pdf

Pensieve header: The Kerler Algebra and the Alexander polynomial.

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In[∘]:=
```
Once[<< KnotTheory`];
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, ■, ■]];
```

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```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

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In[∘]:=

MT =

| □ | a | b | c | d | ka | kb | kc | kd |
|---|---|---|---|---|---|---|---|---|
| a | $a_\$$ | $b_\$$ | 0 | 0 | $ka_\$$ | $kb_\$$ | 0 | 0 |
| b | 0 | 0 | $a_\$$ | $b_\$$ | 0 | 0 | $-ka_\$$ | $-kb_\$$ |
| c | $c_\$$ | $d_\$$ | 0 | 0 | $-kc_\$$ | $-kd_\$$ | 0 | 0 |
| d | 0 | 0 | $c_\$$ | $d_\$$ | 0 | 0 | $kc_\$$ | $kd_\$$ |
| ka | $ka_\$$ | $kb_\$$ | 0 | 0 | $a_\$$ | $b_\$$ | 0 | 0 |
| kb | 0 | 0 | $ka_\$$ | $kb_\$$ | 0 | 0 | $-a_\$$ | $-b_\$$ |
| kc | $kc_\$$ | $kd_\$$ | 0 | 0 | $-c_\$$ | $-d_\$$ | 0 | 0 |
| kd | 0 | 0 | $kc_\$$ | $kd_\$$ | 0 | 0 | $c_\$$ | $d_\$$ |

;

```
ℰ_ // m_{i_,j_→k_} := Expand[ℰ] /.
   Flatten@Table[MT⟦α, 1⟧_i MT⟦1, β⟧_j → (MT⟦α, β⟧ /. $ → k), {α, 2, 9}, {β, 2, 9}];
```

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In[∘]:=
```
KBasis[{i_}] := {a_i, b_i, c_i, d_i, ka_i, kb_i, kc_i, kd_i};
KBasis[{i_, is__}] := Flatten@Outer[Times, KBasis[{i}], KBasis[{is}]]
```

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In[∘]:=
```
η_{i_} := a_i + d_i;
γ_{i_} := ka_i + kd_i;
```

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In[∘]:=
```
lhs = η_1 KBasis[{2}] // m_{1,2→1};
HL[lhs == KBasis[{1}]]
```

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Out[∘]= True

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In[∘]:=
```
lhs = η_1 KBasis[{2}] // Expand // m_{1,2→1};
HL[lhs == KBasis[{1}]]
```

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Out[∘]= True

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In[∘]:=
```
Short[lhs = KBasis[{1, 2, 3}] // m1,2→1 // m1,3→1]
rhs = KBasis[{1, 2, 3}] // m2,3→2 // m1,2→1;
lhs == rhs // HL
```

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Out[∘]//Short= $\{a_1, b_1, 0, 0, ka_1, kb_1, 0, 0, 0, 0, a_1, b_1, 0, 0, -ka_1,$ $\ll 482 \gg, d_1, 0, 0, -kc_1, -kd_1, 0, 0, 0, 0, c_1, d_1, 0, 0, kc_1, kd_1\}$

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Out[∘]= True

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In[∘]:= $R_{i\_,j\_} := a_i\, a_j + d_i\, a_j + T\, a_i\, d_j - (1 - T)\, kc_i\, kb_j - T\, d_i\, d_j$

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In[∘]:=
```
Short[lhs = R1,2 R4,3 R5,6 // m1,4→1 // m2,5→2 // m3,6→3];
rhs = R2,3 R1,4 R5,6 // m1,5→1 // m2,6→2 // m3,4→3;
lhs == rhs // HL
```

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Out[∘]= True

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In[∘]:= $\overline{R}_{i\_,j\_} := a_i\, a_j + d_i\, a_j + T^{-1}\, a_i\, d_j - (1 - T^{-1})\, kc_i\, kb_j - T^{-1}\, d_i\, d_j$

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In[∘]:=
```
Short[lhs = R1,2 R̄3,4 // m1,3→1 // m2,4→2]
rhs = η1 η2 // Expand;
lhs == rhs // HL
```

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Out[∘]//Short= $a_1\, a_2 + a_2\, d_1 + a_1\, d_2 + d_1\, d_2$

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Out[∘]= True

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In[∘]:=
```
Short[lhs = R1,2 R̄3,4 // m1,3→1 // m4,2→2]
rhs = η1 η2 // Expand;
Simplify[lhs - rhs]
```

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Out[∘]//Short= $a_1\, a_2 + a_2\, d_1 + a_1\, d_2 + d_1\, d_2 - 2\, kb_2\, kc_1 + 2\, T\, kb_2\, kc_1$

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Out[∘]= $2 \times (-1 + T)\, kb_2\, kc_1$

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In[∘]:=
```
lhs = R1,4 R̄5,2 γ3 // m2,4→2 // m1,3→1 // m1,5→1
rhs = γ1 η2 // Expand;
lhs == rhs // HL
```

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Out[∘]= $a_2\, ka_1 + d_2\, ka_1 + a_2\, kd_1 + d_2\, kd_1$

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Out[∘]= True

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## RVK and Z

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RVK, rot, Z modified from 2016-09/OneSmidgen.nb. See also in AP/Projects/SL2Invariant/.

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Some details of the code below are at http://drorbn.net/bbs/show?shot=Dror-160920-151350.jpg.

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In[∘]:=
```
RVK::usage =
  "RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings
    xs and a length 2n list of rotation numbers rots. Crossing
    sites are indexed 1 through 2n, and rots⟦k⟧ is the rotation
    between site k-1 and site k. RVK is also a casting operator
    converting to the RVK presentation from other knot presentations.";
```

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In[∘]:=
```
RVK[pd_PD] := Module[{n, xs, x, rots, front = {0}, k},
   n = Length@pd; rots = Table[0, {2 n}];
   xs = Cases[pd, x_X :> { Xp[x⟦4⟧, x⟦1⟧]  PositiveQ@x
                           Xm[x⟦2⟧, x⟦1⟧]  True      ];
   For[k = 0, k < 2 n, ++k, If[k == 0 ∨ FreeQ[front, -k],
     front = Flatten@Replace[front, k → (xs /. {
            Xp[k + 1, l_] | Xm[l_, k + 1] :> {l, k + 1, 1 - l},
            Xp[l_, k + 1] | Xm[k + 1, l_] :> (++rots⟦l⟧; {1 - l, k + 1, l}),
            _Xp | _Xm :> {}
          }), {1}],
     Cases[front, k | -k] /. {k, -k} :> --rots⟦k + 1⟧;
   ]];
   RVK[xs, rots] ];
RVK[K_] := RVK[PD[K]];
```

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In[∘]:=
```
rot_i_[n_] := { η_i  EvenQ[n]
                γ_i  OddQ[n]
```

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In[ ]:=
```
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := Module[{todo, ξ, done, st, c, χ, i, j, k},
  ξ = η₀; done = {0};
  st = Range[0, 2 Length[todo = rvk[[1]]]];
  While[{} =!= todo,
   c = RandomChoice@MaximalBy[todo, Length[done ⋂ {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &];
   {i, j} = List @@ c;
   χ = (c /. {_Xp :> R_{i,j}, _Xm :> R̄_{i,j}}) (ka₀ - kd₀) // m_{j,0→j};
   Do[χ = (rot₀[rvk[[2, k]]] χ) // m_{0,k→k}, {k, {i, j}}];
   ξ *= χ;
   Do[If[MemberQ[done, k], ξ = ξ // m_{st[[k+1]],k+1→st[[k+1]]}; st = st /. st[[k + 2]] → st[[k + 1]]],
    {k, {i, i - 1, j, j - 1}}];
   done = done ⋃ {i - 1, i, j - 1, j};
   todo = DeleteCases[todo, c]
  ];
  Factor@ξ
 ]
```

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In[ ]:=
```
K = Knot[8, 17]; Factor@Alexander[K][T]
Z[K]
```

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KnotTheory: Loading precomputed data in PD4Knots`.

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Out[ ]= $-\frac{1-4T+8T^2-11T^3+8T^4-4T^5+T^6}{T^3}$

*pdf*

Out[ ]= $-\frac{\left(1-4T+8T^2-11T^3+8T^4-4T^5+T^6\right)\,(a_0+d_0)}{T^4}$