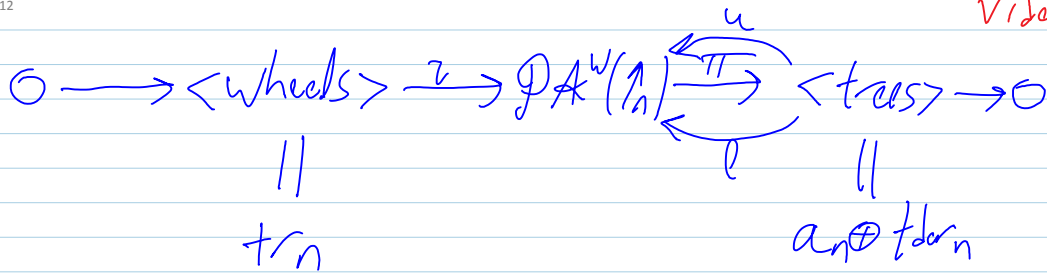


on board:

Next meeting June 20, many videos in between.

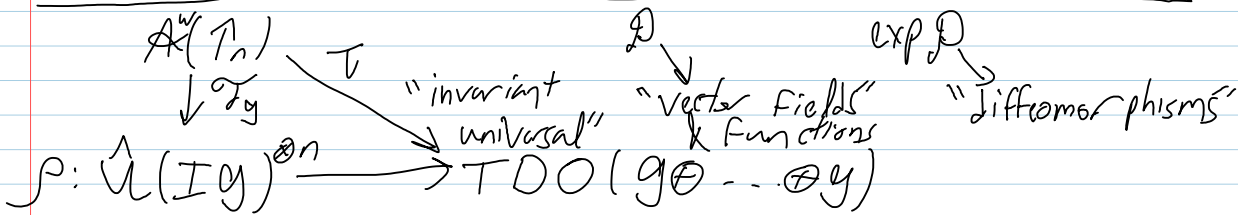


$$\Rightarrow A^W(\mathbb{T}_n) \cong U(\text{tr}_n \rtimes (a_n \oplus \text{td}_n))$$

IF $D \in \langle \text{trees} \rangle$, $\text{div} D := \tau^{-1}(u-l)(D)$

why "div"? why "j"? where

$$j: T\text{Aut}_n = \exp(\text{td}_n) \rightarrow \text{tr}_n \text{ by } j(e^D) = \frac{e^D - 1}{D}(\text{div} D)$$



$$\mathcal{I}g = y^* \rtimes g -$$

* $\psi \in g^* \xrightarrow{\cdot} \hat{\psi}$, multiplication by ψ .

* $x \in g \xrightarrow{\cdot} \text{derivation in the direction of } -\text{ad} x$;

$$\begin{aligned}
 (\rho(x)F)(y) &= df_y([y, x]) = \\
 &= \frac{\partial}{\partial \epsilon} F(y + \epsilon[y, x])|_{\epsilon=0} \\
 &= \frac{\partial}{\partial \epsilon} F(e^{-\epsilon x} y e^{\epsilon x})|_{\epsilon=0} \\
 &=: (\mathcal{I}_x F)(y)
 \end{aligned}$$

[if F is linear, this is $(\rho(x)F)(y) = F([y, x])$]

claim This extends to $U(\mathcal{I}g)$.

Proof 1. $\rho(\psi_1 \psi_2) = \rho(\psi_1) \rho(\psi_2)$ ✓

2. $(\rho(\psi x)F)(y) = \psi(y) F([x, y])$

$$(\rho(x \psi)F)(y) = (\rho(x)(\psi F))(y) = \psi(y) F([x, y]) +$$

$$+F(y) \Psi([x, y])$$

$$(\rho(x \Psi - \Psi x) F)(y) = F(y) \Psi([x, y]) \quad \checkmark$$

$$(\rho([x, \Psi]) F)(y) = [x, \Psi](y) F(y) = -\Psi([x, y]) F(y)$$

3.

ρ maps wheels to functions & trees to v.f. If D is a tree,

claim 1. $\rho(uD)^* = -\rho(lD)$

2. $\rho(\text{div } D) = \text{div}(\rho(uD))$

An Aside on R_3 $\sigma(| \rightarrow |) = \sum \psi_i \otimes x^i$

$$\begin{aligned} (\tau(| \rightarrow |) F)(x, y) &= \sum \psi_i(x) (\delta_{(0, x^i)} F)(x, y) \\ &= (\delta_{(0, x)} F)(x, y) \end{aligned}$$

$$\begin{aligned} (\tau(R_{12}) F)(x, y) &= (\exp(\delta_{(0, x)}) F) = \\ &= F(x, e^{-x} y e^x) =: F(x, y^x) \\ &=: (F \circ \text{IA}_d)(x, y) \end{aligned}$$

$$\begin{aligned} (\tau(R_{12} \cdot R_{13} \cdot R_{23}) F)(x, y, z) &= \\ &= (F \circ \text{IA}_{d_{23}} \circ \text{IA}_{d_{13}} \circ \text{IA}_{d_{12}})(x, y, z) \end{aligned}$$

$$(x, y, z) \xrightarrow{\text{IA}_{d_{12}}} (x, y^x, z) \xrightarrow{\text{IA}_{d_{13}}} (x, y^x, z^x) \xrightarrow{\text{IA}_{d_{23}}}$$

Other side: $(x, y^x, (z^x)^{y^x})$

$$(x, y, z) \xrightarrow{23} (x, y, z^y) \xrightarrow{13} (x, y, (z^y)^x) \xrightarrow{12} (x, y^x, (z^y)^x)$$

This is the Quandle property!

claim $j(e^D) := \frac{e^D - 1}{D} (\operatorname{div} D) = \log(e^{-1D} e^{uD})$

proof-1 Use the Euler trick; need:

$$\exp\left(\frac{e^{tD} - 1}{tD}\right) (\operatorname{div} tD) \stackrel{?}{=} e^{-tD} e^{utD}$$

apply \tilde{E}_t at $t=1$:

$$e^D (\operatorname{div} D) \stackrel{?}{=} e^{-uD} (-1D) e^{uD} + uD$$

- . . . -

proof-2 Interpret $D \rightsquigarrow X = \rho(uD)$, then

$$j(e^D) = \log((e^X)^* e^X) \sim \log(\operatorname{Jac}(e^X))$$

$$\sim \int_0^1 dt e^{tX} \operatorname{div} X \sim \frac{e^X - 1}{X} \operatorname{div} X = \frac{e^D - 1}{D} \operatorname{div} D.$$