4 c_{1,12}}{T^2} + \frac{8 c_{1,12}}{T} + \frac{2 c_{1,13}}{T^2} - \frac{2 c_{1,13}}{T} + T d_{1,5},$$

$$0 == c_{1,6} + c_{1,9} - \frac{c_{1,9}}{T} + c_{1,12} + \frac{c_{1,12}}{T^2} - \frac{2 c_{1,12}}{T} + d_{1,6},\ 0 == \frac{c_{1,9}}{T} - \frac{2 c_{1,12}}{T^2} + \frac{2 c_{1,12}}{T} + d_{1,9},$$

$$0 == 2 c_{1,9} - \frac{2 c_{1,9}}{T} + \frac{c_{1,10}}{T} + 4 c_{1,12} + \frac{4 c_{1,12}}{T^2} - \frac{8 c_{1,12}}{T} - \frac{2 c_{1,13}}{T^2} + \frac{2 c_{1,13}}{T} + T d_{1,10},$$

$$0 == -2 c_{1,9} + \frac{c_{1,9}}{T} + T c_{1,9} + c_{1,10} - \frac{c_{1,10}}{T} + \frac{c_{1,11}}{T} - 6 c_{1,12} - \frac{2 c_{1,12}}{T^2} +$$
$$\frac{6 c_{1,12}}{T} + 2 T c_{1,12} + 2 c_{1,13} + \frac{2 c_{1,13}}{T^2} - \frac{4 c_{1,13}}{T} - \frac{2 c_{1,14}}{T^2} + \frac{2 c_{1,14}}{T} + T^2 d_{1,11},$$

$$0 == \frac{c_{1,12}}{T^2} + d_{1,12},\ 0 == c_{1,9} + T d_{1,9} + 2 T d_{1,12} - 2 T^2 d_{1,12},\ 0 == c_{1,12} + T^2 d_{1,12},$$

$$0 == c_{1,6} + d_{1,6} + d_{1,9} - T d_{1,9} + d_{1,12} - 2 T d_{1,12} + T^2 d_{1,12},\ 0 == -\frac{2 c_{1,12}}{T^2} + \frac{2 c_{1,12}}{T} + \frac{c_{1,13}}{T^2} + T d_{1,13},$$

$$0 == 2 c_{1,9} - \frac{2 c_{1,9}}{T} + \frac{c_{1,10}}{T} + T d_{1,10} + 2 T d_{1,13} - 2 T^2 d_{1,13},\ 0 == 2 c_{1,12} - \frac{2 c_{1,12}}{T} + \frac{c_{1,13}}{T} + T^2 d_{1,13},$$

$$0 == 2 c_{1,6} - \frac{2 c_{1,6}}{T} + \frac{c_{1,7}}{T} + d_{1,7} + d_{1,10} - T d_{1,10} + d_{1,13} - 2 T d_{1,13} + T^2 d_{1,13},$$

$$0 == c_{1,9} + \frac{c_{1,9}}{T^2} - \frac{2 c_{1,9}}{T} - \frac{c_{1,10}}{T^2} + \frac{c_{1,10}}{T} + \frac{c_{1,11}}{T^2} + T d_{1,11} + 2 T d_{1,14} - 2 T^2 d_{1,14},$$

$$0 == c_{1,12} + \frac{c_{1,12}}{T^2} - \frac{2 c_{1,12}}{T} - \frac{c_{1,13}}{T^2} + \frac{c_{1,13}}{T} + \frac{c_{1,14}}{T^2} + T^2 d_{1,14},$$

$$0 == c_{1,6} + \frac{c_{1,6}}{T^2} - \frac{2 c_{1,6}}{T} - \frac{c_{1,7}}{T^2} + \frac{c_{1,7}}{T} + \frac{c_{1,8}}{T^2} + d_{1,8} + d_{1,11} - T d_{1,11} + d_{1,14} - 2 T d_{1,14} + T^2 d_{1,14},$$

$$0 == d_{1,1} - d_{1,4} + 2\,d_{1,12} + e_{1,1},\ 0 == \frac{d_{1,2}}{T} + d_{1,3} + \frac{d_{1,4}}{T} + d_{1,5} - \frac{2\,d_{1,9}}{T} - d_{1,10} - \frac{4\,d_{1,12}}{T} - 2\,d_{1,13} + \frac{e_{1,2}}{T},$$

$$0 == c_{1,6} + \frac{c_{1,7}}{T} + \frac{c_{1,8}}{T^2} + c_{1,9} + \frac{c_{1,10}}{T} + \frac{c_{1,11}}{T^2} + c_{1,12} + \frac{c_{1,13}}{T} + \frac{c_{1,14}}{T^2} + \frac{e_{1,3}}{T^2},$$

$$0 == \frac{d_{1,6}}{T^2} + \frac{d_{1,7}}{T} + d_{1,8} + \frac{d_{1,9}}{T^2} + \frac{d_{1,10}}{T} + d_{1,11} + \frac{d_{1,12}}{T^2} + \frac{d_{1,13}}{T} + d_{1,14} + \frac{e_{1,3}}{T^2},$$

$$0 == c_{1,1} - \frac{c_{1,3}}{T} + \frac{2\,c_{1,8}}{T^2} + e_{1,1} + e_{1,2} - \frac{e_{1,2}}{T} + 2\,e_{1,3} + \frac{2\,e_{1,3}}{T^2} - \frac{4\,e_{1,3}}{T},$$

$$0 == c_{1,2} + \frac{c_{1,3}}{T} + c_{1,4} + \frac{c_{1,5}}{T} - \frac{2\,c_{1,7}}{T} - \frac{4\,c_{1,8}}{T^2} - \frac{c_{1,10}}{T} - \frac{2\,c_{1,11}}{T^2} + \frac{e_{1,2}}{T} - \frac{4\,e_{1,3}}{T^2} + \frac{4\,e_{1,3}}{T},$$

$$0 == -\frac{c_{1,3}}{T} + c_{1,4} + \frac{2\,c_{1,8}}{T^2} - 2\,c_{1,12} + e_{1,1} + e_{1,2} - \frac{e_{1,2}}{T} + 2\,e_{1,3} + \frac{2\,e_{1,3}}{T^2} - \frac{4\,e_{1,3}}{T} - f_{1,1},$$

$$0 == c_{1,1} - c_{1,4} + 2\,c_{1,12} + f_{1,1},\ 0 == e_{1,1} + f_{1,1},\ 0 == e_{1,2} + f_{1,2},$$

$$0 == c_{1,2} - T\,c_{1,2} - c_{1,3} + \frac{c_{1,3}}{T} + c_{1,4} - T\,c_{1,4} - c_{1,5} + \frac{c_{1,5}}{T} - \frac{2\,c_{1,7}}{T} - \frac{4\,c_{1,8}}{T^2} +$$
$$2\,T\,c_{1,9} + c_{1,10} - \frac{c_{1,10}}{T} - \frac{2\,c_{1,11}}{T^2} + 4\,T\,c_{1,12} + 2\,c_{1,13} + \frac{e_{1,2}}{T} - \frac{4\,e_{1,3}}{T^2} + \frac{4\,e_{1,3}}{T} - T\,f_{1,2},$$

$$0 == T\,c_{1,2} + c_{1,3} + T\,c_{1,4} + c_{1,5} - 2\,T\,c_{1,9} - c_{1,10} - 4\,T\,c_{1,12} - 2\,c_{1,13} + T\,f_{1,2},$$

$$0 == -c_{1,2} + T\,c_{1,2} + c_{1,3} - 2\,c_{1,4} + \frac{c_{1,4}}{T} + T\,c_{1,4} + c_{1,5} - \frac{c_{1,5}}{T} + 4\,c_{1,9} - \frac{2\,c_{1,9}}{T} - 2\,T\,c_{1,9} - c_{1,10} + \frac{c_{1,10}}{T} +$$
$$12\,c_{1,12} + \frac{4\,c_{1,12}}{T^2} - \frac{12\,c_{1,12}}{T} - 4\,T\,c_{1,12} - 2\,c_{1,13} - \frac{2\,c_{1,13}}{T^2} + \frac{4\,c_{1,13}}{T} + T\,d_{1,3} - f_{1,2} + T\,f_{1,2},$$

$$0 == e_{1,3} + f_{1,3},\ 0 == -2\,c_{1,6} + 2\,T\,c_{1,6} + c_{1,7} - 4\,c_{1,9} + \frac{2\,c_{1,9}}{T} + 2\,T\,c_{1,9} + c_{1,10} - \frac{c_{1,10}}{T} -$$
$$6\,c_{1,12} - \frac{2\,c_{1,12}}{T^2} + \frac{6\,c_{1,12}}{T} + 2\,T\,c_{1,12} + c_{1,13} + \frac{c_{1,13}}{T^2} - \frac{2\,c_{1,13}}{T} + T\,d_{1,7} - 2\,f_{1,3} + 2\,T\,f_{1,3},$$

$$0 == d_{1,2} + T\,d_{1,3} + d_{1,4} + T\,d_{1,5} - 2\,T\,d_{1,7} - 4\,T^2\,d_{1,8} - T\,d_{1,10} - 2\,T^2\,d_{1,11} + T\,f_{1,2} + 4\,T\,f_{1,3} - 4\,T^2\,f_{1,3},$$

$$0 == c_{1,6} - T^2\,c_{1,6} + \frac{c_{1,7}}{T} - T\,c_{1,7} - c_{1,8} + \frac{c_{1,8}}{T^2} + c_{1,9} - T^2\,c_{1,9} + \frac{c_{1,10}}{T} - T\,c_{1,10} -$$
$$c_{1,11} + \frac{c_{1,11}}{T^2} + c_{1,12} - T^2\,c_{1,12} + \frac{c_{1,13}}{T} - T\,c_{1,13} - c_{1,14} + \frac{c_{1,14}}{T^2} + \frac{e_{1,3}}{T^2} - T^2\,f_{1,3},$$

$$0 == T^2\,c_{1,6} + T\,c_{1,7} + c_{1,8} + T^2\,c_{1,9} + T\,c_{1,10} + c_{1,11} + T^2\,c_{1,12} + T\,c_{1,13} + c_{1,14} + T^2\,f_{1,3},$$

$$0 == d_{1,6} + T\,d_{1,7} + T^2\,d_{1,8} + d_{1,9} + T\,d_{1,10} + T^2\,d_{1,11} + d_{1,12} + T\,d_{1,13} + T^2\,d_{1,14} + T^2\,f_{1,3},$$

$$0 == c_{1,6} - 2\,T\,c_{1,6} + T^2\,c_{1,6} - c_{1,7} + T\,c_{1,7} + c_{1,8} + 3\,c_{1,9} - \frac{c_{1,9}}{T} - 3\,T\,c_{1,9} + T^2\,c_{1,9} - 2\,c_{1,10} +$$
$$\frac{c_{1,10}}{T} + T\,c_{1,10} + c_{1,11} - \frac{c_{1,11}}{T} + 6\,c_{1,12} + \frac{c_{1,12}}{T^2} - \frac{4\,c_{1,12}}{T} - 4\,T\,c_{1,12} + T^2\,c_{1,12} - 3\,c_{1,13} -$$
$$\frac{c_{1,13}}{T^2} + \frac{3\,c_{1,13}}{T} + T\,c_{1,13} + c_{1,14} + \frac{c_{1,14}}{T^2} - \frac{2\,c_{1,14}}{T} + T^2\,d_{1,8} + f_{1,3} - 2\,T\,f_{1,3} + T^2\,f_{1,3},$$

$$0 == d_{1,1} - T\,d_{1,3} + 2\,T^2\,d_{1,8} + f_{1,1} + f_{1,2} - T\,f_{1,2} + 2\,f_{1,3} - 4\,T\,f_{1,3} + 2\,T^2\,f_{1,3}\Big\}$$

In[•]:=
```
{sol} = Solve[eqns, unknowns]
```

Solve: Equations may not give solutions for all "solve" variables.

Out[•]= $$\left\{\left\{c_{1,1}\to -\frac{c_{1,2}}{2}-\frac{c_{1,5}}{2\,T},\ c_{1,3}\to -T\,c_{1,2}-c_{1,5},\ c_{1,4}\to 0,\ c_{1,6}\to 0,\ c_{1,8}\to -\frac{1}{2}\times(1-T)\,c_{1,10},\ c_{1,9}\to 0,\right.\right.$$
$$c_{1,11}\to -T\,c_{1,7}-\frac{1}{2}\times(-1+3\,T)\,c_{1,10},\ c_{1,12}\to 0,\ c_{1,13}\to 0,\ c_{1,14}\to 0,\ d_{1,1}\to \frac{c_{1,2}}{2}+\frac{c_{1,5}}{2\,T},$$
$$d_{1,2}\to -c_{1,2},\ d_{1,3}\to \frac{c_{1,2}}{T}+\frac{c_{1,5}}{T^2},\ d_{1,4}\to 0,\ d_{1,5}\to -\frac{c_{1,5}}{T^2},\ d_{1,6}\to 0,\ d_{1,7}\to -\frac{c_{1,7}}{T}-\frac{(-1+T)\,c_{1,10}}{T^2},$$
$$d_{1,8}\to -\frac{(1-T)\,c_{1,10}}{2\,T^3},\ d_{1,9}\to 0,\ d_{1,10}\to -\frac{c_{1,10}}{T^2},\ d_{1,11}\to \frac{c_{1,7}}{T^2}-\frac{(-1-T)\,c_{1,10}}{2\,T^3},\ d_{1,12}\to 0,\ d_{1,13}\to 0,$$
$$\left.\left.d_{1,14}\to 0,\ e_{1,1}\to -\frac{c_{1,2}}{2}-\frac{c_{1,5}}{2\,T},\ e_{1,2}\to -\frac{c_{1,10}}{T},\ e_{1,3}\to 0,\ f_{1,1}\to \frac{c_{1,2}}{2}+\frac{c_{1,5}}{2\,T},\ f_{1,2}\to \frac{c_{1,10}}{T},\ f_{1,3}\to 0\right\}\right\}$$

In[•]:= `sol /. (a_ → b_) :> (a = b)`

Out[•]= $$\left\{-\frac{c_{1,2}}{2}-\frac{c_{1,5}}{2\,T},\ -T\,c_{1,2}-c_{1,5},\ 0,\ 0,\ -\frac{1}{2}\times(1-T)\,c_{1,10},\ 0,\ -T\,c_{1,7}-\frac{1}{2}\times(-1+3\,T)\,c_{1,10},\ 0,\ 0,\right.$$
$$0,\ \frac{c_{1,2}}{2}+\frac{c_{1,5}}{2\,T},\ -c_{1,2},\ \frac{c_{1,2}}{T}+\frac{c_{1,5}}{T^2},\ 0,\ -\frac{c_{1,5}}{T^2},\ 0,\ -\frac{c_{1,7}}{T}-\frac{(-1+T)\,c_{1,10}}{T^2},\ -\frac{(1-T)\,c_{1,10}}{2\,T^3},$$
$$\left.0,\ -\frac{c_{1,10}}{T^2},\ \frac{c_{1,7}}{T^2}-\frac{(-1-T)\,c_{1,10}}{2\,T^3},\ 0,\ 0,\ 0,\ -\frac{c_{1,2}}{2}-\frac{c_{1,5}}{2\,T},\ -\frac{c_{1,10}}{T},\ 0,\ \frac{c_{1,2}}{2}+\frac{c_{1,5}}{2\,T},\ \frac{c_{1,10}}{T},\ 0\right\}$$

In[•]:= $\left\{R_{1,2},\ \overline{R}_{1,2},\ C_1,\ \overline{C}_1\right\}$

Out[•]= $$\left\{\mathbb{E}_{\{\}\to\{1,2\}}\left[\sqrt{T},\ (-1+T)\,(p_1-p_2)\,x_2,\right.\right.$$
$$\in\text{Series}\left[0,\ -\frac{c_{1,2}}{2}+p_1\,x_1\,c_{1,2}+p_1\,x_2\,(-T\,c_{1,2}-c_{1,5})-\frac{c_{1,5}}{2\,T}+p_2\,x_2\,c_{1,5}+p_1^2\,x_1\,x_2\,c_{1,7}+\right.$$
$$\left.\left.p_1\,p_2\,x_1\,x_2\,c_{1,10}-\frac{1}{2}\times(1-T)\,p_1^2\,x_2^2\,c_{1,10}+p_1\,p_2\,x_2^2\left(-T\,c_{1,7}-\frac{1}{2}\times(-1+3\,T)\,c_{1,10}\right)\right]\right],$$
$$\mathbb{E}_{\{\}\to\{1,2\}}\left[\frac{1}{\sqrt{T}},\ \left(-1+\frac{1}{T}\right)(p_1-p_2)\,x_2,\in\text{Series}\left[0,\right.\right.$$
$$\frac{c_{1,2}}{2}-p_1\,x_1\,c_{1,2}+\frac{c_{1,5}}{2\,T}-\frac{p_2\,x_2\,c_{1,5}}{T^2}+p_1\,x_2\left(\frac{c_{1,2}}{T}+\frac{c_{1,5}}{T^2}\right)-\frac{p_1\,p_2\,x_1\,x_2\,c_{1,10}}{T^2}-$$
$$\left.\left.\frac{(1-T)\,p_1^2\,x_2^2\,c_{1,10}}{2\,T^3}+p_1\,p_2\,x_2^2\left(\frac{c_{1,7}}{T^2}-\frac{(-1-T)\,c_{1,10}}{2\,T^3}\right)+p_1^2\,x_1\,x_2\left(-\frac{c_{1,7}}{T}-\frac{(-1+T)\,c_{1,10}}{T^2}\right)\right]\right],$$
$$\mathbb{E}_{\{\}\to\{1\}}\left[\sqrt{T},\ 0,\ \in\text{Series}\left[0,\ -\frac{c_{1,2}}{2}-\frac{c_{1,5}}{2\,T}-\frac{p_1\,x_1\,c_{1,10}}{T}\right]\right],$$
$$\left.\mathbb{E}_{\{\}\to\{1\}}\left[\frac{1}{\sqrt{T}},\ 0,\ \in\text{Series}\left[0,\ \frac{c_{1,2}}{2}+\frac{c_{1,5}}{2\,T}+\frac{p_1\,x_1\,c_{1,10}}{T}\right]\right]\right\}$$

In[•]:= `Cases[{R_{1,2}, R̄_{1,2}, C_1, C̄_1}, (c | d | e | f)_{$k,_}, ∞] // Union`

Out[•]= $\{c_{1,2},\ c_{1,5},\ c_{1,7},\ c_{1,10}\}$

In[•]:=
```
{c[1,2] = 0, c[1,5] = 0, c[1,7] = 0, c[1,10] = 1};
{R[1,2], Rb[1,2], C[1], Cb[1]}
```

Out[•]=
$$\left\{\mathbb{E}_{\{\}\to\{1,2\}}\left[\sqrt{T},\ (-1+T)\ (p_1-p_2)\ x_2,\right.\right.$$
$$\in\text{Series}\left[0,\ p_1\,p_2\,x_1\,x_2+\frac{1}{2}\times(-1+T)\ p_1^2\,x_2^2+\frac{1}{2}\times(1-3\,T)\ p_1\,p_2\,x_2^2\right]\Big],$$
$$\mathbb{E}_{\{\}\to\{1,2\}}\left[\frac{1}{\sqrt{T}},\ \left(-1+\frac{1}{T}\right)\ (p_1-p_2)\ x_2,\right.$$
$$\left.\in\text{Series}\left[0,\ -\frac{(-1+T)\ p_1^2\,x_1\,x_2}{T^2}-\frac{p_1\,p_2\,x_1\,x_2}{T^2}-\frac{(1-T)\ p_1^2\,x_2^2}{2\,T^3}-\frac{(-1-T)\ p_1\,p_2\,x_2^2}{2\,T^3}\right]\right],$$
$$\left.\mathbb{E}_{\{\}\to\{1\}}\left[\sqrt{T},\ 0,\ \in\text{Series}\left[0,\ -\frac{p_1\,x_1}{T}\right]\right],\ \mathbb{E}_{\{\}\to\{1\}}\left[\frac{1}{\sqrt{T}},\ 0,\ \in\text{Series}\left[0,\ \frac{p_1\,x_1}{T}\right]\right]\right\}$$

In[•]:=
```
RMoves
```

Out[•]=
```
{True, True, True, True, True, True, True, True, True}
```

Some Knot Theory at $k=1

In[•]:=
```
NewBit[K_] := Module[{Alex = Alexander[K][T]},
  T^3 Alex^2/(T - 1) ZF[K][[3, 2]] // Factor]
```

In[•]:=
```
NewBit /@ AllKnots[{3, 5}]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[•]=
$$\left\{2-T+T^2,\ (1+T)\times\left(1-3\,T+T^2\right),\ \frac{4-3\,T+5\,T^2-3\,T^3+3\,T^4-T^5+T^6}{T^2},\ 9-11\,T+7\,T^2-T^3\right\}$$

In[•]:=
```
(*Two knots with equal Alexander, new bit does not agree*)
Alexander[Knot[6, 1]] == Alexander[Knot[9, 46]]
Timing[NewBit[Knot[6, 1]] == NewBit[Knot[9, 46]]]
```

Out[•]=
```
True
```

Out[•]=
$$\left\{23.4375,\ 5-11\,T-T^2+3\,T^3==7-21\,T+9\,T^2+T^3\right\}$$

In[•]:=
```
equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};
Length@Union@Echo[ZF /@ equiv]
```

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

» $\left\{\mathbb{E}_{\{\}\to\{1\}}\left[\frac{T^4}{1-4T+9T^2-15T^3+17T^4-15T^5+9T^6-4T^7+T^8}, 0, \in\text{Series}\left[0, \left(-3+20T-69T^2+161T^3-272T^4+328T^5-225T^6-92T^7+548T^8-952T^9+1113T^{10}-980T^{11}+668T^{12}-349T^{13}+135T^{14}-36T^{15}+5T^{16}\right)\Big/\left(T-8T^2+34T^3-102T^4+235T^5-436T^6+669T^7-860T^8+935T^9-860T^{10}+669T^{11}-436T^{12}+235T^{13}-102T^{14}+34T^{15}-8T^{16}+T^{17}\right)\right]\right],\right.$

$\left.\mathbb{E}_{\{\}\to\{1\}}\left[\frac{T^4}{1-4T+9T^2-15T^3+17T^4-15T^5+9T^6-4T^7+T^8}, 0, \in\text{Series}\left[0, \left(-3+20T-69T^2+161T^3-272T^4+328T^5-225T^6-92T^7+548T^8-952T^9+1113T^{10}-980T^{11}+668T^{12}-349T^{13}+135T^{14}-36T^{15}+5T^{16}\right)\Big/\left(T-8T^2+34T^3-102T^4+235T^5-436T^6+669T^7-860T^8+935T^9-860T^{10}+669T^{11}-436T^{12}+235T^{13}-102T^{14}+34T^{15}-8T^{16}+T^{17}\right)\right]\right]\right\}$

Out[•]= 1

In[•]:=
```
equiv =
  {Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};
Length@Union[ZF /@ equiv]
```

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

Out[•]= 1

Solving for R, C, \$k = 2

In[•]:=
```
$k = 2;
{R1,2, C1}
Short[RMoves, 20]
```

Out[•]=
$$\left\{\mathbb{E}_{\{\}\to\{1,2\}}\left[\sqrt{T},\ (-1+T)\ (p_1-p_2)\ x_2,\right.\right.$$
$$\in\text{Series}\left[0,\ p_1\,p_2\,x_1\,x_2+\frac{1}{2}\times(-1+T)\ p_1^2\,x_2^2+\frac{1}{2}\times(1-3\,T)\ p_1\,p_2\,x_2^2,\right.$$
$$c_{2,1}+p_1\,x_1\,c_{2,2}+p_1\,x_2\,c_{2,3}+p_2\,x_1\,c_{2,4}+p_2\,x_2\,c_{2,5}+p_1^2\,x_1^2\,c_{2,6}+p_1^2\,x_1\,x_2\,c_{2,7}+$$
$$p_1^2\,x_2^2\,c_{2,8}+p_1\,p_2\,x_1^2\,c_{2,9}+p_1\,p_2\,x_1\,x_2\,c_{2,10}+p_1\,p_2\,x_2^2\,c_{2,11}+p_2^2\,x_1^2\,c_{2,12}+p_2^2\,x_1\,x_2\,c_{2,13}+$$
$$p_2^2\,x_2^2\,c_{2,14}+p_1^3\,x_1^3\,c_{2,15}+p_1^3\,x_1^2\,x_2\,c_{2,16}+p_1^3\,x_1\,x_2^2\,c_{2,17}+p_1^3\,x_2^3\,c_{2,18}+p_1^2\,p_2\,x_1^3\,c_{2,19}+$$
$$p_1^2\,p_2\,x_1^2\,x_2\,c_{2,20}+p_1^2\,p_2\,x_1\,x_2^2\,c_{2,21}+p_1^2\,p_2\,x_2^3\,c_{2,22}+p_1\,p_2^2\,x_1^3\,c_{2,23}+p_1\,p_2^2\,x_1^2\,x_2\,c_{2,24}+$$
$$\left.p_1\,p_2^2\,x_1\,x_2^2\,c_{2,25}+p_1\,p_2^2\,x_2^3\,c_{2,26}+p_2^3\,x_1^3\,c_{2,27}+p_2^3\,x_1^2\,x_2\,c_{2,28}+p_2^3\,x_1\,x_2^2\,c_{2,29}+p_2^3\,x_2^3\,c_{2,30}\right]\Big],$$
$$\left.\mathbb{E}_{\{\}\to\{1\}}\left[\sqrt{T},\ 0,\ \in\text{Series}\left[0,\ -\frac{p_1\,x_1}{T},\ e_{2,1}+p_1\,x_1\,e_{2,2}+p_1^2\,x_1^2\,e_{2,3}+p_1^3\,x_1^3\,e_{2,4}\right]\right]\right\}$$

Out[•]//Short=
$$\left\{(-1+T)\ p_1^2\,p_3\,x_1\,x_2\,x_3-3\,T\,p_1\,p_2\,p_3\,x_1\,x_2\,x_3+\right.$$
$$\left(-2\,T+4\,T^2\right)\ p_1\,p_2\,p_3\,x_1\,x_3^2+3\,c_{2,1}+2\,p_1\,x_1\,c_{2,2}+p_2\,x_1\,c_{2,4}+p_3\,x_1\,c_{2,4}+$$
$$T\,p_3\,x_2\,c_{2,4}+p_1\,x_2\,\left(c_{2,2}-T\,c_{2,2}+c_{2,3}+c_{2,4}-T\,c_{2,4}\right)+2\,T\,p_3\,x_3\,c_{2,5}+\text{<<97>>}+$$
$$\frac{1}{2}\,p_1^3\,x_2\,x_3^2\left(1-3\,T+3\,T^2-T^3+2\,T^2\,c_{2,16}-4\,T^3\,c_{2,16}+2\,T^4\,c_{2,16}+2\,c_{2,17}-2\,T\,c_{2,17}-2\,T^2\,c_{2,17}+\right.$$
$$2\,T^3\,c_{2,17}+6\,c_{2,18}-12\,T\,c_{2,18}+6\,T^2\,c_{2,18}+2\,c_{2,21}-6\,T\,c_{2,21}+6\,T^2\,c_{2,21}-2\,T^3\,c_{2,21}+$$
$$\left.2\,c_{2,25}-6\,T\,c_{2,25}+6\,T^2\,c_{2,25}-2\,T^3\,c_{2,25}+2\,c_{2,29}-6\,T\,c_{2,29}+6\,T^2\,c_{2,29}-2\,T^3\,c_{2,29}\right)+$$
$$\frac{1}{2}\,p_1^2\,p_3\,x_2\,x_3^2\left(T-T^3+2\,T\,c_{2,21}-4\,T^2\,c_{2,21}+2\,T^3\,c_{2,21}+4\,T\,c_{2,25}-8\,T^2\,c_{2,25}+\right.$$
$$\left.4\,T^3\,c_{2,25}+6\,T\,c_{2,29}-12\,T^2\,c_{2,29}+6\,T^3\,c_{2,29}\right)+2\,T^3\,p_3^3\,x_3^3\,c_{2,30}+$$
$$p_2^3\,x_2^3\left(T^3\,c_{2,15}+c_{2,30}\right)+p_2^3\,x_2^2\,x_3\left(T^3\,c_{2,16}+T\,c_{2,29}-T^2\,c_{2,29}+3\,c_{2,30}-3\,T\,c_{2,30}\right)+$$
$$p_2^3\,x_2\,x_3^2\left(T^3\,c_{2,17}+T^2\,c_{2,28}-2\,T^3\,c_{2,28}+T^4\,c_{2,28}+2\,T\,c_{2,29}-4\,T^2\,c_{2,29}+2\,T^3\,c_{2,29}+\right.$$
$$\left.3\,c_{2,30}-6\,T\,c_{2,30}+3\,T^2\,c_{2,30}\right)+p_1\,p_3^2\,x_3^3\left(T^2\,c_{2,26}+3\,T^2\,c_{2,30}-3\,T^3\,c_{2,30}\right)+$$
$$\frac{1}{2}\,p_1^3\,x_3^3\left(-1+3\,T-5\,T^2+5\,T^3-2\,T^4+2\,T^3\,c_{2,15}-6\,T^4\,c_{2,15}+6\,T^5\,c_{2,15}-2\,T^6\,c_{2,15}+\right.$$
$$2\,T^2\,c_{2,16}-6\,T^3\,c_{2,16}+6\,T^4\,c_{2,16}-2\,T^5\,c_{2,16}+2\,T\,c_{2,17}-6\,T^2\,c_{2,17}+6\,T^3\,c_{2,17}-2\,T^4\,c_{2,17}+$$
$$4\,c_{2,18}-12\,T\,c_{2,18}+12\,T^2\,c_{2,18}-2\,T^3\,c_{2,18}+2\,c_{2,22}-6\,T\,c_{2,22}+6\,T^2\,c_{2,22}-2\,T^3\,c_{2,22}+$$
$$\left.2\,c_{2,26}-6\,T\,c_{2,26}+6\,T^2\,c_{2,26}-2\,T^3\,c_{2,26}+2\,c_{2,30}-6\,T\,c_{2,30}+6\,T^2\,c_{2,30}-2\,T^3\,c_{2,30}\right)+$$
$$p_2^3\,x_3^3\left(T^3\,c_{2,18}+T^3\,c_{2,27}-3\,T^4\,c_{2,27}+3\,T^5\,c_{2,27}-T^6\,c_{2,27}+T^2\,c_{2,28}-3\,T^3\,c_{2,28}+3\,T^4\,c_{2,28}-\right.$$
$$\left.T^5\,c_{2,28}+T\,c_{2,29}-3\,T^2\,c_{2,29}+3\,T^3\,c_{2,29}-T^4\,c_{2,29}+c_{2,30}-3\,T\,c_{2,30}+3\,T^2\,c_{2,30}-T^3\,c_{2,30}\right)+$$
$$p_1^2\,p_3\,x_3^3\left(T^2-4\,T^3+3\,T^4+T\,c_{2,22}-2\,T^2\,c_{2,22}+2\,T^3\,c_{2,22}+2\,T\,c_{2,26}-4\,T^2\,c_{2,26}+\right.$$
$$\left.2\,T^3\,c_{2,26}+3\,T\,c_{2,30}-6\,T^2\,c_{2,30}+3\,T^3\,c_{2,30}\right)==$$
$$3\,c_{2,1}+\text{<<144>>}+p_2^2\,p_3\ \text{<<1>>}\ \left(T^3\,c_{2,22}+3\,T\,c_{2,30}-6\,T^2\,c_{2,\text{<<2>>}}+3\,T^3\,c_{2,30}\right),\ \text{<<8>>}\Big\}$$

```
In[•]:= unknowns = Cases[{R_{1,2}, R̄_{1,2}, C_1, C̄_1}, (c | d | e | f)_{$k,_}, ∞] // Union
```

```
Out[•]= {c_{2,1}, c_{2,2}, c_{2,3}, c_{2,4}, c_{2,5}, c_{2,6}, c_{2,7}, c_{2,8}, c_{2,9}, c_{2,10}, c_{2,11}, c_{2,12}, c_{2,13}, c_{2,14},
  c_{2,15}, c_{2,16}, c_{2,17}, c_{2,18}, c_{2,19}, c_{2,20}, c_{2,21}, c_{2,22}, c_{2,23}, c_{2,24}, c_{2,25}, c_{2,26}, c_{2,27},
  c_{2,28}, c_{2,29}, c_{2,30}, d_{2,1}, d_{2,2}, d_{2,3}, d_{2,4}, d_{2,5}, d_{2,6}, d_{2,7}, d_{2,8}, d_{2,9}, d_{2,10}, d_{2,11},
  d_{2,12}, d_{2,13}, d_{2,14}, d_{2,15}, d_{2,16}, d_{2,17}, d_{2,18}, d_{2,19}, d_{2,20}, d_{2,21}, d_{2,22}, d_{2,23}, d_{2,24},
  d_{2,25}, d_{2,26}, d_{2,27}, d_{2,28}, d_{2,29}, d_{2,30}, e_{2,1}, e_{2,2}, e_{2,3}, e_{2,4}, f_{2,1}, f_{2,2}, f_{2,3}, f_{2,4}}
```

```
In[•]:= Short[errors = CCF /@ Cases[RMoves, a_ == b_ :> a - b], 25]
```

```
Out[•]//Short= {1/2 ×
   (2 p1 p2 x2 x3 - 2 T p1 p2 x2 x3 - 2 p1^2 p2 x1 x2 x3 + 2 T p1^2 p2 x1 x2 x3 + 2 p1 p2^2 x1 x2 x3 - 2 T p1 p2^2 x1 x2 x3 -
     2 p1^2 p3 x1 x2 x3 + 2 T p1^2 p3 x1 x2 x3 + 2 p1^3 x2^2 x3 - 4 T p1^3 x2^2 x3 + 2 T^2 p1^3 x2^2 x3 - 6 p1^2 p2 x2^2 x3 +
     16 T p1^2 p2 x2^2 x3 - 10 T^2 p1^2 p2 x2^2 x3 + p1 p2^2 x2^2 x3 - 6 T p1 p2^2 x2^2 x3 + 5 T^2 p1 p2^2 x2^2 x3 + 2 T p1 p2 p3 x2^2 x3 -
     2 T^2 p1 p2 p3 x2^2 x3 + p1^2 x3^2 - 2 T p1^2 x3^2 + T^2 p1^2 x3^2 + p1^2 p2 x1 x3^2 - T^2 p1^2 p2 x1 x3^2 - p1^2 p3 x1 x3^2 +
     4 T p1^2 p3 x1 x3^2 - 3 T^2 p1^2 p3 x1 x3^2 - 2 T p1 p2 p3 x1 x3^2 + 2 T^2 p1 p2 p3 x1 x3^2 + p1^3 x2 x3^2 -
     3 T p1^3 x2 x3^2 + 3 T^2 p1^3 x2 x3^2 - T^3 p1^3 x2 x3^2 - 4 p1^2 p2 x2 x3^2 + 14 T p1^2 p2 x2 x3^2 - 16 T^2 p1^2 p2 x2 x3^2 +
     6 T^3 p1^2 p2 x2 x3^2 - 3 T p1 p2^2 x2 x3^2 + 4 T^2 p1 p2^2 x2 x3^2 - T^3 p1 p2^2 x2 x3^2 + T p1^2 p3 x2 x3^2 - T^3 p1^2 p3 x2 x3^2 -
     2 T^2 p1 p2 p3 x2 x3^2 + 2 T^3 p1 p2 p3 x2 x3^2 - p1^3 x3^3 + 3 T p1^3 x3^3 - 5 T^2 p1^3 x3^3 + 5 T^3 p1^3 x3^3 - 2 T^4 p1^3 x3^3 +
     <<974>> + 2 T^2 p3^3 x1 x3^2 c_{2,29} - 2 T^3 p3^3 x1 x3^2 c_{2,29} + 2 p1^3 x2 x3^2 c_{2,29} - 6 T p1^3 x2 x3^2 c_{2,29} +
     6 T^2 p1^3 x2 x3^2 c_{2,29} - 2 T^3 p1^3 x2 x3^2 c_{2,29} - 2 p2^3 x2 x3^2 c_{2,29} + 12 T p2^3 x2 x3^2 c_{2,29} - 20 T^2 p2^3 x2 x3^2 c_{2,29} +
     12 T^3 p2^3 x2 x3^2 c_{2,29} - 2 T^4 p2^3 x2 x3^2 c_{2,29} + 6 T p1^2 p3 x2 x3^2 c_{2,29} - 12 T^2 p1^2 p3 x2 x3^2 c_{2,29} +
     6 T^3 p1^2 p3 x2 x3^2 c_{2,29} - 6 T p2^2 p3 x2 x3^2 c_{2,29} + 18 T^2 p2^2 p3 x2 x3^2 c_{2,29} - 18 T^3 p2^2 p3 x2 x3^2 c_{2,29} +
     6 T^4 p2^2 p3 x2 x3^2 c_{2,29} + 6 T^2 p1 p2^2 x2 x3^2 c_{2,29} - 6 T^3 p1 p2^2 x2 x3^2 c_{2,29} - 6 T^2 p2 p2^2 x2 x3^2 c_{2,29} +
     12 T^3 p2 p2^2 x2 x3^2 c_{2,29} - 6 T^4 p2 p2^2 x2 x3^2 c_{2,29} - 2 T^3 p3^3 x2 x3^2 c_{2,29} + 2 T^4 p3^3 x2 x3^2 c_{2,29} +
     2 T p2^3 x3^3 c_{2,29} - 6 T^2 p2^3 x3^3 c_{2,29} + 6 T^3 p2^3 x3^3 c_{2,29} - 2 T^4 p2^3 x3^3 c_{2,29} + 6 p2^3 x2^2 x3 c_{2,30} -
     6 T p2^3 x2^2 x3 c_{2,30} + 6 p2^3 x2 x3^2 c_{2,30} - 12 T p2^3 x2 x3^2 c_{2,30} + 6 T^2 p2^3 x2 x3^2 c_{2,30} + 2 p1^3 x3^3 c_{2,30} -
     6 T p1^3 x3^3 c_{2,30} + 6 T^2 p1^3 x3^3 c_{2,30} - 2 T^3 p1^3 x3^3 c_{2,30} + 6 T p1^2 p3 x3^3 c_{2,30} - 12 T^2 p1^2 p3 x3^3 c_{2,30} +
     6 T^3 p1^2 p3 x3^3 c_{2,30} - 6 T p2^2 p3 x3^3 c_{2,30} + 12 T^2 p2^2 p3 x3^3 c_{2,30} - 6 T^3 p2^2 p3 x3^3 c_{2,30} + 6 T^2 p1 p3^2 x3^3 c_{2,30} -
     6 T^3 p1 p3^2 x3^3 c_{2,30} - 6 T^2 p2 p3^2 x3^3 c_{2,30} + 6 T^3 p2 p3^2 x3^3 c_{2,30}), <<7>>, <<1>>/(2 <<1>>)}
```

```
In[•]:= Short[#, 10] &[eqns =
  Thread[0 == Union @@ (CoefficientRules[#, {x1, x2, x3, p1, p2, p3}][[ ;; , 2]] & /@ errors)]]
```

```
Out[•]//Short= {0 == c_{2,4} - T c_{2,4}, 0 == -c_{2,4} + T c_{2,4}, <<215>>,
  0 == 1 + 2/T + d_{2,2} + T d_{2,3} + d_{2,4} + T d_{2,5} - 2 T d_{2,7} - 4 T^2 d_{2,8} - T d_{2,10} - 2 T^2 d_{2,11} + 6 T^2 d_{2,17} +
    18 T^3 d_{2,18} + 2 T^2 d_{2,21} + 6 T^3 d_{2,22} + T f_{2,2} + 4 T f_{2,3} - 4 T^2 f_{2,3} + 18 T f_{2,4} - 36 T^2 f_{2,4} + 18 T^3 f_{2,4}}
```

In[•]:=
```
{sol} = Solve[eqns, unknowns]
```

Solve: Equations may not give solutions for all "solve" variables.

Out[•]=

$$\left\{\left\{c_{2,1}\to -\frac{c_{2,2}}{2}-\frac{c_{2,5}}{2\,T},\ c_{2,3}\to -T\,c_{2,2}-c_{2,5},\ c_{2,4}\to 0,\ c_{2,6}\to 0,\ c_{2,8}\to -\frac{1}{2}\times(1-T)\,c_{2,10},\right.\right.$$
$$c_{2,9}\to 0,\ c_{2,11}\to -\frac{1}{2}-T\,c_{2,7}-\frac{1}{2}\times(-1+3\,T)\,c_{2,10},\ c_{2,12}\to 0,\ c_{2,13}\to 0,\ c_{2,14}\to 0,$$
$$c_{2,15}\to 0,\ c_{2,17}\to -((-1+T)\,c_{2,16}),\ c_{2,18}\to -\frac{-1+4\,T-3\,T^2}{6\,T},\ c_{2,19}\to 0,\ c_{2,20}\to -\frac{1}{2\,T},$$
$$c_{2,21}\to -\frac{1-3\,T}{2\,T},\ c_{2,22}\to -\frac{1-11\,T+16\,T^2}{6\,T}-\left(T-T^2\right)c_{2,16},\ c_{2,23}\to 0,\ c_{2,24}\to 0,$$
$$c_{2,25}\to -\frac{1}{2},\ c_{2,26}\to \frac{1}{6}\times(-1+7\,T)-T^2\,c_{2,16},\ c_{2,27}\to 0,\ c_{2,28}\to 0,\ c_{2,29}\to 0,\ c_{2,30}\to 0,$$
$$d_{2,1}\to \frac{c_{2,2}}{2}+\frac{c_{2,5}}{2\,T},\ d_{2,2}\to -c_{2,2},\ d_{2,3}\to \frac{c_{2,2}}{T}+\frac{c_{2,5}}{T^2},\ d_{2,4}\to 0,\ d_{2,5}\to -\frac{c_{2,5}}{T^2},\ d_{2,6}\to 0,$$
$$d_{2,7}\to -\frac{1-T}{T^3}-\frac{c_{2,7}}{T}-\frac{(-1+T)\,c_{2,10}}{T^2},\ d_{2,8}\to -\frac{-1+T}{2\,T^4}-\frac{(1-T)\,c_{2,10}}{2\,T^3},\ d_{2,9}\to 0,\ d_{2,10}\to \frac{1}{T^3}-\frac{c_{2,10}}{T^2},$$
$$d_{2,11}\to -\frac{1}{2\,T^4}+\frac{c_{2,7}}{T^2}-\frac{(-1-T)\,c_{2,10}}{2\,T^3},\ d_{2,12}\to 0,\ d_{2,13}\to 0,\ d_{2,14}\to 0,\ d_{2,15}\to 0,$$
$$d_{2,16}\to -\frac{-1+T}{2\,T^3}-\frac{c_{2,16}}{T},\ d_{2,17}\to -\frac{3-4\,T+T^2}{2\,T^4}-\frac{(-1+T)\,c_{2,16}}{T^2},\ d_{2,18}\to -\frac{-3+4\,T-T^2}{6\,T^5},\ d_{2,19}\to 0,$$
$$d_{2,20}\to -\frac{1}{2\,T^3},\ d_{2,21}\to \frac{2}{T^4},\ d_{2,22}\to -\frac{4+T+T^2}{6\,T^5}-\frac{(1-T)\,c_{2,16}}{T^3},\ d_{2,23}\to 0,\ d_{2,24}\to 0,\ d_{2,25}\to -\frac{1}{2\,T^4},$$
$$d_{2,26}\to -\frac{-1+T}{6\,T^5}+\frac{c_{2,16}}{T^3},\ d_{2,27}\to 0,\ d_{2,28}\to 0,\ d_{2,29}\to 0,\ d_{2,30}\to 0,\ e_{2,1}\to -\frac{c_{2,2}}{2}-\frac{c_{2,5}}{2\,T},$$
$$\left.\left.e_{2,2}\to -\frac{c_{2,10}}{T},\ e_{2,3}\to 0,\ e_{2,4}\to 0,\ f_{2,1}\to \frac{c_{2,2}}{2}+\frac{c_{2,5}}{2\,T},\ f_{2,2}\to -\frac{1}{T^2}+\frac{c_{2,10}}{T},\ f_{2,3}\to 0,\ f_{2,4}\to 0\right\}\right\}$$

In[•]:=
```
sol /. (a_ → b_) :> (a = b)
```

Out[•]=
$$\left\{-\frac{c_{2,2}}{2}-\frac{c_{2,5}}{2\,T},\ -T\,c_{2,2}-c_{2,5},\ 0,\ 0,\ -\frac{1}{2}\times(1-T)\,c_{2,10},\ 0,\ -\frac{1}{2}-T\,c_{2,7}-\frac{1}{2}\times(-1+3\,T)\,c_{2,10},\ 0,\ 0,\ 0,\right.$$
$$0,\ -((-1+T)\,c_{2,16}),\ -\frac{-1+4\,T-3\,T^2}{6\,T},\ 0,\ -\frac{1}{2\,T},\ -\frac{1-3\,T}{2\,T},\ -\frac{1-11\,T+16\,T^2}{6\,T}-\left(T-T^2\right)c_{2,16},$$
$$0,\ 0,\ -\frac{1}{2},\ \frac{1}{6}\times(-1+7\,T)-T^2\,c_{2,16},\ 0,\ 0,\ 0,\ 0,\ \frac{c_{2,2}}{2}+\frac{c_{2,5}}{2\,T},\ -c_{2,2},\ \frac{c_{2,2}}{T}+\frac{c_{2,5}}{T^2},\ 0,$$
$$-\frac{c_{2,5}}{T^2},\ 0,\ -\frac{1-T}{T^3}-\frac{c_{2,7}}{T}-\frac{(-1+T)\,c_{2,10}}{T^2},\ -\frac{-1+T}{2\,T^4}-\frac{(1-T)\,c_{2,10}}{2\,T^3},\ 0,\ \frac{1}{T^3}-\frac{c_{2,10}}{T^2},$$
$$-\frac{1}{2\,T^4}+\frac{c_{2,7}}{T^2}-\frac{(-1-T)\,c_{2,10}}{2\,T^3},\ 0,\ 0,\ 0,\ 0,\ -\frac{-1+T}{2\,T^3}-\frac{c_{2,16}}{T},\ -\frac{3-4\,T+T^2}{2\,T^4}-\frac{(-1+T)\,c_{2,16}}{T^2},$$
$$-\frac{-3+4\,T-T^2}{6\,T^5},\ 0,\ -\frac{1}{2\,T^3},\ \frac{2}{T^4},\ -\frac{4+T+T^2}{6\,T^5}-\frac{(1-T)\,c_{2,16}}{T^3},\ 0,\ 0,\ -\frac{1}{2\,T^4},\ -\frac{-1+T}{6\,T^5}+\frac{c_{2,16}}{T^3},$$
$$\left.0,\ 0,\ 0,\ 0,\ -\frac{c_{2,2}}{2}-\frac{c_{2,5}}{2\,T},\ -\frac{c_{2,10}}{T},\ 0,\ 0,\ \frac{c_{2,2}}{2}+\frac{c_{2,5}}{2\,T},\ -\frac{1}{T^2}+\frac{c_{2,10}}{T},\ 0,\ 0\right\}$$

In[•]:=
```
Cases[{R_{1,2}, \[OverBar]R_{1,2}, C_1, \[OverBar]C_1}, (c | d | e | f)_{$k,_}, ∞] // Union
```

Out[•]=
$$\{c_{2,2},\ c_{2,5},\ c_{2,7},\ c_{2,10},\ c_{2,16}\}$$

In[•]:=
```
{c_{2,2} = 0, c_{2,5} = 0, c_{2,7} = 0, c_{2,10} = 0, c_{2,16} = 0};
{R_{1,2}, \[OverBar]R_{1,2}, C_1, \[OverBar]C_1}
```

Out[•]=
$$\left\{\mathbb{E}_{\{\}\to\{1,2\}}\left[\sqrt{T},\ (-1+T)\,(p_1-p_2)\,x_2,\right.\right.$$
$$\in\text{Series}\left[0,\ p_1\,p_2\,x_1\,x_2+\frac{1}{2}\times(-1+T)\,p_1^2\,x_2^2+\frac{1}{2}\times(1-3\,T)\,p_1\,p_2\,x_2^2,\right.$$
$$-\frac{p_1^2\,p_2\,x_1^2\,x_2}{2\,T}-\frac{1}{2}\,p_1\,p_2\,x_2^2-\frac{(1-3\,T)\,p_1^2\,p_2\,x_1\,x_2^2}{2\,T}-\frac{1}{2}\,p_1\,p_2^2\,x_1\,x_2^2-\frac{\left(-1+4\,T-3\,T^2\right)p_1^3\,x_2^3}{6\,T}-$$
$$\left.\left.\frac{\left(1-11\,T+16\,T^2\right)p_1^2\,p_2\,x_2^3}{6\,T}+\frac{1}{6}\times(-1+7\,T)\,p_1\,p_2^2\,x_2^3\right]\right],\ \mathbb{E}_{\{\}\to\{1,2\}}\left[\frac{1}{\sqrt{T}},\ \left(-1+\frac{1}{T}\right)(p_1-p_2)\,x_2,\right.$$
$$\in\text{Series}\left[0,\ -\frac{(-1+T)\,p_1^2\,x_1\,x_2}{T^2}-\frac{p_1\,p_2\,x_1\,x_2}{T^2}-\frac{(1-T)\,p_1^2\,x_2^2}{2\,T^3}-\frac{(-1-T)\,p_1\,p_2\,x_2^2}{2\,T^3},\ -\frac{(1-T)\,p_1^2\,x_1\,x_2}{T^3}+\right.$$
$$\frac{p_1\,p_2\,x_1\,x_2}{T^3}-\frac{(-1+T)\,p_1^3\,x_1^2\,x_2}{2\,T^3}-\frac{p_1^2\,p_2\,x_1^2\,x_2}{2\,T^3}-\frac{(-1+T)\,p_1^2\,x_2^2}{2\,T^4}-\frac{p_1\,p_2\,x_2^2}{2\,T^4}-\frac{\left(3-4\,T+T^2\right)p_1^3\,x_1\,x_2^2}{2\,T^4}+$$
$$\left.\left.\frac{2\,p_1^2\,p_2\,x_1\,x_2^2}{T^4}-\frac{p_1\,p_2^2\,x_1\,x_2^2}{2\,T^4}-\frac{\left(-3+4\,T-T^2\right)p_1^3\,x_2^3}{6\,T^5}-\frac{\left(4+T+T^2\right)p_1^2\,p_2\,x_2^3}{6\,T^5}-\frac{(-1+T)\,p_1\,p_2^2\,x_2^3}{6\,T^5}\right]\right],$$
$$\left.\mathbb{E}_{\{\}\to\{1\}}\left[\sqrt{T},\ 0,\ \in\text{Series}\left[0,\ -\frac{p_1\,x_1}{T},\ 0\right]\right],\ \mathbb{E}_{\{\}\to\{1\}}\left[\frac{1}{\sqrt{T}},\ 0,\ \in\text{Series}\left[0,\ \frac{p_1\,x_1}{T},\ -\frac{p_1\,x_1}{T^2}\right]\right]\right\}$$

In[•]:=
```
RMoves
```

Out[•]=
```
{True, True, True, True, True, True, True, True, True}
```

Some Knot Theory at $k=2

According to 12XingStats.nb at pensieve://Projects/SL2Invariant/k=2/ the following pair have equal ρ_1 but different ρ_2:

```
In[•]:= equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};
        res21 = ZF /@ equiv
        Simplify[res21[[1]] ≡ res21[[2]]]
```

Out[•]=

$$\left\{\mathbb{E}_{\{\}\to\{1\}}\left[\frac{T^4}{1-4T+9T^2-15T^3+17T^4-15T^5+9T^6-4T^7+T^8},\ 0,\right.\right.$$
$$\in\text{Series}\left[0,\ \left(-3+20T-69T^2+161T^3-272T^4+328T^5-225T^6-92T^7+548T^8-952T^9+1113T^{10}-\right.\right.$$
$$980T^{11}+668T^{12}-349T^{13}+135T^{14}-36T^{15}+5T^{16}\Big)\Big/\left(T-8T^2+34T^3-102T^4+235T^5-436T^6+\right.$$
$$669T^7-860T^8+935T^9-860T^{10}+669T^{11}-436T^{12}+235T^{13}-102T^{14}+34T^{15}-8T^{16}+T^{17}\Big),$$
$$\left(3-40T+264T^2-1128T^3+3437T^4-7552T^5+10297T^6+2304T^7-67324T^8+\right.$$
$$259472T^9-699066T^{10}+1539252T^{11}-2919131T^{12}+4882760T^{13}-7290870T^{14}+$$
$$9779044T^{15}-11816854T^{16}+12877354T^{17}-12651386T^{18}+11191592T^{19}-$$
$$8896165T^{20}+6336738T^{21}-4030390T^{22}+2278962T^{23}-1139320T^{24}+$$
$$\left.\left.500046T^{25}-190857T^{26}+62504T^{27}-17215T^{28}+3862T^{29}-668T^{30}+80T^{31}-5T^{32}\right)\Big/\right.$$
$$\left(2T^2-32T^3+264T^4-1496T^5+6516T^6-23136T^7+69396T^8-180024T^9+410582T^{10}-\right.$$
$$833112T^{11}+1517288T^{12}-2496728T^{13}+3730618T^{14}-5080576T^{15}+6323208T^{16}-$$
$$7205208T^{17}+7524878T^{18}-7205208T^{19}+6323208T^{20}-5080576T^{21}+$$
$$\left.\left.3730618T^{22}-2496728T^{23}+1517288T^{24}-833112T^{25}+410582T^{26}-180024T^{27}+\right.\right.$$
$$\left.\left.69396T^{28}-23136T^{29}+6516T^{30}-1496T^{31}+264T^{32}-32T^{33}+2T^{34}\right)\right]\Big],$$
$$\mathbb{E}_{\{\}\to\{1\}}\left[\frac{T^4}{1-4T+9T^2-15T^3+17T^4-15T^5+9T^6-4T^7+T^8},\right.$$
$$0,$$
$$\in\text{Series}\left[0,\ \left(-3+20T-69T^2+161T^3-272T^4+328T^5-225T^6-92T^7+\right.\right.$$
$$548T^8-952T^9+1113T^{10}-980T^{11}+668T^{12}-349T^{13}+135T^{14}-36T^{15}+5T^{16}\Big)\Big/$$
$$\left(T-8T^2+34T^3-102T^4+235T^5-436T^6+669T^7-860T^8+935T^9-860T^{10}+\right.$$
$$669T^{11}-436T^{12}+235T^{13}-102T^{14}+34T^{15}-8T^{16}+T^{17}\Big),$$
$$\left(3-40T+264T^2-1120T^3+3333T^4-6896T^5+7641T^6+9944T^7-83404T^8+283088T^9-\right.$$
$$716082T^{10}+1514140T^{11}-2796883T^{12}+4607952T^{13}-6839214T^{14}+$$
$$9183044T^{15}-11164950T^{16}+12281354T^{17}-12199730T^{18}+10916784T^{19}-$$
$$8773917T^{20}+6311626T^{21}-4047406T^{22}+2302578T^{23}-1155400T^{24}+$$
$$507686T^{25}-193513T^{26}+63160T^{27}-17319T^{28}+3870T^{29}-668T^{30}+80T^{31}-5T^{32}\Big)\Big/$$
$$\left(2T^2-32T^3+264T^4-1496T^5+6516T^6-23136T^7+69396T^8-180024T^9+410582T^{10}-\right.$$
$$833112T^{11}+1517288T^{12}-2496728T^{13}+3730618T^{14}-5080576T^{15}+6323208T^{16}-$$
$$7205208T^{17}+7524878T^{18}-7205208T^{19}+6323208T^{20}-5080576T^{21}+$$
$$3730618T^{22}-2496728T^{23}+1517288T^{24}-833112T^{25}+410582T^{26}-180024T^{27}+$$
$$\left.\left.\left.\left.69396T^{28}-23136T^{29}+6516T^{30}-1496T^{31}+264T^{32}-32T^{33}+2T^{34}\right)\right]\right]\right\}$$

Out[•]=

$$\frac{(-1+T)\,T\left(1-3T+2T^2+5T^3-12T^4+18T^5-12T^6+5T^7+2T^8-3T^9+T^{10}\right)}{\left(1-T+T^2\right)\times\left(1-3T+5T^2-7T^3+5T^4-3T^5+T^6\right)}==0$$

According to 12XingStats.nb at pensieve://Projects/SL2Invariant/k=2/ the following triple have equal ρ_1 but different ρ_2:

```
In[•]:= equiv =
          {Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]};
        Length@Union[res22 = ZF /@ equiv]
```

```
Out[•]= 3
```

Solving for R, C, \$k = 3

```
In[•]:= $k = 3;
        Short[RMoves, 20]
```

```
Out[•]//Short= { <<1>> }
```

```
In[•]:= unknowns = Cases[{R_{1,2}, R̄_{1,2}, C_1, C̄_1}, (c | d | e | f)_{$k,_}, ∞] // Union
```

```
Out[•]= {c_{3,1}, c_{3,2}, c_{3,3}, c_{3,4}, c_{3,5}, c_{3,6}, c_{3,7}, c_{3,8}, c_{3,9}, c_{3,10}, c_{3,11}, c_{3,12}, c_{3,13}, c_{3,14}, c_{3,15}, c_{3,16},
         c_{3,17}, c_{3,18}, c_{3,19}, c_{3,20}, c_{3,21}, c_{3,22}, c_{3,23}, c_{3,24}, c_{3,25}, c_{3,26}, c_{3,27}, c_{3,28}, c_{3,29}, c_{3,30}, c_{3,31},
         c_{3,32}, c_{3,33}, c_{3,34}, c_{3,35}, c_{3,36}, c_{3,37}, c_{3,38}, c_{3,39}, c_{3,40}, c_{3,41}, c_{3,42}, c_{3,43}, c_{3,44}, c_{3,45},
         c_{3,46}, c_{3,47}, c_{3,48}, c_{3,49}, c_{3,50}, c_{3,51}, c_{3,52}, c_{3,53}, c_{3,54}, c_{3,55}, d_{3,1}, d_{3,2}, d_{3,3}, d_{3,4}, d_{3,5},
         d_{3,6}, d_{3,7}, d_{3,8}, d_{3,9}, d_{3,10}, d_{3,11}, d_{3,12}, d_{3,13}, d_{3,14}, d_{3,15}, d_{3,16}, d_{3,17}, d_{3,18}, d_{3,19}, d_{3,20},
         d_{3,21}, d_{3,22}, d_{3,23}, d_{3,24}, d_{3,25}, d_{3,26}, d_{3,27}, d_{3,28}, d_{3,29}, d_{3,30}, d_{3,31}, d_{3,32}, d_{3,33}, d_{3,34}, d_{3,35},
         d_{3,36}, d_{3,37}, d_{3,38}, d_{3,39}, d_{3,40}, d_{3,41}, d_{3,42}, d_{3,43}, d_{3,44}, d_{3,45}, d_{3,46}, d_{3,47}, d_{3,48}, d_{3,49},
         d_{3,50}, d_{3,51}, d_{3,52}, d_{3,53}, d_{3,54}, d_{3,55}, e_{3,1}, e_{3,2}, e_{3,3}, e_{3,4}, e_{3,5}, f_{3,1}, f_{3,2}, f_{3,3}, f_{3,4}, f_{3,5}}
```

```
In[•]:= Short[errors = CCF /@ Cases[RMoves, a_ == b_ :> a - b], 25]
```

Out[•]//Short=

$$\left\{\frac{1}{24\,T}\left(-24\,p_1^2\,p_2\,x_1\,x_2\,x_3+24\,T\,p_1^2\,p_2\,x_1\,x_2\,x_3+12\,p_1^3\,p_2\,x_1^2\,x_2\,x_3-12\,T\,p_1^3\,p_2\,x_1^2\,x_2\,x_3-48\,p_1^2\,p_2^2\,x_1^2\,x_2\,x_3+\right.\right.$$
$$48\,T\,p_1^2\,p_2^2\,x_1^2\,x_2\,x_3+12\,p_1^3\,p_3\,x_1^2\,x_2\,x_3-12\,T\,p_1^3\,p_3\,x_1^2\,x_2\,x_3+12\,p_1^3\,x_2^2\,x_3-24\,T\,p_1^3\,x_2^2\,x_3+12\,T^2\,p_1^3\,x_2^2\,x_3-$$
$$72\,p_1^2\,p_2\,x_2^2\,x_3+240\,T\,p_1^2\,p_2\,x_2^2\,x_3-168\,T^2\,p_1^2\,p_2\,x_2^2\,x_3-60\,T\,p_1\,p_2^2\,x_2^2\,x_3+60\,T^2\,p_1\,p_2^2\,x_2^2\,x_3+$$
$$96\,p_1^3\,p_2\,x_1\,x_2^2\,x_3-240\,T\,p_1^3\,p_2\,x_1\,x_2^2\,x_3+144\,T^2\,p_1^3\,p_2\,x_1\,x_2^2\,x_3-60\,p_1^2\,p_2^2\,x_1\,x_2^2\,x_3+\text{<<3662>>}+$$
$$144\,T^3\,p_1^4\,x_3^4\,c_{3,55}-96\,T^4\,p_1^4\,x_3^4\,c_{3,55}+24\,T^5\,p_1^4\,x_3^4\,c_{3,55}+96\,T^2\,p_1^3\,p_3\,x_3^4\,c_{3,55}-288\,T^3\,p_1^3\,p_3\,x_3^4\,c_{3,55}+$$
$$288\,T^4\,p_1^3\,p_3\,x_3^4\,c_{3,55}-96\,T^5\,p_1^3\,p_3\,x_3^4\,c_{3,55}-96\,T^2\,p_2^3\,p_3\,x_3^4\,c_{3,55}+288\,T^3\,p_2^3\,p_3\,x_3^4\,c_{3,55}-$$
$$288\,T^4\,p_2^3\,p_3\,x_3^4\,c_{3,55}+96\,T^5\,p_2^3\,p_3\,x_3^4\,c_{3,55}+144\,T^3\,p_1^2\,p_2^2\,x_3^4\,c_{3,55}-288\,T^4\,p_1^2\,p_2^2\,x_3^4\,c_{3,55}+$$
$$144\,T^5\,p_1^2\,p_2^2\,x_3^4\,c_{3,55}-144\,T^3\,p_2^2\,p_2^2\,x_3^4\,c_{3,55}+288\,T^4\,p_2^2\,p_2^2\,x_3^4\,c_{3,55}-144\,T^5\,p_2^2\,p_2^2\,x_3^4\,c_{3,55}+$$
$$\left.96\,T^4\,p_1\,p_3^3\,x_3^4\,c_{3,55}-96\,T^5\,p_1\,p_3^3\,x_3^4\,c_{3,55}-96\,T^4\,p_2\,p_3^3\,x_3^4\,c_{3,55}+96\,T^5\,p_2\,p_3^3\,x_3^4\,c_{3,55}\right),\ \frac{1}{24\,T^6}$$
$$\left(24\,T^3\,p_1\,p_2\,x_1\,x_2-24\,T^3\,p_1^2\,p_2\,x_1^2\,x_2-12\,T^2\,p_1\,p_2\,x_2^2+36\,T^2\,p_1^3\,x_1\,x_2^2-36\,T^3\,p_1^3\,x_1\,x_2^2+60\,T^2\,p_1^2\,p_2\,x_1\,x_2^2+\right.$$
$$\left.\text{<<842>>}+144\,T^{10}\,p_1^2\,p_2^2\,x_2^4\,d_{3,55}+96\,T^9\,p_1\,p_2^3\,x_2^4\,d_{3,55}-96\,T^{10}\,p_1\,p_2^3\,x_2^4\,d_{3,55}+24\,T^{10}\,p_2^4\,x_2^4\,d_{3,55}\right),$$
$$\frac{-p_1\,x_1+T^3\,e_{3,1}+T^3\,p_1\,x_1\,e_{3,2}+\text{<<6>>}+T^3\,p_1^3\,x_1^3\,f_{3,4}+T^3\,p_1^4\,x_1^4\,f_{3,5}}{T^3},$$
$$\frac{\text{<<3127>>}+24\ \text{<<3>>}\ f_{\text{<<1>>}}}{24\,T^4},$$
$$\frac{\text{<<1>>}}{12\,T^6},$$
$$\frac{\text{<<1>>}}{12\,T^2},$$
$$\frac{108-216\,T+\text{<<209>>}+12\,T^7\,p_1^4\,x_1^4\,f_{3,5}}{12\,T^3},$$
$$\frac{1}{12\,T^6}$$
$$\left(-12\,T^2\,p_1\,x_1+18\,T\,p_1^2\,x_1^2+12\,T^2\,p_1^2\,x_1^2+6\,p_1^3\,x_1^3+\text{<<148>>}+\right.$$
$$\left.12\,T^5\,p_1\,x_1\,e_{3,2}+12\,T^4\,p_1^2\,x_1^2\,e_{3,3}+12\,T^3\,p_1^3\,x_1^3\,e_{3,4}+12\,T^2\,p_1^4\,x_1^4\,e_{3,5}\right),$$
$$\frac{1}{12\,T^6}\left(12-144\,T+312\,T^2-180\,T^3-36\,p_1\,x_1+216\,T\,p_1\,x_1+108\,T^2\,p_1\,x_1-408\,T^3\,p_1\,x_1+18\,p_1^2\,x_1^2+\right.$$
$$60\,T\,p_1^2\,x_1^2-600\,T^2\,p_1^2\,x_1^2+444\,T^3\,p_1^2\,x_1^2-2\,p_1^3\,x_1^3-52\,T\,p_1^3\,x_1^3+166\,T^2\,p_1^3\,x_1^3-54\,T^3\,p_1^3\,x_1^3+$$
$$\text{<<171>>}+12\,T^3\,p_1^3\,x_1^3\,e_{3,4}+288\,T^2\,e_{3,5}-1152\,T^3\,e_{3,5}+1728\,T^4\,e_{3,5}-1152\,T^5\,e_{3,5}+288\,T^6\,e_{3,5}-$$
$$1152\,T^2\,p_1\,x_1\,e_{3,5}+3456\,T^3\,p_1\,x_1\,e_{3,5}-3456\,T^4\,p_1\,x_1\,e_{3,5}+1152\,T^5\,p_1\,x_1\,e_{3,5}+864\,T^2\,p_1^2\,x_1^2\,e_{3,5}-$$
$$\left.1728\,T^3\,p_1^2\,x_1^2\,e_{3,5}+864\,T^4\,p_1^2\,x_1^2\,e_{3,5}-192\,T^2\,p_1^3\,x_1^3\,e_{3,5}+192\,T^3\,p_1^3\,x_1^3\,e_{3,5}+12\,T^2\,p_1^4\,x_1^4\,e_{3,5}\right)\Big\}$$

In[•]:=
```
Short[#, 10] &[eqns =
  Thread[0 == Union @@ (CoefficientRules[#, {x1, x2, x3, p1, p2, p3}][[;; , 2]] & /@ errors)]]
```

Out[•]//Short=

$$\Big\{0 == c_{3,4} - T\,c_{3,4},\ 0 == -c_{3,4} + T\,c_{3,4},\ 0 == T\,c_{3,4} - T^2\,c_{3,4},$$

$$\ll 489 \gg,\ 0 == -1 + \frac{9}{T^3} - \frac{18}{T^2} + \frac{10}{T} + d_{3,1} - T\,d_{3,3} + 2\,T^2\,d_{3,8} - 6\,T^3\,d_{3,18} +$$

$$24\,T^4\,d_{3,35} + f_{3,1} + f_{3,2} - T\,f_{3,2} + 2\,f_{3,3} - 4\,T\,f_{3,3} + 2\,T^2\,f_{3,3} + 6\,f_{3,4} - 18\,T\,f_{3,4} +$$

$$18\,T^2\,f_{3,4} - 6\,T^3\,f_{3,4} + 24\,f_{3,5} - 96\,T\,f_{3,5} + 144\,T^2\,f_{3,5} - 96\,T^3\,f_{3,5} + 24\,T^4\,f_{3,5},$$

$$0 == \frac{3}{2} - \frac{4}{T^3} + \frac{23}{T^2} - \frac{25}{T} + d_{3,6} + T\,d_{3,7} + T^2\,d_{3,8} + d_{3,9} + T\,d_{3,10} + T^2\,d_{3,11} + d_{3,12} + T\,d_{3,13} + T^2\,d_{3,14} -$$

$$3\,T\,d_{3,16} - 6\,T^2\,d_{3,17} - 9\,T^3\,d_{3,18} - 2\,T\,d_{3,20} - 4\,T^2\,d_{3,21} - 6\,T^3\,d_{3,22} - T\,d_{3,24} - 2\,T^2\,d_{3,25} -$$

$$3\,T^3\,d_{3,26} + 12\,T^2\,d_{3,33} + 36\,T^3\,d_{3,34} + 72\,T^4\,d_{3,35} + 6\,T^2\,d_{3,38} + 18\,T^3\,d_{3,39} + 36\,T^4\,d_{3,40} + 2\,T^2\,d_{3,43} +$$

$$6\,T^3\,d_{3,44} + 12\,T^4\,d_{3,45} + T^2\,f_{3,3} + 9\,T^2\,f_{3,4} - 9\,T^3\,f_{3,4} + 72\,T^2\,f_{3,5} - 144\,T^3\,f_{3,5} + 72\,T^4\,f_{3,5}\Big\}$$

In[•]:=
```
{sol} = Solve[eqns, unknowns]
```

Solve: Equations may not give solutions for all "solve" variables.

Out[•]=

$$\Big\{\Big\{c_{3,1} \to -\frac{c_{3,2}}{2} - \frac{c_{3,5}}{2\,T},\ c_{3,3} \to -T\,c_{3,2} - c_{3,5},\ c_{3,4} \to 0,\ c_{3,6} \to 0,$$

$$c_{3,8} \to -\frac{1}{2} \times (1 - T)\,c_{3,10},\ c_{3,9} \to 0,\ c_{3,11} \to -T\,c_{3,7} - \frac{1}{2} \times (-1 + 3\,T)\,c_{3,10},\ c_{3,12} \to 0,$$

$$c_{3,13} \to 0,\ c_{3,14} \to 0,\ c_{3,15} \to 0,\ c_{3,17} \to -\left((-1 + T)\,c_{3,16}\right),\ c_{3,18} \to -\frac{1 - T}{6\,T},\ c_{3,19} \to 0,$$

$$c_{3,20} \to 0,\ c_{3,21} \to \frac{1}{2\,T},\ c_{3,22} \to -\frac{-2 + 5\,T}{2\,T} - \left(T - T^2\right)c_{3,16},\ c_{3,23} \to 0,\ c_{3,24} \to 0,$$

$$c_{3,25} \to 0,\ c_{3,26} \to \frac{5}{6} - T^2\,c_{3,16},\ c_{3,27} \to 0,\ c_{3,28} \to 0,\ c_{3,29} \to 0,\ c_{3,30} \to 0,\ c_{3,31} \to 0,$$

$$c_{3,33} \to -\frac{3}{2} \times (-1 + T)\,c_{3,32},\ c_{3,34} \to -\left(\left(-1 + 2\,T - T^2\right)c_{3,32}\right),\ c_{3,35} \to -\frac{1 - 12\,T + 27\,T^2 - 16\,T^3}{24\,T^2},$$

$$c_{3,36} \to 0,\ c_{3,37} \to \frac{1}{6\,T^2},\ c_{3,38} \to -\frac{-1 + 3\,T}{4\,T^2},\ c_{3,39} \to -\frac{-1 + 11\,T - 16\,T^2}{6\,T^2},$$

$$c_{3,40} \to -\frac{-1 + 31\,T - 131\,T^2 + 125\,T^3}{24\,T^2} - \left(T - 2\,T^2 + T^3\right)c_{3,32},\ c_{3,41} \to 0,\ c_{3,42} \to 0,\ c_{3,43} \to \frac{1}{T},$$

$$c_{3,44} \to -\frac{-5 + 23\,T}{6\,T},\ c_{3,45} \to -\frac{-5 + 69\,T - 142\,T^2}{24\,T} + \frac{3}{2} \times (-1 + T)\,T^2\,c_{3,32},\ c_{3,46} \to 0,\ c_{3,47} \to 0,$$

$$c_{3,48} \to 0,\ c_{3,49} \to \frac{1}{6},\ c_{3,50} \to \frac{1}{24} \times (1 - 15\,T) - T^3\,c_{3,32},\ c_{3,51} \to 0,\ c_{3,52} \to 0,\ c_{3,53} \to 0,\ c_{3,54} \to 0,$$

$$c_{3,55} \to 0,\ d_{3,1} \to \frac{c_{3,2}}{2} + \frac{c_{3,5}}{2\,T},\ d_{3,2} \to -c_{3,2},\ d_{3,3} \to \frac{c_{3,2}}{T} + \frac{c_{3,5}}{T^2},\ d_{3,4} \to 0,\ d_{3,5} \to -\frac{c_{3,5}}{T^2},$$

$$d_{3,6} \to 0,\ d_{3,7} \to -\frac{-1 + T}{T^4} - \frac{c_{3,7}}{T} - \frac{(-1 + T)\,c_{3,10}}{T^2},\ d_{3,8} \to -\frac{1 - T}{2\,T^5} - \frac{(1 - T)\,c_{3,10}}{2\,T^3},\ d_{3,9} \to 0,$$

$$d_{3,10} \to -\frac{1}{T^4} - \frac{c_{3,10}}{T^2},\ d_{3,11} \to \frac{1}{2\,T^5} + \frac{c_{3,7}}{T^2} - \frac{(-1 - T)\,c_{3,10}}{2\,T^3},\ d_{3,12} \to 0,\ d_{3,13} \to 0,\ d_{3,14} \to 0,\ d_{3,15} \to 0,$$

$$d_{3,16}\to -\frac{1-T}{T^4}-\frac{c_{3,16}}{T},\ d_{3,17}\to -\frac{-7+9T-2T^2}{2T^5}-\frac{(-1+T)\,c_{3,16}}{T^2},\ d_{3,18}\to -\frac{7-9T+2T^2}{6T^6},\ d_{3,19}\to 0,$$
$$d_{3,20}\to \frac{1}{T^4},\ d_{3,21}\to -\frac{9-T}{2T^5},\ d_{3,22}\to \frac{3}{2T^6}-\frac{(1-T)\,c_{3,16}}{T^3},\ d_{3,23}\to 0,\ d_{3,24}\to 0,\ d_{3,25}\to \frac{1}{T^5},$$
$$d_{3,26}\to -\frac{1}{3T^6}+\frac{c_{3,16}}{T^3},\ d_{3,27}\to 0,\ d_{3,28}\to 0,\ d_{3,29}\to 0,\ d_{3,30}\to 0,\ d_{3,31}\to 0,\ d_{3,32}\to -\frac{-1+T}{6T^4}-\frac{c_{3,32}}{T},$$
$$d_{3,33}\to -\frac{2-3T+T^2}{T^5}-\frac{3\times(-1+T)\,c_{3,32}}{2T^2},\ d_{3,34}\to -\frac{-16+27T-12T^2+T^3}{6T^6}-\frac{\left(1-2T+T^2\right)c_{3,32}}{T^3},$$
$$d_{3,35}\to -\frac{16-27T+12T^2-T^3}{24T^7},\ d_{3,36}\to 0,\ d_{3,37}\to -\frac{1}{6T^4},\ d_{3,38}\to -\frac{-3+T}{T^5},$$
$$d_{3,39}\to \frac{3\times(-3+T)}{2T^6},\ d_{3,40}\to -\frac{-27+5T-T^2-T^3}{24T^7}-\frac{\left(-1+2T-T^2\right)c_{3,32}}{T^4},\ d_{3,41}\to 0,$$
$$d_{3,42}\to 0,\ d_{3,43}\to -\frac{1}{T^5},\ d_{3,44}\to \frac{2}{T^6},\ d_{3,45}\to -\frac{12-T-5T^2}{24T^7}+\frac{3\times(-1+T)\,c_{3,32}}{2T^4},$$
$$d_{3,46}\to 0,\ d_{3,47}\to 0,\ d_{3,48}\to 0,\ d_{3,49}\to -\frac{1}{6T^6},\ d_{3,50}\to -\frac{-1-T}{24T^7}+\frac{c_{3,32}}{T^4},\ d_{3,51}\to 0,$$
$$d_{3,52}\to 0,\ d_{3,53}\to 0,\ d_{3,54}\to 0,\ d_{3,55}\to 0,\ e_{3,1}\to -\frac{c_{3,2}}{2}-\frac{c_{3,5}}{2T},\ e_{3,2}\to -\frac{c_{3,10}}{T},\ e_{3,3}\to 0,$$
$$e_{3,4}\to 0,\ e_{3,5}\to 0,\ f_{3,1}\to \frac{c_{3,2}}{2}+\frac{c_{3,5}}{2T},\ f_{3,2}\to \frac{1}{T^3}+\frac{c_{3,10}}{T},\ f_{3,3}\to 0,\ f_{3,4}\to 0,\ f_{3,5}\to 0\Big\}\Big\}$$

In[•]:=
```
sol /. (a_ → b_) :> (a = b)
```

Out[•]=
$$\left\{-\frac{c_{3,2}}{2}-\frac{c_{3,5}}{2\,T},\ -T\,c_{3,2}-c_{3,5},\ 0,\ 0,\ -\frac{1}{2}\times(1-T)\,c_{3,10},\ 0,\ -T\,c_{3,7}-\frac{1}{2}\times(-1+3\,T)\,c_{3,10},\ 0,\ 0,\ 0,\right.$$
$$0,\ -((-1+T)\,c_{3,16}),\ -\frac{1-T}{6\,T},\ 0,\ 0,\ \frac{1}{2\,T},\ -\frac{-2+5\,T}{2\,T}-\left(T-T^2\right)c_{3,16},\ 0,\ 0,\ 0,\ \frac{5}{6}-T^2\,c_{3,16},\ 0,$$
$$0,\ 0,\ 0,\ 0,\ -\frac{3}{2}\times(-1+T)\,c_{3,32},\ -\left(\left(-1+2\,T-T^2\right)c_{3,32}\right),\ -\frac{1-12\,T+27\,T^2-16\,T^3}{24\,T^2},\ 0,\ \frac{1}{6\,T^2},$$
$$-\frac{-1+3\,T}{4\,T^2},\ -\frac{-1+11\,T-16\,T^2}{6\,T^2},\ -\frac{-1+31\,T-131\,T^2+125\,T^3}{24\,T^2}-\left(T-2\,T^2+T^3\right)c_{3,32},\ 0,\ 0,\ \frac{1}{T},$$
$$-\frac{-5+23\,T}{6\,T},\ -\frac{-5+69\,T-142\,T^2}{24\,T}+\frac{3}{2}\times(-1+T)\,T^2\,c_{3,32},\ 0,\ 0,\ 0,\ \frac{1}{6},\ \frac{1}{24}\times(1-15\,T)-T^3\,c_{3,32},$$
$$0,\ 0,\ 0,\ 0,\ 0,\ \frac{c_{3,2}}{2}+\frac{c_{3,5}}{2\,T},\ -c_{3,2},\ \frac{c_{3,2}}{T}+\frac{c_{3,5}}{T^2},\ 0,\ -\frac{c_{3,5}}{T^2},\ 0,\ -\frac{-1+T}{T^4}-\frac{c_{3,7}}{T}-\frac{(-1+T)\,c_{3,10}}{T^2},$$
$$-\frac{1-T}{2\,T^5}-\frac{(1-T)\,c_{3,10}}{2\,T^3},\ 0,\ -\frac{1}{T^4}-\frac{c_{3,10}}{T^2},\ \frac{1}{2\,T^5}+\frac{c_{3,7}}{T^2}-\frac{(-1-T)\,c_{3,10}}{2\,T^3},\ 0,\ 0,\ 0,\ 0,\ -\frac{1-T}{T^4}-\frac{c_{3,16}}{T},$$
$$-\frac{-7+9\,T-2\,T^2}{2\,T^5}-\frac{(-1+T)\,c_{3,16}}{T^2},\ -\frac{7-9\,T+2\,T^2}{6\,T^6},\ 0,\ \frac{1}{T^4},\ -\frac{9-T}{2\,T^5},\ \frac{3}{2\,T^6}-\frac{(1-T)\,c_{3,16}}{T^3},\ 0,$$
$$0,\ \frac{1}{T^5},\ -\frac{1}{3\,T^6}+\frac{c_{3,16}}{T^3},\ 0,\ 0,\ 0,\ 0,\ 0,\ -\frac{-1+T}{6\,T^4}-\frac{c_{3,32}}{T},\ -\frac{2-3\,T+T^2}{T^5}-\frac{3\times(-1+T)\,c_{3,32}}{2\,T^2},$$
$$-\frac{-16+27\,T-12\,T^2+T^3}{6\,T^6}-\frac{\left(1-2\,T+T^2\right)c_{3,32}}{T^3},\ -\frac{16-27\,T+12\,T^2-T^3}{24\,T^7},\ 0,\ -\frac{1}{6\,T^4},$$
$$-\frac{-3+T}{T^5},\ \frac{3\times(-3+T)}{2\,T^6},\ -\frac{-27+5\,T-T^2-T^3}{24\,T^7}-\frac{\left(-1+2\,T-T^2\right)c_{3,32}}{T^4},\ 0,\ 0,\ -\frac{1}{T^5},$$
$$\frac{2}{T^6},\ -\frac{12-T-5\,T^2}{24\,T^7}+\frac{3\times(-1+T)\,c_{3,32}}{2\,T^4},\ 0,\ 0,\ 0,\ -\frac{1}{6\,T^6},\ -\frac{-1-T}{24\,T^7}+\frac{c_{3,32}}{T^4},\ 0,$$
$$\left.0,\ 0,\ 0,\ 0,\ -\frac{c_{3,2}}{2}-\frac{c_{3,5}}{2\,T},\ -\frac{c_{3,10}}{T},\ 0,\ 0,\ 0,\ \frac{c_{3,2}}{2}+\frac{c_{3,5}}{2\,T},\ \frac{1}{T^3}+\frac{c_{3,10}}{T},\ 0,\ 0,\ 0\right\}$$

In[•]:=
```
Cases[{R1,2, R̄1,2, C1, C̄1}, (c | d | e | f)$k_, ∞] // Union
```

Out[•]=
$$\{c_{3,2},\ c_{3,5},\ c_{3,7},\ c_{3,10},\ c_{3,16},\ c_{3,32}\}$$

In[•]:=
```
{c3,2 = 0, c3,5 = 0, c3,7 = 0, c3,10 = 0, c3,16 = 0, c3,32 = 1};
{R1,2, R̄1,2, C1, C̄1}
```

Out[•]=
$$\left\{\mathbb{E}_{\{\}\to\{1,2\}}\left[\sqrt{T},\ (-1+T)\,(p_1-p_2)\,x_2,\right.\right.$$
$$\in\text{Series}\left[0,\ p_1\,p_2\,x_1\,x_2+\frac{1}{2}\times(-1+T)\,p_1^2\,x_2^2+\frac{1}{2}\times(1-3\,T)\,p_1\,p_2\,x_2^2,\right.$$
$$-\frac{p_1^2\,p_2\,x_1^2\,x_2}{2\,T}-\frac{1}{2}\,p_1\,p_2\,x_2^2-\frac{(1-3\,T)\,p_1^2\,p_2\,x_1\,x_2^2}{2\,T}-\frac{1}{2}\,p_1\,p_2^2\,x_1\,x_2^2-$$
$$\left.\frac{\left(-1+4\,T-3\,T^2\right)p_1^3\,x_2^3}{6\,T}-\frac{\left(1-11\,T+16\,T^2\right)p_1^2\,p_2\,x_2^3}{6\,T}+\frac{1}{6}\times(-1+7\,T)\,p_1\,p_2^2\,x_2^3,\right.$$

$$p_1^4 x_1^3 x_2 + \frac{p_1^3 p_2 x_1^3 x_2}{6 T^2} + \frac{p_1^2 p_2 x_1 x_2^2}{2 T} - \frac{3}{2} \times (-1+T)\, p_1^4 x_1^2 x_2^2 - \frac{(-1+3T)\, p_1^3 p_2 x_1^2 x_2^2}{4 T^2} +$$
$$\frac{p_1^2 p_2^2 x_1^2 x_2^2}{T} - \frac{(1-T)\, p_1^3 x_2^3}{6 T} - \frac{(-2+5T)\, p_1^2 p_2 x_2^3}{2 T} + \frac{5}{6} p_1 p_2^2 x_2^3 + \left(1-2T+T^2\right) p_1^4 x_1 x_2^3 -$$
$$\frac{\left(-1+11T-16T^2\right) p_1^3 p_2 x_1 x_2^3}{6 T^2} - \frac{(-5+23T)\, p_1^2 p_2^2 x_1 x_2^3}{6 T} + \frac{1}{6} p_1 p_2^3 x_1 x_2^3 -$$
$$\frac{\left(1-12T+27T^2-16T^3\right) p_1^4 x_2^4}{24 T^2} + \left(-T+2T^2-T^3 - \frac{-1+31T-131T^2+125T^3}{24 T^2}\right) p_1^3 p_2 x_2^4 +$$
$$\left(\frac{3}{2} \times (-1+T)\, T^2 - \frac{-5+69T-142T^2}{24 T}\right) p_1^2 p_2^2 x_2^4 + \left(\frac{1}{24} \times (1-15T) - T^3\right) p_1 p_2^3 x_2^4 \Big]\Big],$$

$$\mathbb{E}_{\{\}\to\{1,2\}}\Big[\frac{1}{\sqrt{T}}, \left(-1+\frac{1}{T}\right)(p_1-p_2)\, x_2, \in \text{Series}\Big[0, -\frac{(-1+T)\, p_1^2 x_1 x_2}{T^2} - \frac{p_1 p_2 x_1 x_2}{T^2} -$$
$$\frac{(1-T)\, p_1^2 x_2^2}{2 T^3} - \frac{(-1-T)\, p_1 p_2 x_2^2}{2 T^3}, -\frac{(1-T)\, p_1^2 x_1 x_2}{T^3} + \frac{p_1 p_2 x_1 x_2}{T^3} - \frac{(-1+T)\, p_1^3 x_1^2 x_2}{2 T^3} -$$
$$\frac{p_1^2 p_2 x_1^2 x_2}{2 T^3} - \frac{(-1+T)\, p_1^2 x_2^2}{2 T^4} - \frac{p_1 p_2 x_2^2}{2 T^4} - \frac{\left(3-4T+T^2\right) p_1^3 x_1 x_2^2}{2 T^4} + \frac{2 p_1^2 p_2 x_1 x_2^2}{T^4} -$$
$$\frac{p_1 p_2^2 x_1 x_2^2}{2 T^4} - \frac{\left(-3+4T-T^2\right) p_1^3 x_2^3}{6 T^5} - \frac{\left(4+T+T^2\right) p_1^2 p_2 x_2^3}{6 T^5} - \frac{(-1+T)\, p_1 p_2^2 x_2^3}{6 T^5},$$
$$-\frac{(-1+T)\, p_1^2 x_1 x_2}{T^4} - \frac{p_1 p_2 x_1 x_2}{T^4} - \frac{(1-T)\, p_1^3 x_1^2 x_2}{T^4} + \frac{p_1^2 p_2 x_1^2 x_2}{T^4} + \left(-\frac{-1+T}{6 T^4} - \frac{1}{T}\right) p_1^4 x_1^3 x_2 -$$
$$\frac{p_1^3 p_2 x_1^3 x_2}{6 T^4} - \frac{(1-T)\, p_1^2 x_2^2}{2 T^5} + \frac{p_1 p_2 x_2^2}{2 T^5} - \frac{\left(-7+9T-2T^2\right) p_1^3 x_1 x_2^2}{2 T^5} -$$
$$\frac{(9-T)\, p_1^2 p_2 x_1 x_2^2}{2 T^5} + \frac{p_1 p_2^2 x_1 x_2^2}{T^5} + \left(-\frac{3 \times (-1+T)}{2 T^2} - \frac{2-3T+T^2}{T^5}\right) p_1^4 x_1^2 x_2^2 -$$
$$\frac{(-3+T)\, p_1^3 p_2 x_1^2 x_2^2}{T^5} - \frac{p_1^2 p_2^2 x_1^2 x_2^2}{T^5} - \frac{\left(7-9T+2T^2\right) p_1^3 x_2^3}{6 T^6} + \frac{3 p_1^2 p_2 x_2^3}{2 T^6} - \frac{p_1 p_2^2 x_2^3}{3 T^6} +$$
$$\left(-\frac{1-2T+T^2}{T^3} - \frac{-16+27T-12T^2+T^3}{6 T^6}\right) p_1^4 x_1 x_2^3 + \frac{3 \times (-3+T)\, p_1^3 p_2 x_1 x_2^3}{2 T^6} + \frac{2 p_1^2 p_2^2 x_1 x_2^3}{T^6} -$$
$$\frac{p_1 p_2^3 x_1 x_2^3}{6 T^6} - \frac{\left(16-27T+12T^2-T^3\right) p_1^4 x_2^4}{24 T^7} + \left(-\frac{-1+2T-T^2}{T^4} - \frac{-27+5T-T^2-T^3}{24 T^7}\right) p_1^3 p_2 x_2^4 +$$
$$\left(\frac{3 \times (-1+T)}{2 T^4} - \frac{12-T-5T^2}{24 T^7}\right) p_1^2 p_2^2 x_2^4 + \left(-\frac{-1-T}{24 T^7} + \frac{1}{T^4}\right) p_1 p_2^3 x_2^4 \Big]\Big],$$

$$\mathbb{E}_{\{\}\to\{1\}}\Big[\sqrt{T}, 0, \in \text{Series}\Big[0, -\frac{p_1 x_1}{T}, 0, 0\Big]\Big],$$

$$\mathbb{E}_{\{\}\to\{1\}}\Big[\frac{1}{\sqrt{T}}, 0, \in \text{Series}\Big[0, \frac{p_1 x_1}{T}, -\frac{p_1 x_1}{T^2}, \frac{p_1 x_1}{T^3}\Big]\Big]\Big\}$$

In[•]:= **RMoves**

Out[•]= {True, True, True, True, True, True, True, True, True}