

Pensieve header: Perturbing the Heisenberg-algebra knot invariant. Continues “Solving to $\$k=3.nb$ ” at pensieve://Projects/BabyDoPeGDO/.

$\mathbb{E}[\omega, Q, P_eSeries]$ represents ωe^{Q+P} , where ω is a scalar, Q is an ϵ -free quadratic, and $P = \sum_{k=0}^{\$k} P[[k]] \epsilon^k$ is a perturbation (it is ill-advised to include ω in P because then it will have log terms).

Scheme: $\mathbb{E}[_] // \mathbb{E}[_]$ calls FZip or Zip, which are functionally the same. Zip works by handling the quadratic part and calling PZip for the perturbation-only part. PZip works by iteratively solving the synthesis equation. FZip works by encapsulating coefficients, calling Zip, and back-substituting.

Initialization and minor utilities

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Classes\\21-1350-KnotTheory\\Preps"];
Once[<< ". /Common.m"];

```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[*]:= $k=1;
```

```
In[*]:= CCF[_] := ExpandDenominator@ExpandNumerator@Together[_];
CF[_List] := CF /@ _;
CF[_eSeries] := CF /@ _;
CF[_] := Module[
  {vs = Cases[_eSeries, {p | x | pi | xi}_, Infinity] U {p | x | pi | xi}},
  Total[CoefficientRules[Expand[_], vs] /. (ps_ -> c_) -> CCF[c] (Times @@ vs^ps)]
];
CF[_E] := CF /@ _;
CF[_E_sp___[_eS___]] := CF /@ _E_sp[_eS];

```

```
In[*]:= eSeries /: S1_eSeries == S2_eSeries :=
  Length[S1] == Length[S2] & Inner[CF[#1] == CF[#2] &, S1, S2, And];
eSeries[0] := eSeries @@ Table[0, {k, 1};
eSeries /: S1_eSeries + S2_eSeries :=
  eSeries @@ Table[S1[[k]] + S2[[k]], {k, Min[Length@S1, Length@S2]};
eSeries /: S1_eSeries * S2_eSeries := eSeries @@
  Table[Sum[S1[[j+1]] * S2[[k-j+1]], {j, 0, k}], {k, 0, Min[Length@S1, Length@S2] - 1};
eSeries /: c_ * S_eSeries := (c #) & /@ S;
eSeries /: D_vS___ S_eSeries := (s -> D_vS s) /@ S;

```

The Main Program

Variables and their duals:

```
In[*]:= {p*, x*, pi*, xi*} = {pi, xi, p, x};
(vs_List)* := (v -> v*) /@ vs;
(u_{-i})* := (u*)_i;
```

E operations:

```
In[*]:= E /: E[w1_, Q1_, P1_] == E[w2_, Q2_, P2_] := CF[w1 == w2] ^ CF[Q1 == Q2] ^ (P1 == P2);
E /: E[w1_, Q1_, P1_] E[w2_, Q2_, P2_] := E[w1 w2, Q1 + Q2, P1 + P2];
E_{d1 -> r1}[E1s_] == E_{d2 -> r2}[E2s_] ^:= (d1 == d2) ^ (r1 == r2) ^ (E[E1s] == E[E2s]);
E_{d1 -> r1}[E1s_] E_{d2 -> r2}[E2s_] ^:= E_{(d1 U d2) -> (r1 U r2)} @@ (E[E1s] E[E2s]);
E_{dr_}[E_{s_}]_{k_} := E_{dr} @@ E[E_{s_}]_{k_};
```

```
In[*]:= E_{d1 -> r1}[E1s_] // E_{d2 -> r2}[E2s_] := Module[{is = r1 ^ d2, lvs},
  lvs = Flatten@Table[{x_{i}, p_{i}}, {i, is}];
  E_{(d1 U Complement[d2, is]) -> (r2 U Complement[r1, is])} @@ (Zip[lvs U lvs, Times[
    E[E1s] /. Table[(v : x | p)_i -> v_{i}, {i, is}],
    E[E2s] /. Table[(v : xi | pi)_i -> v_{i}, {i, is}]
  ]])
]
```

$[F : \mathcal{E}]_B := \mathbb{E}^{\frac{1}{2} \sum_{i,j \in B} F_{ij} \partial_{z_i} \partial_{z_j}} \mathcal{E}$ and $\langle F : \mathcal{E} \rangle_B := [F : \mathcal{E}]_B|_{z_B \rightarrow 0}$,
 where \mathcal{E} is a docile perturbed Gaussian. The following lemma allows us to restrict to the case where \mathcal{E} has no B - B quadratic part:

Lemma 1. With convergences left to the reader,

$$\langle F : \mathcal{E} \mathbb{E}^{\frac{1}{2} \sum_{i,j \in B} G_{ij} z_i z_j} \rangle_B = \det(1 - GF)^{-1/2} \langle F(1 - GF)^{-1} : \mathcal{E} \rangle_B.$$

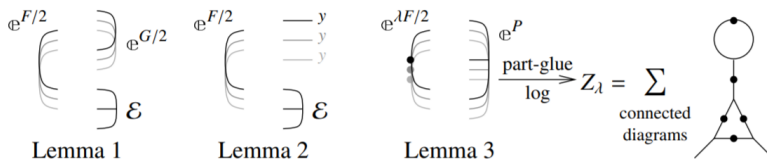
The next lemma dispatches the case where \mathcal{E} has a B -linear part:

Lemma 2. $\langle F : \mathcal{E} \mathbb{E}^{\sum_{i \in B} y_i z_i} \rangle_B = \mathbb{E}^{\frac{1}{2} \sum_{i,j \in B} F_{ij} y_i y_j} \langle F : \mathcal{E}|_{z_B \rightarrow z_B + Fy_B} \rangle_B$

Finally, we deal with the docile perturbation case:

Lemma 3. With an extra variable λ , $Z_\lambda := \log[\lambda F : \mathbb{E}^P]_B$ satisfies and is determined by the following PDE / IVP:

$$Z_0 = P \quad \text{and} \quad \partial_\lambda Z_\lambda = \frac{1}{2} \sum_{i,j \in B} F_{ij} (\partial_{z_i} \partial_{z_j} Z_\lambda + (\partial_{z_i} Z_\lambda)(\partial_{z_j} Z_\lambda)).$$



```
In[*]:= Zip_{vs}[F_, E_] := <F, E> // Zip1_{vs} // Zip2_{vs} // Zip3_{vs}
```

Getting rid of the quadratic.

Lemma 1. With convergences left to the reader,

$$\left\langle F : \mathcal{E} \otimes^{\frac{1}{2} \sum_{i,j \in B} G_{ij} z_i z_j} \right\rangle_B = \det(1 - GF)^{-1/2} \left\langle F(1 - GF)^{-1} : \mathcal{E} \right\rangle_B$$

In[*]:=

```

Zip1_{ } = Identity;
Zip1_{vs_} @ < \mathcal{F}_-, \mathbb{E}[\omega_-, Q_-, P_-] > := Module[{I, F, G, u, v},
  I = IdentityMatrix@Length@vs;
  F = Table[\partial_{u,v} \mathcal{F}, {u, vs*}, {v, vs*}];
  G = Table[\partial_{u,v} Q, {u, vs}, {v, vs}];
  CF /@ {vs*.F.Inverse[I - G.F].vs* / 2,
    \mathbb{E}[PowerExpand@Factor[\omega Det[I - G.F]^{-1/2}], Q - vs.G.vs / 2, P]}
]

```

Getting rid of linear terms.

Lemma 2. $\left\langle F : \mathcal{E} \otimes^{\sum_{i \in B} y_i z_i} \right\rangle_B = \otimes^{\frac{1}{2} \sum_{i,j \in B} F_{ij} y_i y_j} \left\langle F : \mathcal{E}|_{z_B \rightarrow z_B + F y_B} \right\rangle_B$.

In[*]:=

```

Zip2_{ } = Identity;
Zip2_{vs_} @ < \mathcal{F}_-, \mathbb{E}[\omega_-, Q_-, P_-] > := Module[{F, Y, u, v},
  F = Table[\partial_{u,v} \mathcal{F}, {u, vs*}, {v, vs*}];
  Y = Table[\partial_v Q, {v, vs}];
  CF /@ {< \mathcal{F}, \mathbb{E}[\omega, Q - Y.vs + Y.F.Y / 2, P /. Thread[vs \to vs + F.Y]] >}
]

```

Dealing with Feynman diagrams.

Lemma 3. With an extra variable λ , $Z_\lambda := \log[\lambda F : \mathbb{E}^P]_B$ satisfies and is determined by the following PDE / IVP:

$$Z_0 = P \quad \text{and} \quad \partial_\lambda Z_\lambda = \frac{1}{2} \sum_{i,j \in B} F_{ij} (\partial_{z_i} \partial_{z_j} Z_\lambda + (\partial_{z_i} Z_\lambda)(\partial_{z_j} Z_\lambda)).$$

Note that the power m of λ is at most $k - 1 + \frac{2k+2}{2} = 2k$. We write $Z_\lambda = \sum Z[m] \lambda^m$ and then $(m + 1) \partial_\lambda Z[m + 1] = \frac{1}{2} F_{uv} (\partial_u \partial_v Z[m] + \sum_j (\partial_u Z[j]) (\partial_v Z[m - j]))$.

In[*]:=

```

Zip3_{vs_} @ < \mathcal{F}_-, \mathbb{E}[\omega_-, Q_-, P_-] > := Module[{Z, u, v, m, j},
  Z[0] = P;
  For[m = 0, m < 2 $k, ++m,
    Z[m + 1] = CF [
      \frac{1}{2 (m + 1)}
      Sum[\partial_{u*,v*} \mathcal{F} (\partial_{u,v} Z[m] + Sum[(\partial_u Z[j]) (\partial_v Z[m - j]), {j, 0, m}]), {u, vs}, {v, vs}]
    ]
  ];
  \mathbb{E}[\omega, Q, CF[Sum[Z[m], {m, 0, 2 $k}]] /. Table[v \to 0, {v, vs}]]
]

```

Note that the condition $\deg_{z_B} Z_\lambda \leq 4 \deg_{z_e} Z_\lambda - 2 \deg_\lambda Z_\lambda$ is preserved under the above iteration, so if the

initial P satisfies $\deg_{z_B} P \leq 4 \deg_{\epsilon} P$, we only need to sum up to $Z[2 \text{ } \$k]$. Note also that if P is “docile”, $\deg_{z_{\text{all}}} P < 2 \deg_{\epsilon} P + 2$, then so is Z_{λ} .

The Basic Tensors

```

In[*]:=  $\eta_{i\_} := \mathbb{E}_{\{\} \rightarrow \{i\}} [1, \theta, \text{eSeries}[\theta]];$ 
 $m_{i\_ , j\_ \rightarrow k\_} := \mathbb{E}_{\{i, j\} \rightarrow \{k\}} [1, -\xi_i \pi_j + (\pi_i + \pi_j) p_k + (\xi_i + \xi_j) x_k, \text{eSeries}[\theta]]$ 

In[*]:= AllMonomials[{\}, \theta] = {1};
AllMonomials[{\}, d_Integer] /; d > 0 := {};
AllMonomials[{v_, vs___}, d_Integer] :=
  Join@@Table[v^{d-k} AllMonomials[{vs}, k], {k, \theta, d}];
AllMonomials[vs_List, {d_}] := Join@@Table[AllMonomials[vs, k], {k, \theta, d}];

In[*]:= Basis[js_List, m_] := Flatten@Outer[Times,
  AllMonomials[Table[p_j, {j, js}], m], AllMonomials[Table[x_j, {j, js}], m]];
Basis[js_List, {m_}] := Flatten@Table[Basis[js, k], {k, \theta, m}]

In[*]:= Basis[{i, j}, {2}]
Out[*]:= {1, p_i x_i, p_i x_j, p_j x_i, p_j x_j, p_i^2 x_i^2, p_i^2 x_i x_j, p_i^2 x_j^2, p_i p_j x_i^2, p_i p_j x_i x_j, p_i p_j x_j^2, p_j^2 x_i^2, p_j^2 x_i x_j, p_j^2 x_j^2}

In[*]:= GenericCombination[bas_, c_] := bas.Table[c_j, {j, Length@bas}];
GenericCombination[bas_, c_k_] := bas.Table[c_{k,j}, {j, Length@bas}];

In[*]:= GenericCombination[Basis[{i, j}, {2}], c_1]
Out[*]:= c_{1,1} + p_i x_i c_{1,2} + p_i x_j c_{1,3} + p_j x_i c_{1,4} + p_j x_j c_{1,5} + p_i^2 x_i^2 c_{1,6} + p_i^2 x_i x_j c_{1,7} + p_i^2 x_j^2 c_{1,8} +
  p_i p_j x_i^2 c_{1,9} + p_i p_j x_i x_j c_{1,10} + p_i p_j x_j^2 c_{1,11} + p_j^2 x_i^2 c_{1,12} + p_j^2 x_i x_j c_{1,13} + p_j^2 x_j^2 c_{1,14}

In[*]:= R_{i_, j\_} := \mathbb{E}_{\{\} \rightarrow \{i, j\}} [T^{1/2}, (T - 1) (p_i - p_j) x_j,
  eSeries@@Prepend[\theta]@Table[GenericCombination[Basis[{i, j}, {k + 1}], c_k], {k, \$k}]];
\bar{R}_{i_, j\_} := \mathbb{E}_{\{\} \rightarrow \{i, j\}} [T^{-1/2}, (T^{-1} - 1) (p_i - p_j) x_j,
  eSeries@@Prepend[\theta]@Table[GenericCombination[Basis[{i, j}, {k + 1}], d_k], {k, \$k}]];
C_{i\_} := \mathbb{E}_{\{\} \rightarrow \{i\}} [T^{1/2}, \theta, eSeries@@Prepend[\theta]@
  Table[GenericCombination[Basis[{i}, {k + 1}], e_k], {k, \$k}]];
\bar{C}_{i\_} := \mathbb{E}_{\{\} \rightarrow \{i\}} [T^{-1/2}, \theta, eSeries@@Prepend[\theta]@
  Table[GenericCombination[Basis[{i}, {k + 1}], f_k], {k, \$k}]];

```

```

In[*]:= {R1,2, R1,2, C1, C1}
Out[*]:= {E_{{}->{1,2}} [sqrt(T), (-1 + T) (p1 - p2) x2,
  eSeries[0, c1,1 + p1 x1 c1,2 + p1 x2 c1,3 + p2 x1 c1,4 + p2 x2 c1,5 + p1^2 x1^2 c1,6 + p1^2 x1 x2 c1,7 + p1^2 x2^2 c1,8 +
    p1 p2 x1^2 c1,9 + p1 p2 x1 x2 c1,10 + p1 p2 x2^2 c1,11 + p2^2 x1^2 c1,12 + p2^2 x1 x2 c1,13 + p2^2 x2^2 c1,14 ]],
  E_{{}->{1,2}} [1/sqrt(T), (-1 + 1/T) (p1 - p2) x2, eSeries[0,
    d1,1 + p1 x1 d1,2 + p1 x2 d1,3 + p2 x1 d1,4 + p2 x2 d1,5 + p1^2 x1^2 d1,6 + p1^2 x1 x2 d1,7 + p1^2 x2^2 d1,8 +
    p1 p2 x1^2 d1,9 + p1 p2 x1 x2 d1,10 + p1 p2 x2^2 d1,11 + p2^2 x1^2 d1,12 + p2^2 x1 x2 d1,13 + p2^2 x2^2 d1,14 ]],
  E_{{}->{1}} [sqrt(T), 0, eSeries[0, e1,1 + p1 x1 e1,2 + p1^2 x1^2 e1,3 ]],
  E_{{}->{1}} [1/sqrt(T), 0, eSeries[0, f1,1 + p1 x1 f1,2 + p1^2 x1^2 f1,3 ]]}

```

```

In[*]:= RMoves := {
  (R1,2 R4,3 R5,6 // m1,4->1 // m2,5->2 // m3,6->3) == (R2,3 R4,5 R1,6 // m1,4->1 // m2,5->2 // m3,6->3),
  (R1,2 R3,4 // m1,3->1 // m2,4->2) == (eta1 eta2),
  (C1 C2 // m1,2->1) == eta1,
  (R1,4 R5,2 C3 // m2,4->2 // m1,3->1 // m1,5->1) == C1 eta2,
  (C3 R1,2 // m2,3->2 // m2,1->1) == (C3 R1,2 // m1,3->1 // m1,2->1),
  (C2 R1,3 // m1,2->1 // m1,3->1) == eta1, (C2 R3,1 // m1,2->1 // m1,3->1) == eta1,
  (C2 R1,3 // m1,2->1 // m1,3->1) == eta1, (C2 R3,1 // m1,2->1 // m1,3->1) == eta1
}

```

Solving for R, C, \$k = 1

```

In[*]:= $k = 1;
{R1,2, C1}
unknowns = Cases[{R1,2, R1,2, C1, C1}, (c | d | e | f)_{$k,_, infinity} // Union
Out[*]:= {E_{{}->{1,2}} [sqrt(T), (-1 + T) (p1 - p2) x2,
  eSeries[0, c1,1 + p1 x1 c1,2 + p1 x2 c1,3 + p2 x1 c1,4 + p2 x2 c1,5 + p1^2 x1^2 c1,6 + p1^2 x1 x2 c1,7 + p1^2 x2^2 c1,8 +
    p1 p2 x1^2 c1,9 + p1 p2 x1 x2 c1,10 + p1 p2 x2^2 c1,11 + p2^2 x1^2 c1,12 + p2^2 x1 x2 c1,13 + p2^2 x2^2 c1,14 ]],
  E_{{}->{1}} [sqrt(T), 0, eSeries[0, e1,1 + p1 x1 e1,2 + p1^2 x1^2 e1,3 ]]}
Out[*]:= {c1,1, c1,2, c1,3, c1,4, c1,5, c1,6, c1,7, c1,8, c1,9, c1,10, c1,11, c1,12, c1,13, c1,14, d1,1, d1,2, d1,3,
  d1,4, d1,5, d1,6, d1,7, d1,8, d1,9, d1,10, d1,11, d1,12, d1,13, d1,14, e1,1, e1,2, e1,3, f1,1, f1,2, f1,3}

```

In[]:= Short [errors = CCF /@ Cases [RMoves, a_ == b_ => a - b], 25]

$$\begin{aligned}
 \text{Out[]} // \text{Short} = & \left\{ T p_1 x_3 c_{1,2} - T^2 p_1 x_3 c_{1,2} + p_1 x_3 c_{1,3} - T p_1 x_3 c_{1,3} - p_2 x_1 c_{1,4} + T p_2 x_1 c_{1,4} + p_3 x_1 c_{1,4} - T p_3 x_1 c_{1,4} + \right. \\
 & p_1 x_2 c_{1,4} - T p_1 x_2 c_{1,4} - p_2 x_2 c_{1,4} + 2 T p_2 x_2 c_{1,4} - T^2 p_2 x_2 c_{1,4} - T p_3 x_2 c_{1,4} + T^2 p_3 x_2 c_{1,4} + \\
 & T p_2 x_3 c_{1,4} - T^2 p_2 x_3 c_{1,4} + p_1 x_3 c_{1,5} - T p_1 x_3 c_{1,5} - 2 p_1^2 x_1 x_2 c_{1,6} + 2 T p_1^2 x_1 x_2 c_{1,6} + 2 T p_1 p_2 x_2^2 c_{1,6} - \\
 & 2 T^2 p_1 p_2 x_2^2 c_{1,6} + 2 T p_1^2 x_1 x_3 c_{1,6} - 2 T^2 p_1^2 x_1 x_3 c_{1,6} + T^2 p_1^2 x_3^2 c_{1,6} - 2 T^3 p_1^2 x_3^2 c_{1,6} + T^4 p_1^2 x_3^2 c_{1,6} + \\
 & 2 T p_1 p_2 x_2 x_3 c_{1,7} - 2 T^2 p_1 p_2 x_2 x_3 c_{1,7} + T p_1^2 x_3^2 c_{1,7} - 2 T^2 p_1^2 x_3^2 c_{1,7} + T^3 p_1^2 x_3^2 c_{1,7} + 2 p_1^2 x_2 x_3 c_{1,8} - \\
 & 2 T p_1^2 x_2 x_3 c_{1,8} + p_1^2 x_3^2 c_{1,8} - 4 T p_1^2 x_3^2 c_{1,8} + 3 T^2 p_1^2 x_3^2 c_{1,8} + 2 T p_1 p_2 x_3^2 c_{1,8} - 2 T^2 p_1 p_2 x_3^2 c_{1,8} - \\
 & p_1 p_2 x_1^2 c_{1,9} + T p_1 p_2 x_1^2 c_{1,9} + p_1 p_3 x_1^2 c_{1,9} - T p_1 p_3 x_1^2 c_{1,9} - 2 p_1 p_2 x_1 x_2 c_{1,9} + 4 T p_1 p_2 x_1 x_2 c_{1,9} - \\
 & 2 T^2 p_1 p_2 x_1 x_2 c_{1,9} - 2 T p_1 p_3 x_1 x_2 c_{1,9} + 2 T^2 p_1 p_3 x_1 x_2 c_{1,9} + p_1^2 x_2^2 c_{1,9} - 2 T p_1^2 x_2^2 c_{1,9} + \\
 & T^2 p_1^2 x_2^2 c_{1,9} - p_1 p_2 x_2^2 c_{1,9} + 4 T p_1 p_2 x_2^2 c_{1,9} - 4 T^2 p_1 p_2 x_2^2 c_{1,9} + T^3 p_1 p_2 x_2^2 c_{1,9} + T^2 p_1 p_3 x_2^2 c_{1,9} - \\
 & T^3 p_1 p_3 x_2^2 c_{1,9} + 2 T p_1 p_2 x_1 x_3 c_{1,9} - 2 T^2 p_1 p_2 x_1 x_3 c_{1,9} + T^2 p_1 p_2 x_3^2 c_{1,9} - 2 T^3 p_1 p_2 x_3^2 c_{1,9} + \\
 & T^4 p_1 p_2 x_3^2 c_{1,9} + p_1^2 x_2 x_3 c_{1,10} - 2 T p_1^2 x_2 x_3 c_{1,10} + T^2 p_1^2 x_2 x_3 c_{1,10} - p_1 p_2 x_2 x_3 c_{1,10} + \\
 & 4 T p_1 p_2 x_2 x_3 c_{1,10} - 3 T^2 p_1 p_2 x_2 x_3 c_{1,10} + T p_1 p_2 x_3^2 c_{1,10} - 2 T^2 p_1 p_2 x_3^2 c_{1,10} + T^3 p_1 p_2 x_3^2 c_{1,10} + \\
 & 2 p_1 p_2 x_2 x_3 c_{1,11} - 2 T p_1 p_2 x_2 x_3 c_{1,11} + p_1^2 x_3^2 c_{1,11} - 2 T p_1^2 x_3^2 c_{1,11} + T^2 p_1^2 x_3^2 c_{1,11} - p_2^2 x_1^2 c_{1,12} + \\
 & 2 T p_2^2 x_1^2 c_{1,12} - T^2 p_2^2 x_1^2 c_{1,12} - 2 T p_2 p_3 x_1^2 c_{1,12} + 2 T^2 p_2 p_3 x_1^2 c_{1,12} + p_3^2 x_1^2 c_{1,12} - T^2 p_3^2 x_1^2 c_{1,12} - \\
 & 2 p_2^2 x_1 x_2 c_{1,12} + 6 T p_2^2 x_1 x_2 c_{1,12} - 6 T^2 p_2^2 x_1 x_2 c_{1,12} + 2 T^3 p_2^2 x_1 x_2 c_{1,12} - 4 T p_2 p_3 x_1 x_2 c_{1,12} + \\
 & 8 T^2 p_2 p_3 x_1 x_2 c_{1,12} - 4 T^3 p_2 p_3 x_1 x_2 c_{1,12} - 2 T^2 p_3^2 x_1 x_2 c_{1,12} + 2 T^3 p_3^2 x_1 x_2 c_{1,12} + p_1^2 x_2^2 c_{1,12} - \\
 & 2 T p_1^2 x_2^2 c_{1,12} + T^2 p_1^2 x_2^2 c_{1,12} - p_2^2 x_2^2 c_{1,12} + 4 T p_2^2 x_2^2 c_{1,12} - 6 T^2 p_2^2 x_2^2 c_{1,12} + 4 T^3 p_2^2 x_2^2 c_{1,12} - \\
 & T^4 p_2^2 x_2^2 c_{1,12} + 2 T p_1 p_3 x_2^2 c_{1,12} - 2 T^2 p_1 p_3 x_2^2 c_{1,12} - 2 T p_2 p_3 x_2^2 c_{1,12} + 6 T^2 p_2 p_3 x_2^2 c_{1,12} - \\
 & 6 T^3 p_2 p_3 x_2^2 c_{1,12} + 2 T^4 p_2 p_3 x_2^2 c_{1,12} - T^2 p_3^2 x_2^2 c_{1,12} + 2 T^3 p_3^2 x_2^2 c_{1,12} - T^4 p_3^2 x_2^2 c_{1,12} + \\
 & 2 T p_2^2 x_1 x_3 c_{1,12} - 2 T^2 p_2^2 x_1 x_3 c_{1,12} + T^2 p_2^2 x_3^2 c_{1,12} - 2 T^3 p_2^2 x_3^2 c_{1,12} + T^4 p_2^2 x_3^2 c_{1,12} + T p_2^2 x_1 x_3 c_{1,13} - \\
 & T^2 p_2^2 x_1 x_3 c_{1,13} - 2 T p_2 p_3 x_1 x_3 c_{1,13} + 2 T^2 p_2 p_3 x_1 x_3 c_{1,13} + T p_3^2 x_1 x_3 c_{1,13} - T^2 p_3^2 x_1 x_3 c_{1,13} + \\
 & p_1^2 x_2 x_3 c_{1,13} - 2 T p_1^2 x_2 x_3 c_{1,13} + T^2 p_1^2 x_2 x_3 c_{1,13} - p_2^2 x_2 x_3 c_{1,13} + 4 T p_2^2 x_2 x_3 c_{1,13} - \\
 & 4 T^2 p_2^2 x_2 x_3 c_{1,13} + T^3 p_2^2 x_2 x_3 c_{1,13} + 2 T p_1 p_3 x_2 x_3 c_{1,13} - 2 T^2 p_1 p_3 x_2 x_3 c_{1,13} - 2 T p_2 p_3 x_2 x_3 c_{1,13} + \\
 & 4 T^2 p_2 p_3 x_2 x_3 c_{1,13} - 2 T^3 p_2 p_3 x_2 x_3 c_{1,13} - T^2 p_3^2 x_2 x_3 c_{1,13} + T^3 p_3^2 x_2 x_3 c_{1,13} + T p_2^2 x_3^2 c_{1,13} - \\
 & 2 T^2 p_2^2 x_3^2 c_{1,13} + T^3 p_2^2 x_3^2 c_{1,13} + 2 p_2^2 x_2 x_3 c_{1,14} - 2 T p_2^2 x_2 x_3 c_{1,14} + p_1^2 x_3^2 c_{1,14} - 2 T p_1^2 x_3^2 c_{1,14} + \\
 & T^2 p_1^2 x_3^2 c_{1,14} + 2 T p_1 p_3 x_3^2 c_{1,14} - 2 T^2 p_1 p_3 x_3^2 c_{1,14} - 2 T p_2 p_3 x_3^2 c_{1,14} + 2 T^2 p_2 p_3 x_3^2 c_{1,14}, \\
 & \frac{1}{T^2} (T^2 c_{1,1} + T^2 p_1 x_1 c_{1,2} - T p_1 x_2 c_{1,2} + T^2 p_1 x_2 c_{1,2} + T p_1 x_2 c_{1,3} + \ll 89 \gg + \\
 & T^4 p_1^2 x_2^2 d_{1,14} + 2 T^3 p_1 p_2 x_2^2 d_{1,14} - 2 T^4 p_1 p_2 x_2^2 d_{1,14} + T^4 p_2^2 x_2^2 d_{1,14}), \\
 & e_{1,1} + p_1 x_1 e_{1,2} + p_1^2 x_1^2 e_{1,3} + f_{1,1} + p_1 x_1 f_{1,2} + p_1^2 x_1^2 f_{1,3}, \ll 3 \gg, \\
 & d_{1,1} + p_1 x_1 d_{1,2} - T d_{1,3} + \ll 33 \gg + T^2 p_1^2 x_1^2 f_{1,3}, \\
 & \frac{T^2 d_{1,1} + T p_1 x_1 d_{1,2} + T^2 p_1 x_1 d_{1,3} - T^2 d_{1,4} + \ll 19 \gg + T^2 p_1^2 x_1^2 d_{1,14} + T^2 e_{1,1} + T p_1 x_1 e_{1,2} + p_1^2 x_1^2 e_{1,3}}{T^2}, \\
 & \frac{1}{T^2} (T^2 c_{1,1} + T^2 p_1 x_1 c_{1,2} - T c_{1,3} + T p_1 x_1 c_{1,3} + T^2 p_1 x_1 c_{1,4} + T p_1 x_1 c_{1,5} + T^2 p_1^2 x_1^2 c_{1,6} - \\
 & 2 T p_1 x_1 c_{1,7} + T p_1^2 x_1^2 c_{1,7} + 2 c_{1,8} - 4 p_1 x_1 c_{1,8} + p_1^2 x_1^2 c_{1,8} + T^2 p_1^2 x_1^2 c_{1,9} - T p_1 x_1 c_{1,10} + \\
 & T p_1^2 \ll 1 \gg c_{1,10} - \ll 1 \gg + p_1^2 x_1^2 c_{1,11} + T^2 p_1^2 x_1^2 c_{1,12} + T p_1^2 x_1^2 c_{1,13} + p_1^2 x_1^2 c_{1,14} + T^2 e_{1,1} - T e_{1,2} + \\
 & T^2 e_{1,2} + T p_1 x_1 e_{1,2} + 2 e_{1,3} - 4 T e_{1,3} + 2 T^2 e_{1,3} - 4 p_1 x_1 e_{1,3} + 4 T p_1 x_1 e_{1,3} + p_1^2 x_1^2 e_{1,3}) \}
 \end{aligned}$$

In[]:= eqns = Thread[θ == Union@@ (CoefficientRules[#, {x1, x2, x3, p1, p2, p3}][[; ; , 2]] & /@ errors)]

$$\begin{aligned}
 \text{Out[]} = & \left\{ \theta = c_{1,4} - T c_{1,4}, \theta = -c_{1,4} + T c_{1,4}, \theta = T c_{1,4} - T^2 c_{1,4}, \theta = -c_{1,4} + 2 T c_{1,4} - T^2 c_{1,4}, \right. \\
 & \theta = -T c_{1,4} + T^2 c_{1,4}, \theta = T c_{1,2} - T^2 c_{1,2} + c_{1,3} - T c_{1,3} + c_{1,5} - T c_{1,5},
 \end{aligned}$$

$$\begin{aligned}
 \theta &= -2 c_{1,6} + 2 T c_{1,6}, \theta = 2 T c_{1,6} - 2 T^2 c_{1,6}, \theta = c_{1,9} - T c_{1,9}, \\
 \theta &= -c_{1,9} + T c_{1,9}, \theta = 2 T c_{1,9} - 2 T^2 c_{1,9}, \theta = -2 c_{1,9} + 4 T c_{1,9} - 2 T^2 c_{1,9}, \\
 \theta &= -2 T c_{1,9} + 2 T^2 c_{1,9}, \theta = 2 T c_{1,6} - 2 T^2 c_{1,6} - c_{1,9} + 4 T c_{1,9} - 4 T^2 c_{1,9} + T^3 c_{1,9}, \\
 \theta &= 2 T c_{1,8} - 2 T^2 c_{1,8} + T^2 c_{1,9} - 2 T^3 c_{1,9} + T^4 c_{1,9} + T c_{1,10} - 2 T^2 c_{1,10} + T^3 c_{1,10}, \\
 \theta &= 2 T c_{1,7} - 2 T^2 c_{1,7} - c_{1,10} + 4 T c_{1,10} - 3 T^2 c_{1,10} + 2 c_{1,11} - 2 T c_{1,11}, \\
 \theta &= T^2 c_{1,9} - T^3 c_{1,9} + 2 T c_{1,12} - 2 T^2 c_{1,12}, \theta = c_{1,12} - T^2 c_{1,12}, \theta = -c_{1,12} + 2 T c_{1,12} - T^2 c_{1,12}, \\
 \theta &= c_{1,9} - 2 T c_{1,9} + T^2 c_{1,9} + c_{1,12} - 2 T c_{1,12} + T^2 c_{1,12}, \theta = -2 T c_{1,12} + 2 T^2 c_{1,12}, \\
 \theta &= -4 T c_{1,12} + 8 T^2 c_{1,12} - 4 T^3 c_{1,12}, \theta = -2 c_{1,12} + 6 T c_{1,12} - 6 T^2 c_{1,12} + 2 T^3 c_{1,12}, \\
 \theta &= -2 T^2 c_{1,12} + 2 T^3 c_{1,12}, \theta = -T^2 c_{1,12} + 2 T^3 c_{1,12} - T^4 c_{1,12}, \\
 \theta &= -c_{1,12} + 4 T c_{1,12} - 6 T^2 c_{1,12} + 4 T^3 c_{1,12} - T^4 c_{1,12}, \theta = -2 T c_{1,12} + 6 T^2 c_{1,12} - 6 T^3 c_{1,12} + 2 T^4 c_{1,12}, \\
 \theta &= 2 T c_{1,13} - 2 T^2 c_{1,13}, \theta = T c_{1,13} - T^2 c_{1,13}, \theta = 2 T c_{1,12} - 2 T^2 c_{1,12} + T c_{1,13} - T^2 c_{1,13}, \\
 \theta &= 2 c_{1,8} - 2 T c_{1,8} + c_{1,10} - 2 T c_{1,10} + T^2 c_{1,10} + c_{1,13} - 2 T c_{1,13} + T^2 c_{1,13}, \theta = -2 T c_{1,13} + 2 T^2 c_{1,13}, \\
 \theta &= -2 T c_{1,13} + 4 T^2 c_{1,13} - 2 T^3 c_{1,13}, \theta = T^2 c_{1,12} - 2 T^3 c_{1,12} + T^4 c_{1,12} + T c_{1,13} - 2 T^2 c_{1,13} + T^3 c_{1,13}, \\
 \theta &= -T^2 c_{1,13} + T^3 c_{1,13}, \theta = -c_{1,13} + 4 T c_{1,13} - 4 T^2 c_{1,13} + T^3 c_{1,13} + 2 c_{1,14} - 2 T c_{1,14}, \\
 \theta &= 2 T c_{1,14} - 2 T^2 c_{1,14}, \theta = T^2 c_{1,6} - 2 T^3 c_{1,6} + T^4 c_{1,6} + T c_{1,7} - 2 T^2 c_{1,7} + T^3 c_{1,7} + \\
 & \quad c_{1,8} - 4 T c_{1,8} + 3 T^2 c_{1,8} + c_{1,11} - 2 T c_{1,11} + T^2 c_{1,11} + c_{1,14} - 2 T c_{1,14} + T^2 c_{1,14}, \\
 \theta &= -2 T c_{1,14} + 2 T^2 c_{1,14}, \theta = c_{1,1} + d_{1,1}, \theta = c_{1,1} - c_{1,4} + \frac{c_{1,4}}{T} + 2 c_{1,12} + \frac{2 c_{1,12}}{T^2} - \frac{4 c_{1,12}}{T} + d_{1,1}, \\
 \theta &= c_{1,2} + c_{1,4} - \frac{c_{1,4}}{T} - 2 c_{1,9} + \frac{2 c_{1,9}}{T} - 4 c_{1,12} - \frac{4 c_{1,12}}{T^2} + \frac{8 c_{1,12}}{T} + d_{1,2}, \\
 \theta &= \frac{c_{1,4}}{T} + \frac{4 c_{1,12}}{T^2} - \frac{4 c_{1,12}}{T} + d_{1,4}, \theta = c_{1,2} + d_{1,2} + d_{1,4} - T d_{1,4}, \theta = c_{1,4} + T d_{1,4}, \\
 \theta &= c_{1,2} - \frac{c_{1,2}}{T} + \frac{c_{1,3}}{T} + d_{1,3} + d_{1,5} - T d_{1,5}, \theta = c_{1,4} - \frac{c_{1,4}}{T} + \frac{c_{1,5}}{T} + T d_{1,5}, \\
 \theta &= c_{1,4} - \frac{c_{1,4}}{T} + \frac{c_{1,5}}{T} - 4 c_{1,12} - \frac{4 c_{1,12}}{T^2} + \frac{8 c_{1,12}}{T} + \frac{2 c_{1,13}}{T^2} - \frac{2 c_{1,13}}{T} + T d_{1,5}, \\
 \theta &= c_{1,6} + c_{1,9} - \frac{c_{1,9}}{T} + c_{1,12} + \frac{c_{1,12}}{T^2} - \frac{2 c_{1,12}}{T} + d_{1,6}, \theta = \frac{c_{1,9}}{T} - \frac{2 c_{1,12}}{T^2} + \frac{2 c_{1,12}}{T} + d_{1,9}, \\
 \theta &= 2 c_{1,9} - \frac{2 c_{1,9}}{T} + \frac{c_{1,10}}{T} + 4 c_{1,12} + \frac{4 c_{1,12}}{T^2} - \frac{8 c_{1,12}}{T} - \frac{2 c_{1,13}}{T^2} + \frac{2 c_{1,13}}{T} + T d_{1,10}, \\
 \theta &= -2 c_{1,9} + \frac{c_{1,9}}{T} + T c_{1,9} + c_{1,10} - \frac{c_{1,10}}{T} + \frac{c_{1,11}}{T} - 6 c_{1,12} - \frac{2 c_{1,12}}{T^2} + \\
 & \quad \frac{6 c_{1,12}}{T} + 2 T c_{1,12} + 2 c_{1,13} + \frac{2 c_{1,13}}{T^2} - \frac{4 c_{1,13}}{T} - \frac{2 c_{1,14}}{T^2} + \frac{2 c_{1,14}}{T} + T^2 d_{1,11}, \\
 \theta &= \frac{c_{1,12}}{T^2} + d_{1,12}, \theta = c_{1,9} + T d_{1,9} + 2 T d_{1,12} - 2 T^2 d_{1,12}, \theta = c_{1,12} + T^2 d_{1,12}, \\
 \theta &= c_{1,6} + d_{1,6} + d_{1,9} - T d_{1,9} + d_{1,12} - 2 T d_{1,12} + T^2 d_{1,12}, \theta = -\frac{2 c_{1,12}}{T^2} + \frac{2 c_{1,12}}{T} + \frac{c_{1,13}}{T^2} + T d_{1,13}, \\
 \theta &= 2 c_{1,9} - \frac{2 c_{1,9}}{T} + \frac{c_{1,10}}{T} + T d_{1,10} + 2 T d_{1,13} - 2 T^2 d_{1,13}, \theta = 2 c_{1,12} - \frac{2 c_{1,12}}{T} + \frac{c_{1,13}}{T} + T^2 d_{1,13}, \\
 \theta &= 2 c_{1,6} - \frac{2 c_{1,6}}{T} + \frac{c_{1,7}}{T} + d_{1,7} + d_{1,10} - T d_{1,10} + d_{1,13} - 2 T d_{1,13} + T^2 d_{1,13},
 \end{aligned}$$

$$\begin{aligned}
\theta &= c_{1,9} + \frac{c_{1,9}}{T^2} - \frac{2c_{1,9}}{T} - \frac{c_{1,10}}{T^2} + \frac{c_{1,10}}{T} + \frac{c_{1,11}}{T^2} + T d_{1,11} + 2T d_{1,14} - 2T^2 d_{1,14}, \\
\theta &= c_{1,12} + \frac{c_{1,12}}{T^2} - \frac{2c_{1,12}}{T} - \frac{c_{1,13}}{T^2} + \frac{c_{1,13}}{T} + \frac{c_{1,14}}{T^2} + T^2 d_{1,14}, \\
\theta &= c_{1,6} + \frac{c_{1,6}}{T^2} - \frac{2c_{1,6}}{T} - \frac{c_{1,7}}{T^2} + \frac{c_{1,7}}{T} + \frac{c_{1,8}}{T^2} + d_{1,8} + d_{1,11} - T d_{1,11} + d_{1,14} - 2T d_{1,14} + T^2 d_{1,14}, \\
\theta &= d_{1,1} - d_{1,4} + 2d_{1,12} + e_{1,1}, \quad \theta = \frac{d_{1,2}}{T} + d_{1,3} + \frac{d_{1,4}}{T} + d_{1,5} - \frac{2d_{1,9}}{T} - d_{1,10} - \frac{4d_{1,12}}{T} - 2d_{1,13} + \frac{e_{1,2}}{T}, \\
\theta &= c_{1,6} + \frac{c_{1,7}}{T} + \frac{c_{1,8}}{T^2} + c_{1,9} + \frac{c_{1,10}}{T} + \frac{c_{1,11}}{T^2} + c_{1,12} + \frac{c_{1,13}}{T} + \frac{c_{1,14}}{T^2} + \frac{e_{1,3}}{T^2}, \\
\theta &= \frac{d_{1,6}}{T^2} + \frac{d_{1,7}}{T} + d_{1,8} + \frac{d_{1,9}}{T^2} + \frac{d_{1,10}}{T} + d_{1,11} + \frac{d_{1,12}}{T^2} + \frac{d_{1,13}}{T} + d_{1,14} + \frac{e_{1,3}}{T^2}, \\
\theta &= c_{1,1} - \frac{c_{1,3}}{T} + \frac{2c_{1,8}}{T^2} + e_{1,1} + e_{1,2} - \frac{e_{1,2}}{T} + 2e_{1,3} + \frac{2e_{1,3}}{T^2} - \frac{4e_{1,3}}{T}, \\
\theta &= c_{1,2} + \frac{c_{1,3}}{T} + c_{1,4} + \frac{c_{1,5}}{T} - \frac{2c_{1,7}}{T} - \frac{4c_{1,8}}{T^2} - \frac{c_{1,10}}{T} - \frac{2c_{1,11}}{T^2} + \frac{e_{1,2}}{T} - \frac{4e_{1,3}}{T^2} + \frac{4e_{1,3}}{T}, \\
\theta &= -\frac{c_{1,3}}{T} + c_{1,4} + \frac{2c_{1,8}}{T^2} - 2c_{1,12} + e_{1,1} + e_{1,2} - \frac{e_{1,2}}{T} + 2e_{1,3} + \frac{2e_{1,3}}{T^2} - \frac{4e_{1,3}}{T} - f_{1,1}, \\
\theta &= c_{1,1} - c_{1,4} + 2c_{1,12} + f_{1,1}, \quad \theta = e_{1,1} + f_{1,1}, \quad \theta = e_{1,2} + f_{1,2}, \\
\theta &= c_{1,2} - T c_{1,2} - c_{1,3} + \frac{c_{1,3}}{T} + c_{1,4} - T c_{1,4} - c_{1,5} + \frac{c_{1,5}}{T} - \frac{2c_{1,7}}{T} - \frac{4c_{1,8}}{T^2} + \\
&\quad 2T c_{1,9} + c_{1,10} - \frac{c_{1,10}}{T} - \frac{2c_{1,11}}{T^2} + 4T c_{1,12} + 2c_{1,13} + \frac{e_{1,2}}{T} - \frac{4e_{1,3}}{T^2} + \frac{4e_{1,3}}{T} - T f_{1,2}, \\
\theta &= T c_{1,2} + c_{1,3} + T c_{1,4} + c_{1,5} - 2T c_{1,9} - c_{1,10} - 4T c_{1,12} - 2c_{1,13} + T f_{1,2}, \\
\theta &= -c_{1,2} + T c_{1,2} + c_{1,3} - 2c_{1,4} + \frac{c_{1,4}}{T} + T c_{1,4} + c_{1,5} - \frac{c_{1,5}}{T} + 4c_{1,9} - \frac{2c_{1,9}}{T} - 2T c_{1,9} - c_{1,10} + \frac{c_{1,10}}{T} + \\
&\quad 12c_{1,12} + \frac{4c_{1,12}}{T^2} - \frac{12c_{1,12}}{T} - 4T c_{1,12} - 2c_{1,13} - \frac{2c_{1,13}}{T^2} + \frac{4c_{1,13}}{T} + T d_{1,3} - f_{1,2} + T f_{1,2}, \\
\theta &= e_{1,3} + f_{1,3}, \quad \theta = -2c_{1,6} + 2T c_{1,6} + c_{1,7} - 4c_{1,9} + \frac{2c_{1,9}}{T} + 2T c_{1,9} + c_{1,10} - \frac{c_{1,10}}{T} - \\
&\quad 6c_{1,12} - \frac{2c_{1,12}}{T^2} + \frac{6c_{1,12}}{T} + 2T c_{1,12} + c_{1,13} + \frac{c_{1,13}}{T^2} - \frac{2c_{1,13}}{T} + T d_{1,7} - 2f_{1,3} + 2T f_{1,3}, \\
\theta &= d_{1,2} + T d_{1,3} + d_{1,4} + T d_{1,5} - 2T d_{1,7} - 4T^2 d_{1,8} - T d_{1,10} - 2T^2 d_{1,11} + T f_{1,2} + 4T f_{1,3} - 4T^2 f_{1,3}, \\
\theta &= c_{1,6} - T^2 c_{1,6} + \frac{c_{1,7}}{T} - T c_{1,7} - c_{1,8} + \frac{c_{1,8}}{T^2} + c_{1,9} - T^2 c_{1,9} + \frac{c_{1,10}}{T} - T c_{1,10} - \\
&\quad c_{1,11} + \frac{c_{1,11}}{T^2} + c_{1,12} - T^2 c_{1,12} + \frac{c_{1,13}}{T} - T c_{1,13} - c_{1,14} + \frac{c_{1,14}}{T^2} + \frac{e_{1,3}}{T^2} - T^2 f_{1,3}, \\
\theta &= T^2 c_{1,6} + T c_{1,7} + c_{1,8} + T^2 c_{1,9} + T c_{1,10} + c_{1,11} + T^2 c_{1,12} + T c_{1,13} + c_{1,14} + T^2 f_{1,3}, \\
\theta &= d_{1,6} + T d_{1,7} + T^2 d_{1,8} + d_{1,9} + T d_{1,10} + T^2 d_{1,11} + d_{1,12} + T d_{1,13} + T^2 d_{1,14} + T^2 f_{1,3}, \\
\theta &= c_{1,6} - 2T c_{1,6} + T^2 c_{1,6} - c_{1,7} + T c_{1,7} + c_{1,8} + 3c_{1,9} - \frac{c_{1,9}}{T} - 3T c_{1,9} + T^2 c_{1,9} - 2c_{1,10} + \\
&\quad \frac{c_{1,10}}{T} + T c_{1,10} + c_{1,11} - \frac{c_{1,11}}{T} + 6c_{1,12} + \frac{c_{1,12}}{T^2} - \frac{4c_{1,12}}{T} - 4T c_{1,12} + T^2 c_{1,12} - 3c_{1,13} -
\end{aligned}$$

$$\frac{c_{1,13}}{T^2} + \frac{3 c_{1,13}}{T} + T c_{1,13} + c_{1,14} + \frac{c_{1,14}}{T^2} - \frac{2 c_{1,14}}{T} + T^2 d_{1,8} + f_{1,3} - 2 T f_{1,3} + T^2 f_{1,3},$$

$$\theta = d_{1,1} - T d_{1,3} + 2 T^2 d_{1,8} + f_{1,1} + f_{1,2} - T f_{1,2} + 2 f_{1,3} - 4 T f_{1,3} + 2 T^2 f_{1,3}$$

In[*]:= {sol} = Solve[eqns, unknowns]

Solve: Equations may not give solutions for all "solve" variables.

$$\text{Out[*]} = \left\{ \left\{ c_{1,1} \rightarrow -\frac{c_{1,2}}{2} - \frac{c_{1,5}}{2T}, c_{1,3} \rightarrow -T c_{1,2} - c_{1,5}, c_{1,4} \rightarrow \theta, c_{1,6} \rightarrow \theta, c_{1,8} \rightarrow -\frac{1}{2} \times (1-T) c_{1,10}, c_{1,9} \rightarrow \theta, \right. \right.$$

$$c_{1,11} \rightarrow -T c_{1,7} - \frac{1}{2} \times (-1+3T) c_{1,10}, c_{1,12} \rightarrow \theta, c_{1,13} \rightarrow \theta, c_{1,14} \rightarrow \theta, d_{1,1} \rightarrow \frac{c_{1,2}}{2} + \frac{c_{1,5}}{2T},$$

$$d_{1,2} \rightarrow -c_{1,2}, d_{1,3} \rightarrow \frac{c_{1,2}}{T} + \frac{c_{1,5}}{T^2}, d_{1,4} \rightarrow \theta, d_{1,5} \rightarrow -\frac{c_{1,5}}{T^2}, d_{1,6} \rightarrow \theta, d_{1,7} \rightarrow -\frac{c_{1,7}}{T} - \frac{(-1+T) c_{1,10}}{T^2},$$

$$d_{1,8} \rightarrow -\frac{(1-T) c_{1,10}}{2T^3}, d_{1,9} \rightarrow \theta, d_{1,10} \rightarrow -\frac{c_{1,10}}{T^2}, d_{1,11} \rightarrow \frac{c_{1,7}}{T^2} - \frac{(-1-T) c_{1,10}}{2T^3}, d_{1,12} \rightarrow \theta, d_{1,13} \rightarrow \theta,$$

$$\left. \left. d_{1,14} \rightarrow \theta, e_{1,1} \rightarrow -\frac{c_{1,2}}{2} - \frac{c_{1,5}}{2T}, e_{1,2} \rightarrow -\frac{c_{1,10}}{T}, e_{1,3} \rightarrow \theta, f_{1,1} \rightarrow \frac{c_{1,2}}{2} + \frac{c_{1,5}}{2T}, f_{1,2} \rightarrow \frac{c_{1,10}}{T}, f_{1,3} \rightarrow \theta \right\} \right\}$$

In[*]:= sol /. (a_ -> b_) := (a = b)

$$\text{Out[*]} = \left\{ -\frac{c_{1,2}}{2} - \frac{c_{1,5}}{2T}, -T c_{1,2} - c_{1,5}, \theta, \theta, -\frac{1}{2} \times (1-T) c_{1,10}, \theta, -T c_{1,7} - \frac{1}{2} \times (-1+3T) c_{1,10}, \theta, \theta, \right.$$

$$\theta, \frac{c_{1,2}}{2} + \frac{c_{1,5}}{2T}, -c_{1,2}, \frac{c_{1,2}}{T} + \frac{c_{1,5}}{T^2}, \theta, -\frac{c_{1,5}}{T^2}, \theta, -\frac{c_{1,7}}{T} - \frac{(-1+T) c_{1,10}}{T^2}, -\frac{(1-T) c_{1,10}}{2T^3},$$

$$\left. \theta, -\frac{c_{1,10}}{T^2}, \frac{c_{1,7}}{T^2} - \frac{(-1-T) c_{1,10}}{2T^3}, \theta, \theta, \theta, -\frac{c_{1,2}}{2} - \frac{c_{1,5}}{2T}, -\frac{c_{1,10}}{T}, \theta, \frac{c_{1,2}}{2} + \frac{c_{1,5}}{2T}, \frac{c_{1,10}}{T}, \theta \right\}$$

In[*]:= {R1,2, R1,2, C1, C1}

$$\text{Out[*]} = \left\{ E_{\{\} \rightarrow \{1,2\}} \left[\sqrt{T}, (-1+T) (p_1 - p_2) x_2, \right. \right.$$

$$\in \text{Series} \left[\theta, -\frac{c_{1,2}}{2} + p_1 x_1 c_{1,2} + p_1 x_2 (-T c_{1,2} - c_{1,5}) - \frac{c_{1,5}}{2T} + p_2 x_2 c_{1,5} + p_1^2 x_1 x_2 c_{1,7} + \right.$$

$$\left. \left. p_1 p_2 x_1 x_2 c_{1,10} - \frac{1}{2} \times (1-T) p_1^2 x_2^2 c_{1,10} + p_1 p_2 x_2^2 \left(-T c_{1,7} - \frac{1}{2} \times (-1+3T) c_{1,10} \right) \right] \right],$$

$$E_{\{\} \rightarrow \{1,2\}} \left[\frac{1}{\sqrt{T}}, \left(-1 + \frac{1}{T} \right) (p_1 - p_2) x_2, \in \text{Series} \left[\theta, \right. \right.$$

$$\frac{c_{1,2}}{2} - p_1 x_1 c_{1,2} + \frac{c_{1,5}}{2T} - \frac{p_2 x_2 c_{1,5}}{T^2} + p_1 x_2 \left(\frac{c_{1,2}}{T} + \frac{c_{1,5}}{T^2} \right) - \frac{p_1 p_2 x_1 x_2 c_{1,10}}{T^2} -$$

$$\left. \left. \frac{(1-T) p_1^2 x_2^2 c_{1,10}}{2T^3} + p_1 p_2 x_2^2 \left(\frac{c_{1,7}}{T^2} - \frac{(-1-T) c_{1,10}}{2T^3} \right) + p_1^2 x_1 x_2 \left(-\frac{c_{1,7}}{T} - \frac{(-1+T) c_{1,10}}{T^2} \right) \right] \right],$$

$$E_{\{\} \rightarrow \{1\}} \left[\sqrt{T}, \theta, \in \text{Series} \left[\theta, -\frac{c_{1,2}}{2} - \frac{c_{1,5}}{2T} - \frac{p_1 x_1 c_{1,10}}{T} \right] \right],$$

$$E_{\{\} \rightarrow \{1\}} \left[\frac{1}{\sqrt{T}}, \theta, \in \text{Series} \left[\theta, \frac{c_{1,2}}{2} + \frac{c_{1,5}}{2T} + \frac{p_1 x_1 c_{1,10}}{T} \right] \right] \right\}$$

In[*]:= Cases [{R_{1,2}, R̄_{1,2}, C₁, C̄₁}, (c | d | e | f)_{\$k,-}, ∞] // Union

Out[*]:= {C_{1,2}, C_{1,5}, C_{1,7}, C_{1,10}}

In[*]:= {C_{1,2} = 0, C_{1,5} = 0, C_{1,7} = 0, C_{1,10} = 1};
 {R_{1,2}, R̄_{1,2}, C₁, C̄₁}

Out[*]:= {E_{{1}→{1,2}} [√T, (-1 + T) (p₁ - p₂) x₂,
 ∈Series [0, p₁ p₂ x₁ x₂ + $\frac{1}{2} \times (-1 + T) p_1^2 x_2^2 + \frac{1}{2} \times (1 - 3 T) p_1 p_2 x_2^2]]$,
 E_{{1}→{1,2}} [$\frac{1}{\sqrt{T}}$, (-1 + $\frac{1}{T}$) (p₁ - p₂) x₂,
 ∈Series [0, - $\frac{(-1 + T) p_1^2 x_1 x_2}{T^2}$ - $\frac{p_1 p_2 x_1 x_2}{T^2}$ - $\frac{(1 - T) p_1^2 x_2^2}{2 T^3}$ - $\frac{(-1 - T) p_1 p_2 x_2^2}{2 T^3}$]]]},
 E_{{1}→{1}} [√T, 0, ∈Series [0, - $\frac{p_1 x_1}{T}$]]], E_{{1}→{1}} [$\frac{1}{\sqrt{T}}$, 0, ∈Series [0, $\frac{p_1 x_1}{T}$]]] }

In[*]:= RMoves

Out[*]:= {True, True, True, True, True, True, True, True, True}

Some Knot Theory at \$k=1

In[*]:= NewBit [K_] := Module [{Alex = Alexander [K] [T] },

$$T^3 \frac{Alex^2}{T - 1} \text{ZF} [K] [3, 2] // \text{Factor}]$$

In[*]:= NewBit /@ AllKnots [{3, 5}]

KnotTheory: Loading precomputed data in PD4Knots`.

Out[*]:= { 2 - T + T², (1 + T) × (1 - 3 T + T²), $\frac{4 - 3 T + 5 T^2 - 3 T^3 + 3 T^4 - T^5 + T^6}{T^2}$, 9 - 11 T + 7 T² - T³ }

In[*]:= (*Two knots with equal Alexander, new bit does not agree*)

Alexander [Knot [6, 1]] == Alexander [Knot [9, 46]]

Timing [NewBit [Knot [6, 1]] == NewBit [Knot [9, 46]]]

Out[*]:= True

Out[*]:= {19.0938, 5 - 11 T - T² + 3 T³ == 7 - 21 T + 9 T² + T³ }

In[]:= **equiv** = {**Knot**[10, 106], **Knot**[12, **NonAlternating**, 369]};
Length@Union@Echo[ZF /@ **equiv**]

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

$$\gg \left\{ E_{\{\} \rightarrow \{1\}} \left[\frac{T^4}{1 - 4 T + 9 T^2 - 15 T^3 + 17 T^4 - 15 T^5 + 9 T^6 - 4 T^7 + T^8}, \theta, \right. \right. \\ \left. \in \text{Series} \left[\theta, \left(-3 + 20 T - 69 T^2 + 161 T^3 - 272 T^4 + 328 T^5 - 225 T^6 - 92 T^7 + 548 T^8 - 952 T^9 + 1113 T^{10} - \right. \right. \right. \\ \left. \left. \left. 980 T^{11} + 668 T^{12} - 349 T^{13} + 135 T^{14} - 36 T^{15} + 5 T^{16} \right) / \left(T - 8 T^2 + 34 T^3 - 102 T^4 + 235 T^5 - 436 T^6 + \right. \right. \right. \\ \left. \left. \left. 669 T^7 - 860 T^8 + 935 T^9 - 860 T^{10} + 669 T^{11} - 436 T^{12} + 235 T^{13} - 102 T^{14} + 34 T^{15} - 8 T^{16} + T^{17} \right) \right] \right], \\ E_{\{\} \rightarrow \{1\}} \left[\frac{T^4}{1 - 4 T + 9 T^2 - 15 T^3 + 17 T^4 - 15 T^5 + 9 T^6 - 4 T^7 + T^8}, \theta, \right. \\ \left. \in \text{Series} \left[\theta, \left(-3 + 20 T - 69 T^2 + 161 T^3 - 272 T^4 + 328 T^5 - 225 T^6 - 92 T^7 + 548 T^8 - 952 T^9 + 1113 T^{10} - \right. \right. \right. \\ \left. \left. \left. 980 T^{11} + 668 T^{12} - 349 T^{13} + 135 T^{14} - 36 T^{15} + 5 T^{16} \right) / \left(T - 8 T^2 + 34 T^3 - 102 T^4 + 235 T^5 - 436 T^6 + \right. \right. \right. \\ \left. \left. \left. 669 T^7 - 860 T^8 + 935 T^9 - 860 T^{10} + 669 T^{11} - 436 T^{12} + 235 T^{13} - 102 T^{14} + 34 T^{15} - 8 T^{16} + T^{17} \right) \right] \right] \left. \right\}$$

Out[]:= 1

In[]:= **equiv** =
 {**Knot**[12, **Alternating**, 427], **Knot**[12, **Alternating**, 435], **Knot**[12, **Alternating**, 990]};
Length@Union[ZF /@ **equiv**]

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

Out[]:= 1

Solving for R, C, \$k = 2

In[]:= **\$k** = 2;
 {**R**_{1,2}, **C**₁}
Short[**RMoves**, 20]

$$\text{Out[]:= } \left\{ E_{\{\} \rightarrow \{1,2\}} \left[\sqrt{T}, (-1 + T) (p_1 - p_2) x_2, \right. \right. \\ \left. \in \text{Series} \left[\theta, p_1 p_2 x_1 x_2 + \frac{1}{2} \times (-1 + T) p_1^2 x_2^2 + \frac{1}{2} \times (1 - 3 T) p_1 p_2 x_2^2, \right. \right. \\ \left. \left. \left. c_{2,1} + p_1 x_1 c_{2,2} + p_1 x_2 c_{2,3} + p_2 x_1 c_{2,4} + p_2 x_2 c_{2,5} + p_1^2 x_1^2 c_{2,6} + p_1^2 x_1 x_2 c_{2,7} + \right. \right. \right. \\ \left. \left. \left. p_1^2 x_2^2 c_{2,8} + p_1 p_2 x_1^2 c_{2,9} + p_1 p_2 x_1 x_2 c_{2,10} + p_1 p_2 x_2^2 c_{2,11} + p_2^2 x_1^2 c_{2,12} + p_2^2 x_1 x_2 c_{2,13} + \right. \right. \right. \\ \left. \left. \left. p_2^2 x_2^2 c_{2,14} + p_1^3 x_1^3 c_{2,15} + p_1^3 x_1^2 x_2 c_{2,16} + p_1^3 x_1 x_2^2 c_{2,17} + p_1^3 x_2^3 c_{2,18} + p_1^2 p_2 x_1^3 c_{2,19} + \right. \right. \right. \\ \left. \left. \left. p_1^2 p_2 x_1^2 x_2 c_{2,20} + p_1^2 p_2 x_1 x_2^2 c_{2,21} + p_1^2 p_2 x_2^3 c_{2,22} + p_1 p_2^2 x_1^3 c_{2,23} + p_1 p_2^2 x_1^2 x_2 c_{2,24} + \right. \right. \right. \\ \left. \left. \left. p_1 p_2^2 x_1 x_2^2 c_{2,25} + p_1 p_2^2 x_2^3 c_{2,26} + p_2^3 x_1^3 c_{2,27} + p_2^3 x_1^2 x_2 c_{2,28} + p_2^3 x_1 x_2^2 c_{2,29} + p_2^3 x_2^3 c_{2,30} \right] \right], \\ E_{\{\} \rightarrow \{1\}} \left[\sqrt{T}, \theta, \in \text{Series} \left[\theta, -\frac{p_1 x_1}{T}, e_{2,1} + p_1 x_1 e_{2,2} + p_1^2 x_1^2 e_{2,3} + p_1^3 x_1^3 e_{2,4} \right] \right] \left. \right\}$$

$$\begin{aligned}
 \text{Out[*]]/Short= } & \left\{ (-1 + T) p_1^2 p_3 x_1 x_2 x_3 - 3 T p_1 p_2 p_3 x_1 x_2 x_3 + (-2 T + 4 T^2) p_1 p_2 p_3 x_1 x_3^2 + 3 c_{2,1} + \right. \\
 & 2 p_1 x_1 c_{2,2} + p_2 x_1 c_{2,4} + p_3 x_1 c_{2,4} + T p_3 x_2 c_{2,4} + p_1 x_2 (c_{2,2} - T c_{2,2} + c_{2,3} + c_{2,4} - T c_{2,4}) + \\
 & 2 T p_3 x_3 c_{2,5} + p_2 x_2 (T c_{2,2} + c_{2,5}) + p_1 x_3 (T c_{2,2} - T^2 c_{2,2} + 2 c_{2,3} - T c_{2,3} + c_{2,5} - T c_{2,5}) + \\
 & p_2 x_3 (T c_{2,3} + T c_{2,4} - T^2 c_{2,4} + c_{2,5} - T c_{2,5}) + 2 p_1^2 x_1^2 c_{2,6} + p_1^2 x_1 x_2 c_{2,7} + \\
 & p_1^2 x_1 x_3 (2 T c_{2,6} - 2 T^2 c_{2,6} + c_{2,7}) + p_1 p_2 x_1^2 c_{2,9} + p_1 p_3 x_1^2 c_{2,9} + T^2 p_2 p_3 x_2^2 c_{2,9} + p_1 p_2 x_1 x_2 c_{2,10} + \\
 & T p_1 p_3 x_1 x_3 c_{2,10} + T^2 p_2 p_3 x_2 x_3 c_{2,10} + p_1 p_2 x_1 x_3 (2 T c_{2,9} - 2 T^2 c_{2,9} + c_{2,10} - T c_{2,10}) + \\
 & T^2 p_2 p_3 x_3^2 c_{2,11} + p_1 p_2 x_2^2 (2 T c_{2,6} - 2 T^2 c_{2,6} + T c_{2,9} - T^2 c_{2,9} + c_{2,11}) + \ll 68 \gg + p_1^2 p_3 x_3^3 \\
 & (T c_{2,19} - 2 T^2 c_{2,19} + T^3 c_{2,19} + 2 T c_{2,23} - 4 T^2 c_{2,23} + 2 T^3 c_{2,23} + 3 T c_{2,27} - 6 T^2 c_{2,27} + 3 T^3 c_{2,27}) + \\
 & p_2^3 x_1^2 x_2 c_{2,28} + T p_3^3 x_1^2 x_3 c_{2,28} + T^3 p_3^3 x_2^2 x_3 c_{2,28} + p_2^3 x_1^2 x_3 (3 T c_{2,27} - 3 T^2 c_{2,27} + c_{2,28} - T c_{2,28}) + \\
 & p_1 p_3^2 x_2^2 x_3 (T^2 c_{2,24} - T^3 c_{2,24} + 3 T^2 c_{2,28} - 3 T^3 c_{2,28}) + p_1^3 x_2^2 x_3 (1 - 2 T + T^2 + c_{2,16} - 3 T c_{2,16} + \\
 & 3 T^2 c_{2,16} - T^3 c_{2,16} + T c_{2,17} - T^2 c_{2,17} + 3 c_{2,18} - 3 T c_{2,18} + c_{2,20} - 3 T c_{2,20} + 3 T^2 c_{2,20} - \\
 & T^3 c_{2,20} + c_{2,24} - 3 T c_{2,24} + 3 T^2 c_{2,24} - T^3 c_{2,24} + c_{2,28} - 3 T c_{2,28} + 3 T^2 c_{2,28} - T^3 c_{2,28}) + \\
 & p_1^2 p_3 x_2^2 x_3 (T - T^2 + T c_{2,20} - 2 T^2 c_{2,20} + T^3 c_{2,20} + 2 T c_{2,24} - 4 T^2 c_{2,24} + 2 T^3 c_{2,24} + \\
 & 3 T c_{2,28} - 6 T^2 c_{2,28} + 3 T^3 c_{2,28}) + p_2^3 x_1 x_2^2 c_{2,29} + T^2 p_3^3 x_1 x_3^2 c_{2,29} + \\
 & T^3 p_3^3 x_2 x_3^2 c_{2,29} + p_2^3 x_1 x_2 x_3 (2 T c_{2,28} - 2 T^2 c_{2,28} + 2 c_{2,29} - 2 T c_{2,29}) + p_2^3 x_1 x_3^2 \\
 & (3 T^2 c_{2,27} - 6 T^3 c_{2,27} + 3 T^4 c_{2,27} + 2 T c_{2,28} - 4 T^2 c_{2,28} + 2 T^3 c_{2,28} + c_{2,29} - 2 T c_{2,29} + T^2 c_{2,29}) + \\
 & p_1 p_3^2 x_2 x_3^2 (T^2 c_{2,25} - T^3 c_{2,25} + 3 T^2 c_{2,29} - 3 T^3 c_{2,29}) + \\
 & \frac{1}{2} p_1^3 x_2 x_3^2 (1 - 3 T + 3 T^2 - T^3 + 2 T^2 c_{2,16} - 4 T^3 c_{2,16} + 2 T^4 c_{2,16} + 2 c_{2,17} - 2 T c_{2,17} - 2 T^2 c_{2,17} + \\
 & 2 T^3 c_{2,17} + 6 c_{2,18} - 12 T c_{2,18} + 6 T^2 c_{2,18} + 2 c_{2,21} - 6 T c_{2,21} + 6 T^2 c_{2,21} - 2 T^3 c_{2,21} + \\
 & 2 c_{2,25} - 6 T c_{2,25} + 6 T^2 c_{2,25} - 2 T^3 c_{2,25} + 2 c_{2,29} - 6 T c_{2,29} + 6 T^2 c_{2,29} - 2 T^3 c_{2,29}) + \\
 & \frac{1}{2} p_1^2 p_3 x_2 x_3^2 (T - T^3 + 2 T c_{2,21} - 4 T^2 c_{2,21} + 2 T^3 c_{2,21} + 4 T c_{2,25} - 8 T^2 c_{2,25} + \\
 & 4 T^3 c_{2,25} + 6 T c_{2,29} - 12 T^2 c_{2,29} + 6 T^3 c_{2,29}) + 2 T^3 p_3^3 x_3^3 c_{2,30} + \\
 & p_2^3 x_2^3 (T^3 c_{2,15} + c_{2,30}) + p_2^3 x_2^2 x_3 (T^3 c_{2,16} + T c_{2,29} - T^2 c_{2,29} + 3 c_{2,30} - 3 T c_{2,30}) + \\
 & p_2^2 x_2 x_3^2 (T^3 c_{2,17} + T^2 c_{2,28} - 2 T^3 c_{2,28} + T^4 c_{2,28} + 2 T c_{2,29} - 4 T^2 c_{2,29} + 2 T^3 c_{2,29} + \\
 & 3 c_{2,30} - 6 T c_{2,30} + 3 T^2 c_{2,30}) + p_1 p_3^2 x_3^3 (T^2 c_{2,26} + 3 T^2 c_{2,30} - 3 T^3 c_{2,30}) + \\
 & \frac{1}{2} p_1^3 x_3^3 (-1 + 3 T - 5 T^2 + 5 T^3 - 2 T^4 + 2 T^3 c_{2,15} - 6 T^4 c_{2,15} + 6 T^5 c_{2,15} - 2 T^6 c_{2,15} + \\
 & 2 T^2 c_{2,16} - 6 T^3 c_{2,16} + 6 T^4 c_{2,16} - 2 T^5 c_{2,16} + 2 T c_{2,17} - 6 T^2 c_{2,17} + 6 T^3 c_{2,17} - 2 T^4 c_{2,17} + \\
 & 4 c_{2,18} - 12 T c_{2,18} + 12 T^2 c_{2,18} - 2 T^3 c_{2,18} + 2 c_{2,22} - 6 T c_{2,22} + 6 T^2 c_{2,22} - 2 T^3 c_{2,22} + \\
 & 2 c_{2,26} - 6 T c_{2,26} + 6 T^2 c_{2,26} - 2 T^3 c_{2,26} + 2 c_{2,30} - 6 T c_{2,30} + 6 T^2 c_{2,30} - 2 T^3 c_{2,30}) + \\
 & p_2^2 x_3^3 (T^3 c_{2,18} + T^3 c_{2,27} - 3 T^4 c_{2,27} + 3 T^5 c_{2,27} - T^6 c_{2,27} + T^2 c_{2,28} - 3 T^3 c_{2,28} + 3 T^4 c_{2,28} - \\
 & T^5 c_{2,28} + T c_{2,29} - 3 T^2 c_{2,29} + 3 T^3 c_{2,29} - T^4 c_{2,29} + c_{2,30} - 3 T c_{2,30} + 3 T^2 c_{2,30} - T^3 c_{2,30}) + \\
 & p_1^2 p_3 x_3^3 (T^2 - 4 T^3 + 3 T^4 + T c_{2,22} - 2 T^2 c_{2,22} + 2 T^3 c_{2,22} + 2 T c_{2,26} - 4 T^2 c_{2,26} + 2 T^3 c_{2,26} + \\
 & 3 T c_{2,30} - 6 T^2 c_{2,30} + 3 T^3 c_{2,30}) = 3 c_{\ll 1 \gg} + \ll 144 \gg + \ll 1 \gg, \ll 8 \gg \}
 \end{aligned}$$

In[*]:= unknowns = Cases [{ R_{1,2}, $\bar{R}_{1,2}$, C₁, \bar{C}_1 }, (c | d | e | f)_{\$k,_}, ∞] // Union

Out[*]:= { C_{2,1}, C_{2,2}, C_{2,3}, C_{2,4}, C_{2,5}, C_{2,6}, C_{2,7}, C_{2,8}, C_{2,9}, C_{2,10}, C_{2,11}, C_{2,12}, C_{2,13}, C_{2,14},
 C_{2,15}, C_{2,16}, C_{2,17}, C_{2,18}, C_{2,19}, C_{2,20}, C_{2,21}, C_{2,22}, C_{2,23}, C_{2,24}, C_{2,25}, C_{2,26}, C_{2,27},
 C_{2,28}, C_{2,29}, C_{2,30}, d_{2,1}, d_{2,2}, d_{2,3}, d_{2,4}, d_{2,5}, d_{2,6}, d_{2,7}, d_{2,8}, d_{2,9}, d_{2,10}, d_{2,11},
 d_{2,12}, d_{2,13}, d_{2,14}, d_{2,15}, d_{2,16}, d_{2,17}, d_{2,18}, d_{2,19}, d_{2,20}, d_{2,21}, d_{2,22}, d_{2,23}, d_{2,24},
 d_{2,25}, d_{2,26}, d_{2,27}, d_{2,28}, d_{2,29}, d_{2,30}, e_{2,1}, e_{2,2}, e_{2,3}, e_{2,4}, f_{2,1}, f_{2,2}, f_{2,3}, f_{2,4} }

In[*]:= Short[errors = CCF /@ Cases[RMoves, a_ == b_ => a - b], 25]

$$\text{Out[*]//Short} = \left\{ \frac{1}{2} \times \right.$$

$$\begin{aligned} & (2 p_1 p_2 x_2 x_3 - 2 T p_1 p_2 x_2 x_3 - 2 p_1^2 p_2 x_1 x_2 x_3 + 2 T p_1^2 p_2 x_1 x_2 x_3 + 2 p_1 p_2^2 x_1 x_2 x_3 - 2 T p_1 p_2^2 x_1 x_2 x_3 - \\ & 2 p_1^2 p_3 x_1 x_2 x_3 + 2 T p_1^2 p_3 x_1 x_2 x_3 + 2 p_1^3 x_2^2 x_3 - 4 T p_1^3 x_2^2 x_3 + 2 T^2 p_1^3 x_2^2 x_3 - 6 p_1^2 p_2 x_2^2 x_3 + \\ & 16 T p_1^2 p_2 x_2^2 x_3 - 10 T^2 p_1^2 p_2 x_2^2 x_3 + p_1 p_2^2 x_2^2 x_3 - 6 T p_1 p_2^2 x_2^2 x_3 + 5 T^2 p_1 p_2^2 x_2^2 x_3 + 2 T p_1 p_2 p_3 x_2^2 x_3 - \\ & 2 T^2 p_1 p_2 p_3 x_2^2 x_3 + p_1^2 x_3^2 - 2 T p_1^2 x_3^2 + T^2 p_1^2 x_3^2 + p_1^2 p_2 x_1 x_3^2 - T^2 p_1^2 p_2 x_1 x_3^2 - p_1^2 p_3 x_1 x_3^2 + \\ & 4 T p_1^2 p_3 x_1 x_3^2 - 3 T^2 p_1^2 p_3 x_1 x_3^2 - 2 T p_1 p_2 p_3 x_1 x_3^2 + 2 T^2 p_1 p_2 p_3 x_1 x_3^2 + p_1^3 x_2 x_3^2 - 3 T p_1^3 x_2 x_3^2 + \\ & 3 T^2 p_1^3 x_2 x_3^2 - T^3 p_1^3 x_2 x_3^2 - 4 p_1^2 p_2 x_2 x_3^2 + 14 T p_1^2 p_2 x_2 x_3^2 - 16 T^2 p_1^2 p_2 x_2 x_3^2 + 6 T^3 p_1^2 p_2 x_2 x_3^2 - \\ & 3 T p_1 p_2^2 x_2 x_3^2 + 4 T^2 p_1 p_2^2 x_2 x_3^2 - T^3 p_1 p_2^2 x_2 x_3^2 + T p_1^2 p_3 x_2 x_3^2 - T^3 p_1^2 p_3 x_2 x_3^2 - 2 T^2 p_1 p_2 p_3 x_2 x_3^2 + \\ & 2 T^3 p_1 p_2 p_3 x_2 x_3^2 - p_1^3 x_3^3 + 3 T p_1^3 x_3^3 - 5 T^2 p_1^3 x_3^3 + 5 T^3 p_1^3 x_3^3 - 2 T^4 p_1^3 x_3^3 + T p_1^2 p_2 x_3^3 - 2 T^2 p_1^2 p_2 x_3^3 - \\ & T^3 p_1^2 p_2 x_3^3 + 2 T^4 p_1^2 p_2 x_3^3 + T^2 p_1 p_2^2 x_3^3 - T^4 p_1 p_2^2 x_3^3 + 2 T^2 p_1^2 p_3 x_3^3 - 8 T^3 p_1^2 p_3 x_3^3 + 6 T^4 p_1^2 p_3 x_3^3 - \\ & 2 T^2 p_1 p_2 p_3 x_3^3 + 8 T^3 p_1 p_2 p_3 x_3^3 - 6 T^4 p_1 p_2 p_3 x_3^3 + 2 T p_1 x_3 c_{2,2} - 2 T^2 p_1 x_3 c_{2,2} + 2 p_1 x_3 c_{2,3} - \\ & 2 T p_1 x_3 c_{2,3} - 2 p_2 x_1 c_{2,4} + 2 T p_2 x_1 c_{2,4} + 2 p_3 x_1 c_{2,4} - 2 T p_3 x_1 c_{2,4} + 2 p_1 x_2 c_{2,4} - 2 T p_1 x_2 c_{2,4} - \\ & 2 p_2 x_2 c_{2,4} + 4 T p_2 x_2 c_{2,4} - 2 T^2 p_2 x_2 c_{2,4} - 2 T p_3 x_2 c_{2,4} + 2 T^2 p_3 x_2 c_{2,4} + 2 T p_2 x_3 c_{2,4} - \\ & 2 T^2 p_2 x_3 c_{2,4} + 2 p_1 x_3 c_{2,5} - 2 T p_1 x_3 c_{2,5} - 4 p_1^2 x_1 x_2 c_{2,6} + 4 T p_1^2 x_1 x_2 c_{2,6} + 4 T p_1 p_2 x_2^2 c_{2,6} - \\ & 4 T^2 p_1 p_2 x_2^2 c_{2,6} + 4 T p_1^2 x_1 x_3 c_{2,6} - 4 T^2 p_1^2 x_1 x_3 c_{2,6} + \ll 860 \gg + 6 T p_1^2 p_3 x_2^2 x_3 c_{2,28} - \\ & 12 T^2 p_1^2 p_3 x_2^2 x_3 c_{2,28} + 6 T^3 p_1^2 p_3 x_2^2 x_3 c_{2,28} - 6 T p_2^2 p_3 x_2^2 x_3 c_{2,28} + 24 T^2 p_2^2 p_3 x_2^2 x_3 c_{2,28} - \\ & 36 T^3 p_2^2 p_3 x_2^2 x_3 c_{2,28} + 24 T^4 p_2^2 p_3 x_2^2 x_3 c_{2,28} - 6 T^5 p_2^2 p_3 x_2^2 x_3 c_{2,28} + 6 T^2 p_1 p_3^2 x_2^2 x_3 c_{2,28} - \\ & 6 T^3 p_1 p_3^2 x_2^2 x_3 c_{2,28} - 6 T^2 p_2 p_3^2 x_2^2 x_3 c_{2,28} + 18 T^3 p_2 p_3^2 x_2^2 x_3 c_{2,28} - 18 T^4 p_2 p_3^2 x_2^2 x_3 c_{2,28} + \\ & 6 T^5 p_2 p_3^2 x_2^2 x_3 c_{2,28} - 2 T^3 p_3^2 x_2^2 x_3 c_{2,28} + 4 T^4 p_3^2 x_2^2 x_3 c_{2,28} - 2 T^5 p_3^2 x_2^2 x_3 c_{2,28} + \\ & 4 T p_2^3 x_1 x_3^2 c_{2,28} - 8 T^2 p_2^3 x_1 x_3^2 c_{2,28} + 4 T^3 p_2^3 x_1 x_3^2 c_{2,28} + 2 T^2 p_2^3 x_2 x_3^2 c_{2,28} - 4 T^3 p_2^3 x_2 x_3^2 c_{2,28} + \\ & 2 T^4 p_2^3 x_2 x_3^2 c_{2,28} + 2 T^2 p_2^3 x_3^3 c_{2,28} - 6 T^3 p_2^3 x_3^3 c_{2,28} + 6 T^4 p_2^3 x_3^3 c_{2,28} - 2 T^5 p_2^3 x_3^3 c_{2,28} + \\ & 4 p_2^3 x_1 x_2 x_3 c_{2,29} - 4 T p_2^3 x_1 x_2 x_3 c_{2,29} + 2 T p_2^3 x_2^2 x_3 c_{2,29} - 2 T^2 p_2^3 x_2^2 x_3 c_{2,29} + 2 T p_2^3 x_1 x_3^2 c_{2,29} - \\ & 4 T^2 p_2^3 x_1 x_3^2 c_{2,29} + 2 T^3 p_2^3 x_1 x_3^2 c_{2,29} - 6 T p_2^2 p_3 x_1 x_3^2 c_{2,29} + 12 T^2 p_2^2 p_3 x_1 x_3^2 c_{2,29} - \\ & 6 T^3 p_2^2 p_3 x_1 x_3^2 c_{2,29} - 6 T^2 p_2 p_3^2 x_1 x_3^2 c_{2,29} + 6 T^3 p_2 p_3^2 x_1 x_3^2 c_{2,29} + 2 T^2 p_3^3 x_1 x_3^2 c_{2,29} - \\ & 2 T^3 p_3^3 x_1 x_3^2 c_{2,29} + 2 p_1^3 x_2 x_3^2 c_{2,29} - 6 T p_1^3 x_2 x_3^2 c_{2,29} + 6 T^2 p_1^3 x_2 x_3^2 c_{2,29} - 2 T^3 p_1^3 x_2 x_3^2 c_{2,29} - \\ & 2 p_2^3 x_2 x_3^2 c_{2,29} + 12 T p_2^3 x_2 x_3^2 c_{2,29} - 20 T^2 p_2^3 x_2 x_3^2 c_{2,29} + 12 T^3 p_2^3 x_2 x_3^2 c_{2,29} - 2 T^4 p_2^3 x_2 x_3^2 c_{2,29} + \\ & 6 T p_1^2 p_3 x_2 x_3^2 c_{2,29} - 12 T^2 p_1^2 p_3 x_2 x_3^2 c_{2,29} + 6 T^3 p_1^2 p_3 x_2 x_3^2 c_{2,29} - 6 T p_2^2 p_3 x_2 x_3^2 c_{2,29} + \\ & 18 T^2 p_2^2 p_3 x_2 x_3^2 c_{2,29} - 18 T^3 p_2^2 p_3 x_2 x_3^2 c_{2,29} + 6 T^4 p_2^2 p_3 x_2 x_3^2 c_{2,29} + 6 T^2 p_1 p_3^2 x_2 x_3^2 c_{2,29} - \\ & 6 T^3 p_1 p_3^2 x_2 x_3^2 c_{2,29} - 6 T^2 p_2 p_3^2 x_2 x_3^2 c_{2,29} + 12 T^3 p_2 p_3^2 x_2 x_3^2 c_{2,29} - 6 T^4 p_2 p_3^2 x_2 x_3^2 c_{2,29} - \\ & 2 T^3 p_3^3 x_2 x_3^2 c_{2,29} + 2 T^4 p_3^3 x_2 x_3^2 c_{2,29} + 2 T p_2^3 x_3^3 c_{2,29} - 6 T^2 p_2^3 x_3^3 c_{2,29} + 6 T^3 p_2^3 x_3^3 c_{2,29} - \\ & 2 T^4 p_2^3 x_3^3 c_{2,29} + 6 p_2^3 x_2^2 x_3 c_{2,30} - 6 T p_2^3 x_2^2 x_3 c_{2,30} + 6 p_2^3 x_2 x_3^2 c_{2,30} - 12 T p_2^3 x_2 x_3^2 c_{2,30} + \\ & 6 T^2 p_2^3 x_2 x_3^2 c_{2,30} + 2 p_1^3 x_3^3 c_{2,30} - 6 T p_1^3 x_3^3 c_{2,30} + 6 T^2 p_1^3 x_3^3 c_{2,30} - 2 T^3 p_1^3 x_3^3 c_{2,30} + \\ & 6 T p_1^2 p_3 x_3^3 c_{2,30} - 12 T^2 p_1^2 p_3 x_3^3 c_{2,30} + 6 T^3 p_1^2 p_3 x_3^3 c_{2,30} - 6 T p_2^2 p_3 x_3^3 c_{2,30} + 12 T^2 p_2^2 p_3 x_3^3 c_{2,30} - \\ & 6 T^3 p_2^2 p_3 x_3^3 c_{2,30} + 6 T^2 p_1 p_3^2 x_3^3 c_{2,30} - 6 T^3 p_1 p_3^2 x_3^3 c_{2,30} - 6 T^2 p_2 p_3^2 x_3^3 c_{2,30} + 6 T^3 p_2 p_3^2 x_3^3 c_{2,30}) , \\ & \left. \frac{\ll 1 \gg}{2 \ll 1 \gg} , \ll 5 \gg , \frac{\ll 1 \gg}{\ll 1 \gg} , \frac{\ll 106 \gg + 2 \ll 4 \gg}{2 T^4} \right\} \end{aligned}$$

In[]:= Short[#, 10] &[eqns = Thread[0 == Union @@ (CoefficientRules[#, {x1, x2, x3, p1, p2, p3}][[; ; , 2] & /@ errors)]]]

$$\begin{aligned}
 \text{Out[]/Short} = \left\{ \begin{aligned}
 &\theta = c_{2,4} - T c_{2,4}, \quad \theta = -c_{2,4} + T c_{2,4}, \quad \theta = T c_{2,4} - T^2 c_{2,4}, \\
 &\theta = -c_{2,4} + 2 T c_{2,4} - T^2 c_{2,4}, \quad \theta = -T c_{2,4} + T^2 c_{2,4}, \quad \ll 209 \gg, \\
 &\theta = -\frac{1}{2} + \frac{T}{2} + T^3 c_{2,15} + T^2 c_{2,16} + T c_{2,17} + c_{2,18} + T^3 c_{2,19} + T^2 c_{2,20} + T c_{2,21} + \\
 &\quad c_{2,22} + T^3 c_{2,23} + T^2 c_{2,24} + T c_{2,25} + c_{2,26} + T^3 c_{2,27} + T^2 c_{2,28} + T c_{2,29} + c_{2,30} + T^3 f_{2,4}, \\
 &\theta = \frac{1}{2} - \frac{1}{2T} + d_{2,15} + T d_{2,16} + T^2 d_{2,17} + T^3 d_{2,18} + d_{2,19} + T d_{2,20} + T^2 d_{2,21} + T^3 d_{2,22} + \\
 &\quad d_{2,23} + T d_{2,24} + T^2 d_{2,25} + T^3 d_{2,26} + d_{2,27} + T d_{2,28} + T^2 d_{2,29} + T^3 d_{2,30} + T^3 f_{2,4}, \\
 &\theta = -1 - \frac{1}{T^2} + \frac{3}{2T} + \frac{T}{2} - c_{2,15} + 3 T c_{2,15} - 3 T^2 c_{2,15} + T^3 c_{2,15} + c_{2,16} - 2 T c_{2,16} + T^2 c_{2,16} - c_{2,17} + \\
 &\quad T c_{2,17} + c_{2,18} - 4 c_{2,19} + \frac{c_{2,19}}{T} + 6 T c_{2,19} - 4 T^2 c_{2,19} + T^3 c_{2,19} + 3 c_{2,20} - \frac{c_{2,20}}{T} - 3 T c_{2,20} + \\
 &\quad T^2 c_{2,20} - 2 c_{2,21} + \frac{c_{2,21}}{T} + T c_{2,21} + c_{2,22} - \frac{c_{2,22}}{T} - 10 c_{2,23} - \frac{c_{2,23}}{T^2} + \frac{5 c_{2,23}}{T} + 10 T c_{2,23} - \\
 &\quad 5 T^2 c_{2,23} + T^3 c_{2,23} + 6 c_{2,24} + \frac{c_{2,24}}{T^2} - \frac{4 c_{2,24}}{T} - 4 T c_{2,24} + T^2 c_{2,24} - 3 c_{2,25} - \frac{c_{2,25}}{T^2} + \frac{3 c_{2,25}}{T} + \\
 &\quad T c_{2,25} + c_{2,26} + \frac{c_{2,26}}{T^2} - \frac{2 c_{2,26}}{T} - 20 c_{2,27} + \frac{c_{2,27}}{T^3} - \frac{6 c_{2,27}}{T^2} + \frac{15 c_{2,27}}{T} + 15 T c_{2,27} - 6 T^2 c_{2,27} + \\
 &\quad T^3 c_{2,27} + 10 c_{2,28} - \frac{c_{2,28}}{T^3} + \frac{5 c_{2,28}}{T^2} - \frac{10 c_{2,28}}{T} - 5 T c_{2,28} + T^2 c_{2,28} - 4 c_{2,29} + \frac{c_{2,29}}{T^3} - \frac{4 c_{2,29}}{T^2} + \\
 &\quad \frac{6 c_{2,29}}{T} + T c_{2,29} + c_{2,30} - \frac{c_{2,30}}{T^3} + \frac{3 c_{2,30}}{T^2} - \frac{3 c_{2,30}}{T} + T^3 d_{2,18} - f_{2,4} + 3 T f_{2,4} - 3 T^2 f_{2,4} + T^3 f_{2,4}, \\
 &\theta = 1 + \frac{2}{T} + d_{2,2} + T d_{2,3} + d_{2,4} + T d_{2,5} - 2 T d_{2,7} - 4 T^2 d_{2,8} - T d_{2,10} - 2 T^2 d_{2,11} + 6 T^2 d_{2,17} + \\
 &\quad 18 T^3 d_{2,18} + 2 T^2 d_{2,21} + 6 T^3 d_{2,22} + T f_{2,2} + 4 T f_{2,3} - 4 T^2 f_{2,3} + 18 T f_{2,4} - 36 T^2 f_{2,4} + 18 T^3 f_{2,4} \}
 \end{aligned} \right.
 \end{aligned}$$

In[]:= {sol} = Solve[eqns, unknowns]

Solve: Equations may not give solutions for all "solve" variables.

$$\text{Out[]} = \left\{ \left\{ \begin{array}{l} c_{2,1} \rightarrow -\frac{c_{2,2}}{2} - \frac{c_{2,5}}{2T}, c_{2,3} \rightarrow -T c_{2,2} - c_{2,5}, c_{2,4} \rightarrow 0, c_{2,6} \rightarrow 0, c_{2,8} \rightarrow -\frac{1}{2} \times (1-T) c_{2,10}, \\ c_{2,9} \rightarrow 0, c_{2,11} \rightarrow -\frac{1}{2} - T c_{2,7} - \frac{1}{2} \times (-1+3T) c_{2,10}, c_{2,12} \rightarrow 0, c_{2,13} \rightarrow 0, c_{2,14} \rightarrow 0, \\ c_{2,15} \rightarrow 0, c_{2,17} \rightarrow -((-1+T) c_{2,16}), c_{2,18} \rightarrow -\frac{-1+4T-3T^2}{6T}, c_{2,19} \rightarrow 0, c_{2,20} \rightarrow -\frac{1}{2T}, \\ c_{2,21} \rightarrow -\frac{1-3T}{2T}, c_{2,22} \rightarrow -\frac{1-11T+16T^2}{6T} - (T-T^2) c_{2,16}, c_{2,23} \rightarrow 0, c_{2,24} \rightarrow 0, \\ c_{2,25} \rightarrow -\frac{1}{2}, c_{2,26} \rightarrow \frac{1}{6} \times (-1+7T) - T^2 c_{2,16}, c_{2,27} \rightarrow 0, c_{2,28} \rightarrow 0, c_{2,29} \rightarrow 0, c_{2,30} \rightarrow 0, \\ d_{2,1} \rightarrow \frac{c_{2,2}}{2} + \frac{c_{2,5}}{2T}, d_{2,2} \rightarrow -c_{2,2}, d_{2,3} \rightarrow \frac{c_{2,2}}{T} + \frac{c_{2,5}}{T^2}, d_{2,4} \rightarrow 0, d_{2,5} \rightarrow -\frac{c_{2,5}}{T^2}, d_{2,6} \rightarrow 0, \\ d_{2,7} \rightarrow -\frac{1-T}{T^3} - \frac{c_{2,7}}{T} - \frac{(-1+T) c_{2,10}}{T^2}, d_{2,8} \rightarrow -\frac{-1+T}{2T^4} - \frac{(1-T) c_{2,10}}{2T^3}, d_{2,9} \rightarrow 0, d_{2,10} \rightarrow \frac{1}{T^3} - \frac{c_{2,10}}{T^2}, \\ d_{2,11} \rightarrow -\frac{1}{2T^4} + \frac{c_{2,7}}{T^2} - \frac{(-1-T) c_{2,10}}{2T^3}, d_{2,12} \rightarrow 0, d_{2,13} \rightarrow 0, d_{2,14} \rightarrow 0, d_{2,15} \rightarrow 0, \\ d_{2,16} \rightarrow -\frac{-1+T}{2T^3} - \frac{c_{2,16}}{T}, d_{2,17} \rightarrow -\frac{3-4T+T^2}{2T^4} - \frac{(-1+T) c_{2,16}}{T^2}, d_{2,18} \rightarrow -\frac{-3+4T-T^2}{6T^5}, d_{2,19} \rightarrow 0, \\ d_{2,20} \rightarrow -\frac{1}{2T^3}, d_{2,21} \rightarrow \frac{2}{T^4}, d_{2,22} \rightarrow -\frac{4+T+T^2}{6T^5} - \frac{(1-T) c_{2,16}}{T^3}, d_{2,23} \rightarrow 0, d_{2,24} \rightarrow 0, d_{2,25} \rightarrow -\frac{1}{2T^4}, \\ d_{2,26} \rightarrow -\frac{-1+T}{6T^5} + \frac{c_{2,16}}{T^3}, d_{2,27} \rightarrow 0, d_{2,28} \rightarrow 0, d_{2,29} \rightarrow 0, d_{2,30} \rightarrow 0, e_{2,1} \rightarrow -\frac{c_{2,2}}{2} - \frac{c_{2,5}}{2T}, \\ e_{2,2} \rightarrow -\frac{c_{2,10}}{T}, e_{2,3} \rightarrow 0, e_{2,4} \rightarrow 0, f_{2,1} \rightarrow \frac{c_{2,2}}{2} + \frac{c_{2,5}}{2T}, f_{2,2} \rightarrow -\frac{1}{T^2} + \frac{c_{2,10}}{T}, f_{2,3} \rightarrow 0, f_{2,4} \rightarrow 0 \end{array} \right\} \right\}$$

In[*]:= sol /. (a_ -> b_) :-> (a = b)

$$\text{Out[*]} = \left\{ -\frac{c_{2,2}}{2} - \frac{c_{2,5}}{2T}, -T c_{2,2} - c_{2,5}, \theta, \theta, -\frac{1}{2} \times (1-T) c_{2,10}, \theta, -\frac{1}{2} - T c_{2,7} - \frac{1}{2} \times (-1+3T) c_{2,10}, \theta, \theta, \theta, \right.$$

$$\theta, -((-1+T) c_{2,16}), -\frac{-1+4T-3T^2}{6T}, \theta, -\frac{1}{2T}, -\frac{1-3T}{2T}, -\frac{1-11T+16T^2}{6T} - (T-T^2) c_{2,16},$$

$$\theta, \theta, -\frac{1}{2}, \frac{1}{6} \times (-1+7T) - T^2 c_{2,16}, \theta, \theta, \theta, \theta, \frac{c_{2,2}}{2} + \frac{c_{2,5}}{2T}, -c_{2,2}, \frac{c_{2,2}}{T} + \frac{c_{2,5}}{T^2}, \theta,$$

$$-\frac{c_{2,5}}{T^2}, \theta, -\frac{1-T}{T^3} \frac{c_{2,7}}{T} - \frac{(-1+T) c_{2,10}}{T^2}, -\frac{-1+T}{2T^4} - \frac{(1-T) c_{2,10}}{2T^3}, \theta, \frac{1}{T^3} - \frac{c_{2,10}}{T^2},$$

$$-\frac{1}{2T^4} + \frac{c_{2,7}}{T^2} - \frac{(-1-T) c_{2,10}}{2T^3}, \theta, \theta, \theta, \theta, -\frac{-1+T}{2T^3} - \frac{c_{2,16}}{T}, -\frac{3-4T+T^2}{2T^4} - \frac{(-1+T) c_{2,16}}{T^2},$$

$$-\frac{-3+4T-T^2}{6T^5}, \theta, -\frac{1}{2T^3}, \frac{2}{T^4}, -\frac{4+T+T^2}{6T^5} - \frac{(1-T) c_{2,16}}{T^3}, \theta, \theta, -\frac{1}{2T^4}, -\frac{-1+T}{6T^5} + \frac{c_{2,16}}{T^3},$$

$$\theta, \theta, \theta, \theta, -\frac{c_{2,2}}{2} - \frac{c_{2,5}}{2T}, -\frac{c_{2,10}}{T}, \theta, \theta, \frac{c_{2,2}}{2} + \frac{c_{2,5}}{2T}, -\frac{1}{T^2} + \frac{c_{2,10}}{T}, \theta, \theta \left. \right\}$$

In[*]:= Cases[{R1,2, R1,2, C1, C1}, (c | d | e | f)_{sk,-}, \infty] // Union

Out[*]= {C2,2, C2,5, C2,7, C2,10, C2,16}

In[*]:= {C2,2 = 0, C2,5 = 0, C2,7 = 0, C2,10 = 0, C2,16 = 0};
{R1,2, R1,2, C1, C1}

$$\text{Out[*]} = \left\{ \mathbb{E}_{\{1\} \rightarrow \{1,2\}} \left[\sqrt{T}, (-1+T) (p_1 - p_2) x_2, \right. \right.$$

$$\in \text{Series} \left[\theta, p_1 p_2 x_1 x_2 + \frac{1}{2} \times (-1+T) p_1^2 x_2^2 + \frac{1}{2} \times (1-3T) p_1 p_2 x_2^2, \right.$$

$$-\frac{p_1^2 p_2 x_1^2 x_2}{2T} - \frac{1}{2} p_1 p_2 x_2^2 - \frac{(1-3T) p_1^2 p_2 x_1 x_2^2}{2T} - \frac{1}{2} p_1 p_2^2 x_1 x_2^2 - \frac{(-1+4T-3T^2) p_1^3 x_2^3}{6T} -$$

$$\left. \frac{(1-11T+16T^2) p_1^2 p_2 x_2^3}{6T} + \frac{1}{6} \times (-1+7T) p_1 p_2^2 x_2^3 \right], \mathbb{E}_{\{1\} \rightarrow \{1,2\}} \left[\frac{1}{\sqrt{T}}, \left(-1 + \frac{1}{T} \right) (p_1 - p_2) x_2, \right.$$

$$\in \text{Series} \left[\theta, -\frac{(-1+T) p_1^2 x_1 x_2}{T^2} - \frac{p_1 p_2 x_1 x_2}{T^2} - \frac{(1-T) p_1^2 x_2^2}{2T^3} - \frac{(-1-T) p_1 p_2 x_2^2}{2T^3}, -\frac{(1-T) p_1^2 x_1 x_2}{T^3} + \right.$$

$$\frac{p_1 p_2 x_1 x_2}{T^3} - \frac{(-1+T) p_1^3 x_1^2 x_2}{2T^3} - \frac{p_1^2 p_2 x_1^2 x_2}{2T^3} - \frac{(-1+T) p_1^2 x_2^2}{2T^4} - \frac{p_1 p_2 x_2^2}{2T^4} - \frac{(3-4T+T^2) p_1^3 x_1 x_2^2}{2T^4} +$$

$$\left. \frac{2 p_1^2 p_2 x_1 x_2^2}{T^4} - \frac{p_1 p_2^2 x_1 x_2^2}{2T^4} - \frac{(-3+4T-T^2) p_1^3 x_2^3}{6T^5} - \frac{(4+T+T^2) p_1^2 p_2 x_2^3}{6T^5} - \frac{(-1+T) p_1 p_2^2 x_2^3}{6T^5} \right],$$

$$\mathbb{E}_{\{1\} \rightarrow \{1\}} \left[\sqrt{T}, \theta, \in \text{Series} \left[\theta, -\frac{p_1 x_1}{T}, \theta \right], \mathbb{E}_{\{1\} \rightarrow \{1\}} \left[\frac{1}{\sqrt{T}}, \theta, \in \text{Series} \left[\theta, \frac{p_1 x_1}{T}, -\frac{p_1 x_1}{T^2} \right] \right] \right\}$$

In[*]:= RMoves

Out[*]= {True, True, True, True, True, True, True, True, True}

Some Knot Theory at $\$k=2$

According to 12XingStats.nb at pensieve://Projects/SL2Invariant/k=2/ the following pair have equal ρ_1 but different ρ_2 :

```
In[ ]:= equiv = {Knot[10, 106], Knot[12, NonAlternating, 369]};
res21 = ZF /@ equiv
Simplify[res21[[1]] == res21[[2]]]
```

$$\text{Out[]} = \left\{ \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\frac{T^4}{1 - 4T + 9T^2 - 15T^3 + 17T^4 - 15T^5 + 9T^6 - 4T^7 + T^8}, \theta, \right. \right. \\ \text{eSeries} \left[\theta, \left(-3 + 20T - 69T^2 + 161T^3 - 272T^4 + 328T^5 - 225T^6 - 92T^7 + 548T^8 - 952T^9 + 1113T^{10} - \right. \right. \\ \left. \left. 980T^{11} + 668T^{12} - 349T^{13} + 135T^{14} - 36T^{15} + 5T^{16} \right) / \left(T - 8T^2 + 34T^3 - 102T^4 + 235T^5 - 436T^6 + \right. \right. \\ \left. \left. 669T^7 - 860T^8 + 935T^9 - 860T^{10} + 669T^{11} - 436T^{12} + 235T^{13} - 102T^{14} + 34T^{15} - 8T^{16} + T^{17} \right), \right. \\ \left(3 - 40T + 264T^2 - 1128T^3 + 3437T^4 - 7552T^5 + 10297T^6 + 2304T^7 - 67324T^8 + \right. \\ \left. 259472T^9 - 699066T^{10} + 1539252T^{11} - 2919131T^{12} + 4882760T^{13} - 7290870T^{14} + \right. \\ \left. 9779044T^{15} - 11816854T^{16} + 12877354T^{17} - 12651386T^{18} + 11191592T^{19} - \right. \\ \left. 8896165T^{20} + 6336738T^{21} - 4030390T^{22} + 2278962T^{23} - 1139320T^{24} + \right. \\ \left. 500046T^{25} - 190857T^{26} + 62504T^{27} - 17215T^{28} + 3862T^{29} - 668T^{30} + 80T^{31} - 5T^{32} \right) / \\ \left(2T^2 - 32T^3 + 264T^4 - 1496T^5 + 6516T^6 - 23136T^7 + 69396T^8 - 180024T^9 + 410582T^{10} - \right. \\ \left. 833112T^{11} + 1517288T^{12} - 2496728T^{13} + 3730618T^{14} - 5080576T^{15} + 6323208T^{16} - \right. \\ \left. 7205208T^{17} + 7524878T^{18} - 7205208T^{19} + 6323208T^{20} - 5080576T^{21} + \right. \\ \left. 3730618T^{22} - 2496728T^{23} + 1517288T^{24} - 833112T^{25} + 410582T^{26} - 180024T^{27} + \right. \\ \left. 69396T^{28} - 23136T^{29} + 6516T^{30} - 1496T^{31} + 264T^{32} - 32T^{33} + 2T^{34} \right) \left. \right],$$

$$\mathbb{E}_{\{\} \rightarrow \{1\}} \left[\frac{T^4}{1 - 4T + 9T^2 - 15T^3 + 17T^4 - 15T^5 + 9T^6 - 4T^7 + T^8}, \right. \\ \theta, \\ \text{eSeries} \left[\theta, \left(-3 + 20T - 69T^2 + 161T^3 - 272T^4 + 328T^5 - 225T^6 - 92T^7 + \right. \right. \\ \left. \left. 548T^8 - 952T^9 + 1113T^{10} - 980T^{11} + 668T^{12} - 349T^{13} + 135T^{14} - 36T^{15} + 5T^{16} \right) / \right. \\ \left(T - 8T^2 + 34T^3 - 102T^4 + 235T^5 - 436T^6 + 669T^7 - 860T^8 + 935T^9 - 860T^{10} + \right. \\ \left. 669T^{11} - 436T^{12} + 235T^{13} - 102T^{14} + 34T^{15} - 8T^{16} + T^{17} \right), \\ \left(3 - 40T + 264T^2 - 1120T^3 + 3333T^4 - 6896T^5 + 7641T^6 + 9944T^7 - 83404T^8 + 283088T^9 - \right. \\ \left. 716082T^{10} + 1514140T^{11} - 2796883T^{12} + 4607952T^{13} - 6839214T^{14} + \right. \\ \left. 9183044T^{15} - 11164950T^{16} + 12281354T^{17} - 12199730T^{18} + 10916784T^{19} - \right. \\ \left. 8773917T^{20} + 6311626T^{21} - 4047406T^{22} + 2302578T^{23} - 1155400T^{24} + \right. \\ \left. 507686T^{25} - 193513T^{26} + 63160T^{27} - 17319T^{28} + 3870T^{29} - 668T^{30} + 80T^{31} - 5T^{32} \right) / \\ \left(2T^2 - 32T^3 + 264T^4 - 1496T^5 + 6516T^6 - 23136T^7 + 69396T^8 - 180024T^9 + 410582T^{10} - \right. \\ \left. 833112T^{11} + 1517288T^{12} - 2496728T^{13} + 3730618T^{14} - 5080576T^{15} + 6323208T^{16} - \right. \\ \left. 7205208T^{17} + 7524878T^{18} - 7205208T^{19} + 6323208T^{20} - 5080576T^{21} + \right. \\ \left. 3730618T^{22} - 2496728T^{23} + 1517288T^{24} - 833112T^{25} + 410582T^{26} - 180024T^{27} + \right. \\ \left. 69396T^{28} - 23136T^{29} + 6516T^{30} - 1496T^{31} + 264T^{32} - 32T^{33} + 2T^{34} \right) \left. \right] \left. \right\}$$

```
Out[ ]:= False
```

According to 12XingStats.nb at pensieve://Projects/SL2Invariant/k=2/ the following triple have equal ρ_1 but different ρ_2 :

```
In[ ]:= equiv =
  {Knot [12, Alternating, 427], Knot [12, Alternating, 435], Knot [12, Alternating, 990]};
  Length@Union[res22 = ZF /@equiv]
```

```
Out[ ]:= 3
```

Solving for R, C, \$k = 3

```
In[ ]:= $k = 3;
  Short [RMoves, 20]
```

$$\begin{aligned}
 \text{Out[]}/\text{Short} = & \left\{ -p_1 p_2 p_3 x_1 x_2 x_3 + \frac{(1-T) p_1^3 p_3 x_1^2 x_2 x_3}{2T} + 4 p_1^2 p_2 p_3 x_1^2 x_2 x_3 + \frac{(1-4T+3T^2) p_1^3 p_3 x_1 x_2^2 x_3}{2T} + \right. \\
 & \frac{1}{2} \times (11-21T) p_1^2 p_2 p_3 x_1 x_2^2 x_3 + 4T p_1 p_2^2 p_3 x_1 x_2^2 x_3 + \frac{1}{2} \times (-1+5T) p_1 p_2 p_3 x_1 x_3^2 + \\
 & (3-5T) p_1^2 p_2 p_3 x_1^2 x_3^2 + \frac{(1-4T+3T^2) p_1^3 p_3 x_1 x_2 x_3^2}{2T} + \frac{1}{2} \times (15-41T+20T^2) p_1^2 p_2 p_3 x_1 x_2 x_3^2 + \\
 & \frac{1}{2} \times (11T-21T^2) p_1 p_2^2 p_3 x_1 x_2 x_3^2 + \ll 250 \gg + 2T^4 p_3^4 x_3^4 c_{3,55} + p_2^4 x_2^4 (T^4 c_{3,31} + c_{3,55}) + \\
 & p_2^4 x_2^3 x_3 (T^4 c_{3,32} + T c_{3,54} - T^2 c_{3,54} + 4 c_{3,55} - 4T c_{3,55}) + p_2^4 x_2^2 x_3^2 (T^4 c_{3,33} + T^2 c_{3,53} - 2T^3 c_{3,53} + \\
 & T^4 c_{3,53} + 3T c_{3,54} - 6T^2 c_{3,54} + 3T^3 c_{3,54} + 6 c_{3,55} - 12T c_{3,55} + 6T^2 c_{3,55}) + p_2^4 x_2 x_3^3 \\
 & (T^4 c_{3,34} + T^3 c_{3,52} - 3T^4 c_{3,52} + 3T^5 c_{3,52} - T^6 c_{3,52} + 2T^2 c_{3,53} - 6T^3 c_{3,53} + 6T^4 c_{3,53} - 2T^5 c_{3,53} + \\
 & 3T c_{3,54} - 9T^2 c_{3,54} + 9T^3 c_{3,54} - 3T^4 c_{3,54} + 4 c_{3,55} - 12T c_{3,55} + 12T^2 c_{3,55} - 4T^3 c_{3,55}) + \frac{1}{24} p_1^3 \\
 & p_3 x_3^4 (-7 + 69T - 188T^2 + 364T^3 - 477T^4 + 239T^5 + 24T c_{3,40} - 72T^2 c_{3,40} + 72T^3 c_{3,40} + 48T c_{3,45} - \\
 & 144T^2 c_{3,45} + 144T^3 c_{3,45} - 48T^4 c_{3,45} + 72T c_{3,50} - 216T^2 c_{3,50} + 216T^3 c_{3,50} - 72T^4 c_{3,50} + \\
 & 96T c_{3,55} - 288T^2 c_{3,55} + 288T^3 c_{3,55} - 96T^4 c_{3,55}) + p_1 p_3^3 x_3^4 (T^3 c_{3,50} + 4T^3 c_{3,55} - 4T^4 c_{3,55}) + \\
 & p_2^4 x_3^4 (T^4 c_{3,35} + T^4 c_{3,51} - 4T^5 c_{3,51} + 6T^6 c_{3,51} - 4T^7 c_{3,51} + T^8 c_{3,51} + T^3 c_{3,52} - 4T^4 c_{3,52} + \\
 & 6T^5 c_{3,52} - 4T^6 c_{3,52} + T^7 c_{3,52} + T^2 c_{3,53} - 4T^3 c_{3,53} + 6T^4 c_{3,53} - 4T^5 c_{3,53} + T^6 c_{3,53} + T c_{3,54} - \\
 & 4T^2 c_{3,54} + 6T^3 c_{3,54} - 4T^4 c_{3,54} + T^5 c_{3,54} + c_{3,55} - 4T c_{3,55} + 6T^2 c_{3,55} - 4T^3 c_{3,55} + T^4 c_{3,55}) + \\
 & \frac{1}{24} p_1^2 p_3^2 x_3^4 (7T^2 - 97T^3 + 329T^4 - 239T^5 + 24T^2 c_{3,45} - 48T^3 c_{3,45} + 48T^4 c_{3,45} + \\
 & 72T^2 c_{3,50} - 144T^3 c_{3,50} + 72T^4 c_{3,50} + 144T^2 c_{3,55} - 288T^3 c_{3,55} + 144T^4 c_{3,55}) + \\
 & \frac{1}{12T} p_1^4 x_3^4 (5 - 19T + 13T^2 + 38T^3 - 89T^4 + 77T^5 - 25T^6 + 12T^5 c_{3,31} - 48T^6 c_{3,31} + 72T^7 c_{3,31} - \\
 & 48T^8 c_{3,31} + 12T^9 c_{3,31} + 12T^4 c_{3,32} - 48T^5 c_{3,32} + 72T^6 c_{3,32} - 48T^7 c_{3,32} + 12T^8 c_{3,32} + \\
 & 12T^3 c_{3,33} - 48T^4 c_{3,33} + 72T^5 c_{3,33} - 48T^6 c_{3,33} + 12T^7 c_{3,33} + 12T^2 c_{3,34} - 48T^3 c_{3,34} + \\
 & 72T^4 c_{3,34} - 48T^5 c_{3,34} + 12T^6 c_{3,34} + 24T c_{3,35} - 96T^2 c_{3,35} + 144T^3 c_{3,35} - 96T^4 c_{3,35} + \\
 & 36T^5 c_{3,35} + 12T c_{3,40} - 48T^2 c_{3,40} + 72T^3 c_{3,40} - 48T^4 c_{3,40} + 12T^5 c_{3,40} + 12T c_{3,45} - \\
 & 48T^2 c_{3,45} + 72T^3 c_{3,45} - 48T^4 c_{3,45} + 12T^5 c_{3,45} + 12T c_{3,50} - 48T^2 c_{3,50} + 72T^3 c_{3,50} - \\
 & 48T^4 c_{3,50} + 12T^5 c_{3,50} + 12T c_{3,55} - 48T^2 c_{3,55} + 72T^3 c_{3,55} - 48T^4 c_{3,55} + 12T^5 c_{3,55}) = \\
 & 3 c_{3,1} + 2 p_1 x_1 c_{3,2} + p_1 x_3 c_{3,3} + p_1 x_2 (c_{3,2} - T c_{3,2} + c_{3,3}) + T p_3 x_1 c_{3,4} +
 \end{aligned}$$

$$\begin{aligned}
 & p_2 x_1 (2 c_{3,4} - T c_{3,4}) + p_3 x_2 (2 T c_{3,4} - T^2 c_{3,4}) + \\
 & 2 T p_3 x_3 c_{3,5} + \langle\langle 355 \rangle\rangle + \\
 & p_2^3 p_3 x_2 x_3^3 (T^4 c_{3,39} + 4 T c_{3,54} - 16 T^2 c_{3,54} + 24 T^3 c_{3,54} - 16 T^4 c_{3,54} + 4 T^5 c_{3,54}) + \\
 & p_2 p_3^2 x_2 x_3^3 (T^4 c_{3,49} + 4 T^3 c_{3,54} - 8 T^4 c_{3,54} + 4 T^5 c_{3,54}) + 2 T^4 p_3^4 x_3^4 c_{3,55} + \\
 & p_2^4 x_2^4 (T^4 c_{3,31} + c_{3,51} - 8 T c_{3,51} + 28 T^2 c_{3,51} - 56 T^3 c_{3,51} + 70 T^4 c_{3,51} - 56 T^5 c_{3,51} + \\
 & \quad 28 T^6 c_{3,51} - 8 T^7 c_{3,51} + T^8 c_{3,51} + c_{3,55}) + p_2 p_3^3 x_3^4 (T^4 c_{3,50} + 4 T^3 c_{3,55} - 4 T^4 c_{3,55}) + \\
 & p_2^3 p_3 x_3^4 (T^4 c_{3,40} + 4 T c_{3,55} - 12 T^2 c_{3,55} + 12 T^3 c_{3,55} - 4 T^4 c_{3,55}) + \\
 & p_2^4 x_3^4 (T^4 c_{3,35} + c_{3,55} - 4 T c_{3,55} + 6 T^2 c_{3,55} - 4 T^3 c_{3,55} + T^4 c_{3,55}) + \\
 & p_2^2 p_3^2 x_3^4 (T^4 c_{3,45} + 6 T^2 c_{3,55} - 12 T^3 c_{3,55} + 6 T^4 c_{3,55}), \\
 & \langle\langle 7 \rangle\rangle, \frac{p_1^4 \langle\langle 1 \rangle\rangle (\langle\langle 1 \rangle\rangle)}{12 T^5} + \langle\langle 3 \rangle\rangle + \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle} = \theta \}
 \end{aligned}$$

In[*]:= unknowns = Cases [{R_{1,2}, R̄_{1,2}, C₁, C̄₁}, (c | d | e | f) \$k,_, ∞] // Union

Out[*]= {C_{3,1}, C_{3,2}, C_{3,3}, C_{3,4}, C_{3,5}, C_{3,6}, C_{3,7}, C_{3,8}, C_{3,9}, C_{3,10}, C_{3,11}, C_{3,12}, C_{3,13}, C_{3,14}, C_{3,15}, C_{3,16}, C_{3,17}, C_{3,18}, C_{3,19}, C_{3,20}, C_{3,21}, C_{3,22}, C_{3,23}, C_{3,24}, C_{3,25}, C_{3,26}, C_{3,27}, C_{3,28}, C_{3,29}, C_{3,30}, C_{3,31}, C_{3,32}, C_{3,33}, C_{3,34}, C_{3,35}, C_{3,36}, C_{3,37}, C_{3,38}, C_{3,39}, C_{3,40}, C_{3,41}, C_{3,42}, C_{3,43}, C_{3,44}, C_{3,45}, C_{3,46}, C_{3,47}, C_{3,48}, C_{3,49}, C_{3,50}, C_{3,51}, C_{3,52}, C_{3,53}, C_{3,54}, C_{3,55}, d_{3,1}, d_{3,2}, d_{3,3}, d_{3,4}, d_{3,5}, d_{3,6}, d_{3,7}, d_{3,8}, d_{3,9}, d_{3,10}, d_{3,11}, d_{3,12}, d_{3,13}, d_{3,14}, d_{3,15}, d_{3,16}, d_{3,17}, d_{3,18}, d_{3,19}, d_{3,20}, d_{3,21}, d_{3,22}, d_{3,23}, d_{3,24}, d_{3,25}, d_{3,26}, d_{3,27}, d_{3,28}, d_{3,29}, d_{3,30}, d_{3,31}, d_{3,32}, d_{3,33}, d_{3,34}, d_{3,35}, d_{3,36}, d_{3,37}, d_{3,38}, d_{3,39}, d_{3,40}, d_{3,41}, d_{3,42}, d_{3,43}, d_{3,44}, d_{3,45}, d_{3,46}, d_{3,47}, d_{3,48}, d_{3,49}, d_{3,50}, d_{3,51}, d_{3,52}, d_{3,53}, d_{3,54}, d_{3,55}, e_{3,1}, e_{3,2}, e_{3,3}, e_{3,4}, e_{3,5}, f_{3,1}, f_{3,2}, f_{3,3}, f_{3,4}, f_{3,5}}

In[*]:= Short [errors = CCF /@ Cases [RMoves, a_ == b_ => a - b], 25]

$$\begin{aligned}
 \text{Out[*]//Short} = & \left\{ \frac{1}{24 T} \left(-24 p_1^2 p_2 x_1 x_2 x_3 + 24 T p_1^2 p_2 x_1 x_2 x_3 + 12 p_1^3 p_2 x_1^2 x_2 x_3 - 12 T p_1^3 p_2 x_1^2 x_2 x_3 - 48 p_1^2 p_2^2 x_1^2 x_2 x_3 + \right. \right. \\
 & 48 T p_1^2 p_2^2 x_1^2 x_2 x_3 + 12 p_1^3 p_3 x_1^2 x_2 x_3 - 12 T p_1^3 p_3 x_1^2 x_2 x_3 + 12 p_1^3 x_2^2 x_3 - 24 T p_1^3 x_2^2 x_3 + 12 T^2 p_1^3 x_2^2 x_3 - \\
 & 72 p_1^2 p_2 x_2^2 x_3 + 240 T p_1^2 p_2 x_2^2 x_3 - 168 T^2 p_1^2 p_2 x_2^2 x_3 - 60 T p_1 p_2^2 x_2^2 x_3 + 60 T^2 p_1 p_2^2 x_2^2 x_3 + \\
 & 96 p_1^3 p_2 x_1 x_2^2 x_3 - 240 T p_1^3 p_2 x_1 x_2^2 x_3 + 144 T^2 p_1^3 p_2 x_1 x_2^2 x_3 - 60 p_1^2 p_2^2 x_1 x_2^2 x_3 + 288 T p_1^2 p_2^2 x_1 x_2^2 x_3 - \\
 & 228 T^2 p_1^2 p_2^2 x_1 x_2^2 x_3 - 12 T p_1 p_2^3 x_1 x_2^2 x_3 + 12 T^2 p_1 p_2^3 x_1 x_2^2 x_3 + 12 p_1^3 p_3 x_1 x_2^2 x_3 - 24 T p_1^3 p_3 x_1 x_2^2 x_3 + \\
 & 12 T^2 p_1^3 p_3 x_1 x_2^2 x_3 - 36 p_1^4 x_2^3 x_3 + 132 T p_1^4 x_2^3 x_3 - 156 T^2 p_1^4 x_2^3 x_3 + 60 T^3 p_1^4 x_2^3 x_3 + 96 p_1^3 p_2 x_2^3 x_3 - \\
 & 540 T p_1^3 p_2 x_2^3 x_3 + 864 T^2 p_1^3 p_2 x_2^3 x_3 - 420 T^3 p_1^3 p_2 x_2^3 x_3 - 20 p_1^2 p_2^2 x_2^3 x_3 + 264 T p_1^2 p_2^2 x_2^3 x_3 - \\
 & 708 T^2 p_1^2 p_2^2 x_2^3 x_3 + \langle\langle 3611 \rangle\rangle + 24 T^6 p_3^4 x_2 x_3^3 c_{3,54} + 24 T^2 p_2^4 x_3^4 c_{3,54} - 96 T^3 p_2^4 x_3^4 c_{3,54} + \\
 & 144 T^4 p_2^4 x_3^4 c_{3,54} - 96 T^5 p_2^4 x_3^4 c_{3,54} + 24 T^6 p_2^4 x_3^4 c_{3,54} + 96 T p_2^4 x_2^3 x_3 c_{3,55} - 96 T^2 p_2^4 x_2^3 x_3 c_{3,55} + \\
 & 144 T p_2^4 x_2^3 x_3 c_{3,55} - 288 T^2 p_2^4 x_2^3 x_3 c_{3,55} + 144 T^3 p_2^4 x_2^3 x_3 c_{3,55} + 96 T p_2^4 x_2 x_3^3 c_{3,55} - \\
 & 288 T^2 p_2^4 x_2 x_3^3 c_{3,55} + 288 T^3 p_2^4 x_2 x_3^3 c_{3,55} - 96 T^4 p_2^4 x_2 x_3^3 c_{3,55} + 24 T p_1^4 x_3^4 c_{3,55} - 96 T^2 p_1^4 x_3^4 c_{3,55} + \\
 & 144 T^3 p_1^4 x_3^4 c_{3,55} - 96 T^4 p_1^4 x_3^4 c_{3,55} + 24 T^5 p_1^4 x_3^4 c_{3,55} + 96 T^2 p_1^3 p_3 x_3^4 c_{3,55} - 288 T^3 p_1^3 p_3 x_3^4 c_{3,55} + \\
 & 288 T^4 p_1^3 p_3 x_3^4 c_{3,55} - 96 T^5 p_1^3 p_3 x_3^4 c_{3,55} - 96 T^2 p_2^3 p_3 x_3^4 c_{3,55} + 288 T^3 p_2^3 p_3 x_3^4 c_{3,55} - \\
 & 288 T^4 p_2^3 p_3 x_3^4 c_{3,55} + 96 T^5 p_2^3 p_3 x_3^4 c_{3,55} + 144 T^3 p_1^2 p_3^2 x_3^4 c_{3,55} - 288 T^4 p_1^2 p_3^2 x_3^4 c_{3,55} + \\
 & 144 T^5 p_1^2 p_3^2 x_3^4 c_{3,55} - 144 T^3 p_2^2 p_3^2 x_3^4 c_{3,55} + 288 T^4 p_2^2 p_3^2 x_3^4 c_{3,55} - 144 T^5 p_2^2 p_3^2 x_3^4 c_{3,55} + \\
 & 96 T^4 p_1 p_3^3 x_3^4 c_{3,55} - 96 T^5 p_1 p_3^3 x_3^4 c_{3,55} - 96 T^4 p_2 p_3^3 x_3^4 c_{3,55} + 96 T^5 p_2 p_3^3 x_3^4 c_{3,55} \left. \right), \\
 & \frac{1}{24 T^6} \left(24 T^3 p_1 p_2 x_1 x_2 - 24 T^3 p_1^2 p_2 x_1^2 x_2 - 12 T^2 p_1 p_2 x_2^2 + 36 T^2 p_1^3 x_1 x_2^2 - 36 T^3 p_1^3 x_1 x_2^2 + \right. \\
 & \left. 60 T^2 p_1^2 p_2 x_1 x_2^2 + 24 T^3 p_1^2 p_2 x_1 x_2^2 - 24 T^3 p_1 p_2^2 x_1 x_2^2 - 24 T^2 p_1^4 x_1^2 x_2^2 + \right.
 \end{aligned}$$

$$\begin{aligned}
 & \langle\langle 828 \rangle\rangle + 96 T^7 p_1^3 p_2 x_2^4 d_{3,55} - 288 T^8 p_1^3 p_2 x_2^4 d_{3,55} + 288 T^9 p_1^3 p_2 x_2^4 d_{3,55} - \\
 & 96 T^{10} p_1^3 p_2 x_2^4 d_{3,55} + 144 T^8 p_1^2 p_2^2 x_2^4 d_{3,55} - 288 T^9 p_1^2 p_2^2 x_2^4 d_{3,55} + \\
 & 144 T^{10} p_1^2 p_2^2 x_2^4 d_{3,55} + 96 T^9 p_1 p_2^3 x_2^4 d_{3,55} - 96 T^{10} p_1 p_2^3 x_2^4 d_{3,55} + 24 T^{10} p_2^4 x_2^4 d_{3,55} \Big), \\
 & \frac{1}{T^3} \left(-p_1 x_1 + T^3 e_{3,1} + T^3 p_1 x_1 e_{3,2} + T^3 p_1^2 x_1^2 e_{3,3} + \langle\langle 1 \rangle\rangle + \langle\langle 1 \rangle\rangle + \langle\langle 1 \rangle\rangle + \right. \\
 & \left. T^3 p_1 x_1 f_{3,2} + T^3 p_1^2 x_1^2 f_{3,3} + T^3 p_1^3 x_1^3 f_{3,4} + T^3 p_1^4 x_1^4 f_{3,5} \right), \\
 & \frac{\langle\langle 3127 \rangle\rangle + 24 T^8 \langle\langle 1 \rangle\rangle x_2^4 f_{3,5}}{24 T^4}, \frac{\langle\langle 1 \rangle\rangle}{12 T^6}, \\
 & \frac{\langle\langle 157 \rangle\rangle + 12 \langle\langle 4 \rangle\rangle}{12 T^2}, \\
 & \frac{108 - 216 T + \langle\langle 209 \rangle\rangle + 12 T^7 p_1^4 x_1^4 f_{3,5}}{12 T^3}, \\
 & \frac{1}{12 T^6} \\
 & \left(-12 T^2 p_1 x_1 + 18 T p_1^2 x_1^2 + 12 T^2 p_1^2 x_1^2 + 6 p_1^3 x_1^3 + 12 T p_1^3 x_1^3 - 8 T^2 p_1^3 x_1^3 + \right. \\
 & \left. 2 p_1^4 x_1^4 - 9 T p_1^4 x_1^4 + \langle\langle 139 \rangle\rangle + 12 T^5 p_1^4 x_1^4 d_{3,54} + 12 T^6 p_1^4 x_1^4 d_{3,55} + 12 T^6 e_{3,1} + \right. \\
 & \left. 12 T^5 p_1 x_1 e_{3,2} + 12 T^4 p_1^2 x_1^2 e_{3,3} + 12 T^3 p_1^3 x_1^3 e_{3,4} + 12 T^2 p_1^4 x_1^4 e_{3,5} \right), \\
 & \frac{1}{12 T^6} \left(12 - 144 T + 312 T^2 - 180 T^3 - 36 p_1 x_1 + 216 T p_1 x_1 + 108 T^2 p_1 x_1 - 408 T^3 p_1 x_1 + 18 p_1^2 x_1^2 + \right. \\
 & \left. 60 T p_1^2 x_1^2 - 600 T^2 p_1^2 x_1^2 + 444 T^3 p_1^2 x_1^2 - 2 p_1^3 x_1^3 - 52 T p_1^3 x_1^3 + 166 T^2 p_1^3 x_1^3 - 54 T^3 p_1^3 x_1^3 + 5 T p_1^4 x_1^4 - \right. \\
 & \left. 9 T^2 p_1^4 x_1^4 - 2 T^3 p_1^4 x_1^4 + 12 T^6 c_{3,1} + 12 T^6 p_1 x_1 c_{3,2} - 12 T^5 c_{3,3} + 12 T^5 p_1 x_1 c_{3,3} + 12 T^6 p_1 x_1 c_{3,4} + \right. \\
 & \left. 12 T^5 p_1 x_1 c_{3,5} + 12 T^6 p_1^2 x_1^2 c_{3,6} - 24 T^5 p_1 x_1 c_{3,7} + 12 T^5 p_1^2 x_1^2 c_{3,7} + \langle\langle 137 \rangle\rangle + 24 T^6 e_{3,3} - \right. \\
 & \left. 48 T^4 p_1 x_1 e_{3,3} + 48 T^5 p_1 x_1 e_{3,3} + 12 T^4 p_1^2 x_1^2 e_{3,3} - 72 T^3 e_{3,4} + 216 T^4 e_{3,4} - 216 T^5 e_{3,4} + 72 T^6 e_{3,4} + \right. \\
 & \left. 216 T^3 p_1 x_1 e_{3,4} - 432 T^4 p_1 x_1 e_{3,4} + 216 T^5 p_1 x_1 e_{3,4} - 108 T^3 p_1^2 x_1^2 e_{3,4} + 108 T^4 p_1^2 x_1^2 e_{3,4} + \right. \\
 & \left. 12 T^3 p_1^3 x_1^3 e_{3,4} + 288 T^2 e_{3,5} - 1152 T^3 e_{3,5} + 1728 T^4 e_{3,5} - 1152 T^5 e_{3,5} + 288 T^6 e_{3,5} - \right. \\
 & \left. 1152 T^2 p_1 x_1 e_{3,5} + 3456 T^3 p_1 x_1 e_{3,5} - 3456 T^4 p_1 x_1 e_{3,5} + 1152 T^5 p_1 x_1 e_{3,5} + 864 T^2 p_1^2 x_1^2 e_{3,5} - \right. \\
 & \left. 1728 T^3 p_1^2 x_1^2 e_{3,5} + 864 T^4 p_1^2 x_1^2 e_{3,5} - 192 T^2 p_1^3 x_1^3 e_{3,5} + 192 T^3 p_1^3 x_1^3 e_{3,5} + 12 T^2 p_1^4 x_1^4 e_{3,5} \right) \Big\}
 \end{aligned}$$

In[]:= Short [#, 10] & [eqns =

Thread [0 == Union @@ (CoefficientRules [#, { x1, x2, x3, p1, p2, p3 }] [; , 2] & /@ errors)]]

Out[]:= Short = { 0 == c3,4 - T c3,4, 0 == -c3,4 + T c3,4, 0 == T c3,4 - T^2 c3,4,

⟨⟨ 489 ⟩⟩, 0 == -1 + $\frac{9}{T^3} - \frac{18}{T^2} + \frac{10}{T} + d_{3,1} - T d_{3,3} + 2 T^2 d_{3,8} - 6 T^3 d_{3,18} +$

$24 T^4 d_{3,35} + f_{3,1} + f_{3,2} - T f_{3,2} + 2 f_{3,3} - 4 T f_{3,3} + 2 T^2 f_{3,3} + 6 f_{3,4} - 18 T f_{3,4} +$
 $18 T^2 f_{3,4} - 6 T^3 f_{3,4} + 24 f_{3,5} - 96 T f_{3,5} + 144 T^2 f_{3,5} - 96 T^3 f_{3,5} + 24 T^4 f_{3,5},$

$0 == \frac{3}{2} - \frac{4}{T^3} + \frac{23}{T^2} - \frac{25}{T} + d_{3,6} + T d_{3,7} + T^2 d_{3,8} + d_{3,9} + T d_{3,10} + T^2 d_{3,11} + d_{3,12} + T d_{3,13} + T^2 d_{3,14} -$

$3 T d_{3,16} - 6 T^2 d_{3,17} - 9 T^3 d_{3,18} - 2 T d_{3,20} - 4 T^2 d_{3,21} - 6 T^3 d_{3,22} - T d_{3,24} - 2 T^2 d_{3,25} -$
 $3 T^3 d_{3,26} + 12 T^2 d_{3,33} + 36 T^3 d_{3,34} + 72 T^4 d_{3,35} + 6 T^2 d_{3,38} + 18 T^3 d_{3,39} + 36 T^4 d_{3,40} + 2 T^2 d_{3,43} +$
 $6 T^3 d_{3,44} + 12 T^4 d_{3,45} + T^2 f_{3,3} + 9 T^2 f_{3,4} - 9 T^3 f_{3,4} + 72 T^2 f_{3,5} - 144 T^3 f_{3,5} + 72 T^4 f_{3,5} \Big\}$

In[]:= { sol } = Solve [eqns, unknowns]

Solve: Equations may not give solutions for all "solve" variables.

$$\begin{aligned}
 \text{Out[*]} = & \left\{ \left\{ c_{3,1} \rightarrow -\frac{c_{3,2}}{2} - \frac{c_{3,5}}{2T}, c_{3,3} \rightarrow -T c_{3,2} - c_{3,5}, c_{3,4} \rightarrow 0, c_{3,6} \rightarrow 0, \right. \right. \\
 & c_{3,8} \rightarrow -\frac{1}{2} \times (1-T) c_{3,10}, c_{3,9} \rightarrow 0, c_{3,11} \rightarrow -T c_{3,7} - \frac{1}{2} \times (-1+3T) c_{3,10}, c_{3,12} \rightarrow 0, \\
 & c_{3,13} \rightarrow 0, c_{3,14} \rightarrow 0, c_{3,15} \rightarrow 0, c_{3,17} \rightarrow -((-1+T) c_{3,16}), c_{3,18} \rightarrow -\frac{1-T}{6T}, c_{3,19} \rightarrow 0, \\
 & c_{3,20} \rightarrow 0, c_{3,21} \rightarrow \frac{1}{2T}, c_{3,22} \rightarrow -\frac{-2+5T}{2T} - (T-T^2) c_{3,16}, c_{3,23} \rightarrow 0, c_{3,24} \rightarrow 0, \\
 & c_{3,25} \rightarrow 0, c_{3,26} \rightarrow \frac{5}{6} - T^2 c_{3,16}, c_{3,27} \rightarrow 0, c_{3,28} \rightarrow 0, c_{3,29} \rightarrow 0, c_{3,30} \rightarrow 0, c_{3,31} \rightarrow 0, \\
 & c_{3,33} \rightarrow \frac{3}{2} \times (-1+T) c_{3,32}, c_{3,34} \rightarrow -((-1+2T-T^2) c_{3,32}), c_{3,35} \rightarrow -\frac{1-12T+27T^2-16T^3}{24T^2}, \\
 & c_{3,36} \rightarrow 0, c_{3,37} \rightarrow \frac{1}{6T^2}, c_{3,38} \rightarrow -\frac{-1+3T}{4T^2}, c_{3,39} \rightarrow -\frac{-1+11T-16T^2}{6T^2}, \\
 & c_{3,40} \rightarrow -\frac{-1+31T-131T^2+125T^3}{24T^2} - (T-2T^2+T^3) c_{3,32}, c_{3,41} \rightarrow 0, c_{3,42} \rightarrow 0, c_{3,43} \rightarrow \frac{1}{T}, \\
 & c_{3,44} \rightarrow -\frac{-5+23T}{6T}, c_{3,45} \rightarrow -\frac{-5+69T-142T^2}{24T} + \frac{3}{2} \times (-1+T) T^2 c_{3,32}, c_{3,46} \rightarrow 0, c_{3,47} \rightarrow 0, \\
 & c_{3,48} \rightarrow 0, c_{3,49} \rightarrow \frac{1}{6}, c_{3,50} \rightarrow \frac{1}{24} \times (1-15T) - T^3 c_{3,32}, c_{3,51} \rightarrow 0, c_{3,52} \rightarrow 0, c_{3,53} \rightarrow 0, c_{3,54} \rightarrow 0, \\
 & c_{3,55} \rightarrow 0, d_{3,1} \rightarrow \frac{c_{3,2}}{2} + \frac{c_{3,5}}{2T}, d_{3,2} \rightarrow -c_{3,2}, d_{3,3} \rightarrow \frac{c_{3,2}}{T} + \frac{c_{3,5}}{T^2}, d_{3,4} \rightarrow 0, d_{3,5} \rightarrow -\frac{c_{3,5}}{T^2}, \\
 & d_{3,6} \rightarrow 0, d_{3,7} \rightarrow -\frac{-1+T}{T^4} - \frac{c_{3,7}}{T} - \frac{(-1+T) c_{3,10}}{T^2}, d_{3,8} \rightarrow -\frac{1-T}{2T^5} - \frac{(1-T) c_{3,10}}{2T^3}, d_{3,9} \rightarrow 0, \\
 & d_{3,10} \rightarrow -\frac{1}{T^4} - \frac{c_{3,10}}{T^2}, d_{3,11} \rightarrow \frac{1}{2T^5} + \frac{c_{3,7}}{T^2} - \frac{(-1-T) c_{3,10}}{2T^3}, d_{3,12} \rightarrow 0, d_{3,13} \rightarrow 0, d_{3,14} \rightarrow 0, d_{3,15} \rightarrow 0, \\
 & d_{3,16} \rightarrow -\frac{1-T}{T^4} - \frac{c_{3,16}}{T}, d_{3,17} \rightarrow -\frac{-7+9T-2T^2}{2T^5} - \frac{(-1+T) c_{3,16}}{T^2}, d_{3,18} \rightarrow -\frac{7-9T+2T^2}{6T^6}, d_{3,19} \rightarrow 0, \\
 & d_{3,20} \rightarrow \frac{1}{T^4}, d_{3,21} \rightarrow -\frac{9-T}{2T^5}, d_{3,22} \rightarrow \frac{3}{2T^6} - \frac{(1-T) c_{3,16}}{T^3}, d_{3,23} \rightarrow 0, d_{3,24} \rightarrow 0, d_{3,25} \rightarrow \frac{1}{T^5}, \\
 & d_{3,26} \rightarrow -\frac{1}{3T^6} + \frac{c_{3,16}}{T^3}, d_{3,27} \rightarrow 0, d_{3,28} \rightarrow 0, d_{3,29} \rightarrow 0, d_{3,30} \rightarrow 0, d_{3,31} \rightarrow 0, d_{3,32} \rightarrow -\frac{-1+T}{6T^4} - \frac{c_{3,32}}{T}, \\
 & d_{3,33} \rightarrow -\frac{2-3T+T^2}{T^5} - \frac{3 \times (-1+T) c_{3,32}}{2T^2}, d_{3,34} \rightarrow -\frac{-16+27T-12T^2+T^3}{6T^6} - \frac{(1-2T+T^2) c_{3,32}}{T^3}, \\
 & d_{3,35} \rightarrow -\frac{16-27T+12T^2-T^3}{24T^7}, d_{3,36} \rightarrow 0, d_{3,37} \rightarrow -\frac{1}{6T^4}, d_{3,38} \rightarrow -\frac{-3+T}{T^5}, \\
 & d_{3,39} \rightarrow \frac{3 \times (-3+T)}{2T^6}, d_{3,40} \rightarrow -\frac{-27+5T-T^2-T^3}{24T^7} - \frac{(-1+2T-T^2) c_{3,32}}{T^4}, d_{3,41} \rightarrow 0, \\
 & d_{3,42} \rightarrow 0, d_{3,43} \rightarrow -\frac{1}{T^5}, d_{3,44} \rightarrow \frac{2}{T^6}, d_{3,45} \rightarrow -\frac{12-T-5T^2}{24T^7} + \frac{3 \times (-1+T) c_{3,32}}{2T^4},
 \end{aligned}$$

$$\begin{aligned}
 & d_{3,46} \rightarrow 0, d_{3,47} \rightarrow 0, d_{3,48} \rightarrow 0, d_{3,49} \rightarrow -\frac{1}{6 T^6}, d_{3,50} \rightarrow -\frac{-1-T}{24 T^7} + \frac{c_{3,32}}{T^4}, d_{3,51} \rightarrow 0, \\
 & d_{3,52} \rightarrow 0, d_{3,53} \rightarrow 0, d_{3,54} \rightarrow 0, d_{3,55} \rightarrow 0, e_{3,1} \rightarrow -\frac{c_{3,2}}{2} - \frac{c_{3,5}}{2 T}, e_{3,2} \rightarrow -\frac{c_{3,10}}{T}, e_{3,3} \rightarrow 0, \\
 & e_{3,4} \rightarrow 0, e_{3,5} \rightarrow 0, f_{3,1} \rightarrow \frac{c_{3,2}}{2} + \frac{c_{3,5}}{2 T}, f_{3,2} \rightarrow \frac{1}{T^3} + \frac{c_{3,10}}{T}, f_{3,3} \rightarrow 0, f_{3,4} \rightarrow 0, f_{3,5} \rightarrow 0 \}
 \end{aligned}$$

In[*]:= sol /. (a_ -> b_) :-> (a = b)

$$\begin{aligned}
 \text{Out[*]} = & \left\{ -\frac{c_{3,2}}{2} - \frac{c_{3,5}}{2 T}, -T c_{3,2} - c_{3,5}, 0, 0, -\frac{1}{2} \times (1-T) c_{3,10}, 0, -T c_{3,7} - \frac{1}{2} \times (-1+3 T) c_{3,10}, 0, 0, 0, \right. \\
 & 0, -((-1+T) c_{3,16}), -\frac{1-T}{6 T}, 0, 0, \frac{1}{2 T}, -\frac{-2+5 T}{2 T} - (T-T^2) c_{3,16}, 0, 0, 0, \frac{5}{6} - T^2 c_{3,16}, 0, \\
 & 0, 0, 0, 0, -\frac{3}{2} \times (-1+T) c_{3,32}, -((-1+2 T-T^2) c_{3,32}), -\frac{1-12 T+27 T^2-16 T^3}{24 T^2}, 0, \frac{1}{6 T^2}, \\
 & -\frac{-1+3 T}{4 T^2}, -\frac{-1+11 T-16 T^2}{6 T^2}, -\frac{-1+31 T-131 T^2+125 T^3}{24 T^2} - (T-2 T^2+T^3) c_{3,32}, 0, 0, \frac{1}{T}, \\
 & -\frac{-5+23 T}{6 T}, -\frac{-5+69 T-142 T^2}{24 T} + \frac{3}{2} \times (-1+T) T^2 c_{3,32}, 0, 0, 0, \frac{1}{6}, \frac{1}{24} \times (1-15 T) - T^3 c_{3,32}, \\
 & 0, 0, 0, 0, 0, \frac{c_{3,2}}{2} + \frac{c_{3,5}}{2 T}, -c_{3,2}, \frac{c_{3,2}}{T} + \frac{c_{3,5}}{T^2}, 0, -\frac{c_{3,5}}{T^2}, 0, -\frac{-1+T}{T^4} - \frac{c_{3,7}}{T} - \frac{(-1+T) c_{3,10}}{T^2}, \\
 & -\frac{1-T}{2 T^5} - \frac{(1-T) c_{3,10}}{2 T^3}, 0, -\frac{1}{T^4} - \frac{c_{3,10}}{T^2}, \frac{1}{2 T^5} + \frac{c_{3,7}}{T^2} - \frac{(-1-T) c_{3,10}}{2 T^3}, 0, 0, 0, 0, -\frac{1-T}{T^4} - \frac{c_{3,16}}{T}, \\
 & -\frac{-7+9 T-2 T^2}{2 T^5} - \frac{(-1+T) c_{3,16}}{T^2}, -\frac{7-9 T+2 T^2}{6 T^6}, 0, \frac{1}{T^4}, -\frac{9-T}{2 T^5}, \frac{3}{2 T^6} - \frac{(1-T) c_{3,16}}{T^3}, 0, \\
 & 0, \frac{1}{T^5}, -\frac{1}{3 T^6} + \frac{c_{3,16}}{T^3}, 0, 0, 0, 0, 0, -\frac{-1+T}{6 T^4} - \frac{c_{3,32}}{T}, -\frac{2-3 T+T^2}{T^5} - \frac{3 \times (-1+T) c_{3,32}}{2 T^2}, \\
 & -\frac{-16+27 T-12 T^2+T^3}{6 T^6} - \frac{(1-2 T+T^2) c_{3,32}}{T^3}, -\frac{16-27 T+12 T^2-T^3}{24 T^7}, 0, -\frac{1}{6 T^4}, \\
 & -\frac{-3+T}{T^5}, \frac{3 \times (-3+T)}{2 T^6}, -\frac{-27+5 T-T^2-T^3}{24 T^7} - \frac{(-1+2 T-T^2) c_{3,32}}{T^4}, 0, 0, -\frac{1}{T^5}, \\
 & \frac{2}{T^6}, -\frac{12-T-5 T^2}{24 T^7} + \frac{3 \times (-1+T) c_{3,32}}{2 T^4}, 0, 0, 0, -\frac{1}{6 T^6}, -\frac{-1-T}{24 T^7} + \frac{c_{3,32}}{T^4}, 0, \\
 & 0, 0, 0, 0, -\frac{c_{3,2}}{2} - \frac{c_{3,5}}{2 T}, -\frac{c_{3,10}}{T}, 0, 0, 0, \frac{c_{3,2}}{2} + \frac{c_{3,5}}{2 T}, \frac{1}{T^3} + \frac{c_{3,10}}{T}, 0, 0, 0 \}
 \end{aligned}$$

In[*]:= Cases[{R1,2, R1,2, C1, C1}, (c | d | e | f)_{sk,-}, inf] // Union

Out[*]:= {c3,2, c3,5, c3,7, c3,10, c3,16, c3,32}

In[*]:= {c3,2 = 0, c3,5 = 0, c3,7 = 0, c3,10 = 0, c3,16 = 0, c3,32 = 1};
 {R1,2, R1,2, C1, C1}

Out[*]:= {E_{1,2} -> {1,2} [sqrt(T), (-1+T) (p1 - p2) x2,
 eSeries[0, p1 p2 x1 x2 + 1/2 * (-1+T) p1^2 x2^2 + 1/2 * (1-3 T) p1 p2 x2^2,

$$\begin{aligned}
 & -\frac{p_1^2 p_2 x_1^2 x_2}{2 T} - \frac{1}{2} p_1 p_2 x_2^2 - \frac{(1-3 T) p_1^2 p_2 x_1 x_2^2}{2 T} - \frac{1}{2} p_1 p_2^2 x_1 x_2^2 - \\
 & \frac{(-1+4 T-3 T^2) p_1^3 x_2^3}{6 T} - \frac{(1-11 T+16 T^2) p_1^2 p_2 x_2^3}{6 T} + \frac{1}{6} \times (-1+7 T) p_1 p_2^2 x_2^3, \\
 & p_1^4 x_1^3 x_2 + \frac{p_1^3 p_2 x_1^3 x_2}{6 T^2} + \frac{p_1^2 p_2 x_1 x_2^2}{2 T} - \frac{3}{2} \times (-1+T) p_1^4 x_1^2 x_2^2 - \frac{(-1+3 T) p_1^3 p_2 x_1^2 x_2^2}{4 T^2} + \\
 & \frac{p_1^2 p_2^2 x_1^2 x_2^2}{T} - \frac{(1-T) p_1^3 x_2^3}{6 T} - \frac{(-2+5 T) p_1^2 p_2 x_2^3}{2 T} + \frac{5}{6} p_1 p_2^2 x_2^3 + (1-2 T+T^2) p_1^4 x_1 x_2^2 - \\
 & \frac{(-1+11 T-16 T^2) p_1^3 p_2 x_1 x_2^3}{6 T^2} - \frac{(-5+23 T) p_1^2 p_2^2 x_1 x_2^3}{6 T} + \frac{1}{6} p_1 p_2^3 x_1 x_2^3 - \\
 & \frac{(1-12 T+27 T^2-16 T^3) p_1^4 x_2^4}{24 T^2} + \left(-T+2 T^2-T^3 - \frac{-1+31 T-131 T^2+125 T^3}{24 T^2} \right) p_1^3 p_2 x_2^4 + \\
 & \left(\frac{3}{2} \times (-1+T) T^2 - \frac{-5+69 T-142 T^2}{24 T} \right) p_1^2 p_2^2 x_2^4 + \left(\frac{1}{24} \times (1-15 T) - T^3 \right) p_1 p_2^3 x_2^4 \Big], \\
 & \mathbb{E}_{\{1\} \rightarrow \{1,2\}} \left[\frac{1}{\sqrt{T}}, \left(-1 + \frac{1}{T} \right) (p_1 - p_2) x_2, \in \text{Series} \left[0, -\frac{(-1+T) p_1^2 x_1 x_2}{T^2} - \frac{p_1 p_2 x_1 x_2}{T^2} - \right. \right. \\
 & \left. \frac{(1-T) p_1^2 x_2^2}{2 T^3} - \frac{(-1-T) p_1 p_2 x_2^2}{2 T^3}, -\frac{(1-T) p_1^2 x_1 x_2}{T^3} + \frac{p_1 p_2 x_1 x_2}{T^3} - \frac{(-1+T) p_1^3 x_1^2 x_2}{2 T^3} - \right. \\
 & \left. \frac{p_1^2 p_2 x_1^2 x_2}{2 T^3} - \frac{(-1+T) p_1^2 x_2^2}{2 T^4} - \frac{p_1 p_2 x_2^2}{2 T^4} - \frac{(3-4 T+T^2) p_1^3 x_1 x_2^2}{2 T^4} + \frac{2 p_1^2 p_2 x_1 x_2^2}{T^4} - \right. \\
 & \left. \frac{p_1 p_2^2 x_1 x_2^2}{2 T^4} - \frac{(-3+4 T-T^2) p_1^3 x_2^3}{6 T^5} - \frac{(4+T+T^2) p_1^2 p_2 x_2^3}{6 T^5} - \frac{(-1+T) p_1 p_2^2 x_2^3}{6 T^5}, \right. \\
 & \left. -\frac{(-1+T) p_1^2 x_1 x_2}{T^4} - \frac{p_1 p_2 x_1 x_2}{T^4} - \frac{(1-T) p_1^3 x_1^2 x_2}{T^4} + \frac{p_1^2 p_2 x_1^2 x_2}{T^4} + \left(-\frac{-1+T}{6 T^4} - \frac{1}{T} \right) p_1^4 x_1^3 x_2 - \right. \\
 & \left. \frac{p_1^3 p_2 x_1^3 x_2}{6 T^4} - \frac{(1-T) p_1^2 x_2^2}{2 T^5} + \frac{p_1 p_2 x_2^2}{2 T^5} - \frac{(-7+9 T-2 T^2) p_1^3 x_1 x_2^2}{2 T^5} - \right. \\
 & \left. \frac{(9-T) p_1^2 p_2 x_1 x_2^2}{2 T^5} + \frac{p_1 p_2^2 x_1 x_2^2}{T^5} + \left(-\frac{3 \times (-1+T)}{2 T^2} - \frac{2-3 T+T^2}{T^5} \right) p_1^4 x_1^2 x_2^2 - \right. \\
 & \left. \frac{(-3+T) p_1^3 p_2 x_1^2 x_2^2}{T^5} - \frac{p_1^2 p_2^2 x_1^2 x_2^2}{T^5} - \frac{(7-9 T+2 T^2) p_1^3 x_2^3}{6 T^6} + \frac{3 p_1^2 p_2 x_2^3}{2 T^6} - \frac{p_1 p_2^2 x_2^3}{3 T^6} + \right. \\
 & \left(-\frac{1-2 T+T^2}{T^3} - \frac{-16+27 T-12 T^2+T^3}{6 T^6} \right) p_1^4 x_1 x_2^2 + \frac{3 \times (-3+T) p_1^3 p_2 x_1 x_2^3}{2 T^6} + \frac{2 p_1^2 p_2^2 x_1 x_2^3}{T^6} - \\
 & \frac{p_1 p_2^3 x_1 x_2^3}{6 T^6} - \frac{(16-27 T+12 T^2-T^3) p_1^4 x_2^4}{24 T^7} + \left(-\frac{-1+2 T-T^2}{T^4} - \frac{-27+5 T-T^2-T^3}{24 T^7} \right) p_1^3 p_2 x_2^4 + \\
 & \left(\frac{3 \times (-1+T)}{2 T^4} - \frac{12-T-5 T^2}{24 T^7} \right) p_1^2 p_2^2 x_2^4 + \left(-\frac{-1-T}{24 T^7} + \frac{1}{T^4} \right) p_1 p_2^3 x_2^4 \Big],
 \end{aligned}$$

$$E_{\{\} \rightarrow \{1\}} \left[\sqrt{T}, 0, \text{Series} \left[0, -\frac{p_1 x_1}{T}, 0, 0 \right] \right],$$

$$E_{\{\} \rightarrow \{1\}} \left[\frac{1}{\sqrt{T}}, 0, \text{Series} \left[0, \frac{p_1 x_1}{T}, -\frac{p_1 x_1}{T^2}, \frac{p_1 x_1}{T^3} \right] \right] \}$$

In[*]:= **RMoves**

Out[*]:= {True, True, True, True, True, True, True, True, True}