Feb 22 - 3.1 through 3.4: w-Knots, the basics, and 3.5: Jacobi diagrams, trees and wheels

February-26-12
4:52 PM

Continue using the 120222 handout!

Remark 3.5. We have a split-exact sequence

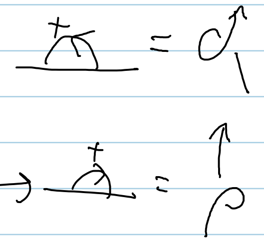
$$0 \to \mathbb{Z}^2 \to \left\{\begin{matrix}\text{long}\\ \text{v-knots}\end{matrix}\right\} \to \left\{\begin{matrix}\text{unframed}\\ \text{long}\\ \text{v-knots}\end{matrix}\right\} \to 1$$

with splitting

$sl = (sl_L, sl_R) : \left\{\begin{matrix}\text{long}\\ \text{v-knots}\end{matrix}\right\} \to \mathbb{Z}^2$ by signed counting of left/right arrows.

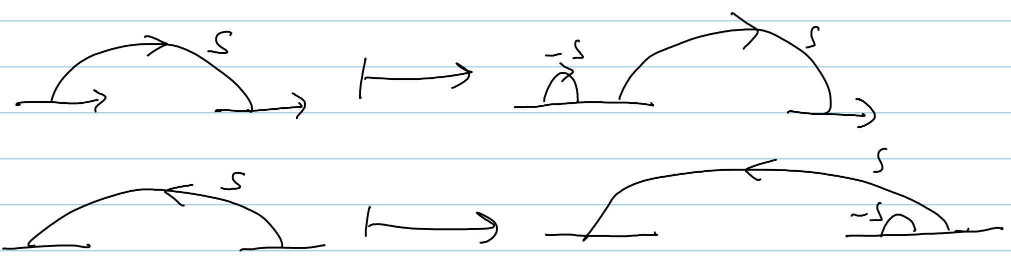
$\iota : \binom{1}{0} \mapsto$

$\binom{0}{1} \mapsto$

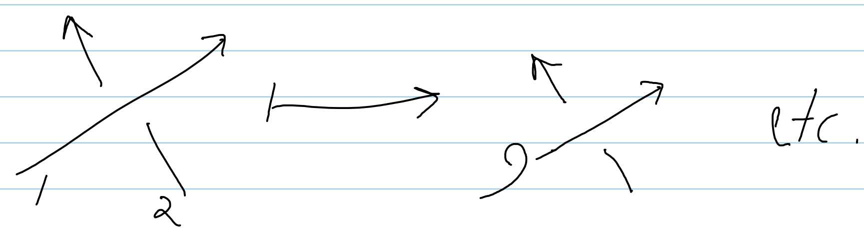


$$\begin{tikzcd} 0 \arrow[r] & \underset{A}{\mathbb{Z}^2} \arrow[r, shift left, "\iota"] & \underset{B}{\left\{\begin{matrix}\text{long}\\ \text{v-knots}\end{matrix}\right\}} \arrow[l, shift left, "sl"] \arrow[r, shift left, "\pi"] & \underset{C}{\left\{\begin{matrix}\text{unframed}\\ \text{long}\\ \text{v-knots}\end{matrix}\right\}} \arrow[l, shift left, "h"] \arrow[r] & 1 \end{tikzcd}$$

h:



That is,



etc.

(Alternatively, add the kinks at the start, counted by sl).

<u>Prop</u> Everything is well defined, and sl & h make homotopies that prove the sequence exact:

$$\pi \circ h = I_C \qquad sl \circ \iota = I_A \qquad h \circ \pi \# \iota \circ sl = I_B$$

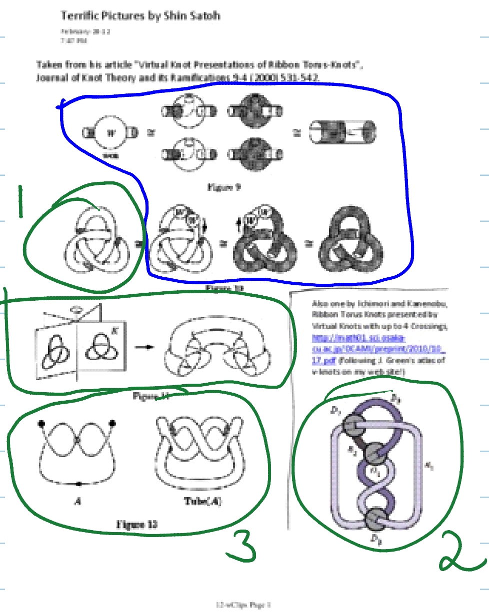
Order of proceedings:

1. sl is well-defined and additive.
2. ι is well-defined and additive

2. v is well-defined and invariant. $\mathcal{P}$ R1's /

3. h is well-defined.
4. Said identities hold.

Remark 3.7



Do green, barely mention blue.

3.2 – Finite type invariants.

Proposition 3.9. $\mathcal{A}^v$ & $\mathcal{A}^w$ are graded co-commutative bi-algebras & the Milnor–Moore theorem applies.

m		0	1	2	3	4	5	6	7
$\dim \mathcal{G}_m \mathcal{A}^-(\uparrow)$	$u \mid v$	1 | 1	1 | 2	2 | 7	3 | 27	6 | 139	10 |?	19 |?	33 |?
	w	1	2	4	7	12	19	30	45
$\dim \mathcal{G}_m \mathcal{L}ie^-(\uparrow)$	$u \mid v$	1 | 1	1 | 2	2 | 7	3 | 27	6 | ≥ 128	10 |?	19 |?	33 |?
	w	1	2	4	7	12	19	30	45
$\dim \mathcal{G}_m \mathcal{A}^{r-}(\uparrow)$	$u \mid v$	1 | 1	0 | 0	1 | 2	1 | 7	3 | 42	4 |?	9 |?	14 |?
	w	1	0	1	1	2	2	4	4
$\dim \mathcal{G}_m \mathcal{P}^-(\uparrow)$	$u \mid v$	0 | 0	1 | 2	1 | 4	1 | 15	2 | 82	3 |?	5 |?	8 |?
	w	0	2	1	1	1	1	1	1
$\dim \mathcal{G}_m \mathcal{A}^-(\bigcirc)$	$u \mid v$	1 | 1	1 | 1	2 | 2	3 | 5	6 | 19	10 | 77	19 |?	33 |?
	w	1	1	1	1	1	1	1	1
$\dim \mathcal{G}_m \mathcal{A}^{r-}(\bigcirc)$	$u \mid v$	1 | 1	0 | 0	1 | 0	1 | 1	3 | 4	4 | 17	9 |?	14 |?
	w	1	0	0	0	0	0	0	0

All u-numbers are long-known to deg ~12, by very hard computations.

v-numbers by me & [BHLR]; hard computations

w-numbers are known precisely to all degrees, as we shall see later.

Then do .

There is a w-expansion and it is easy?!

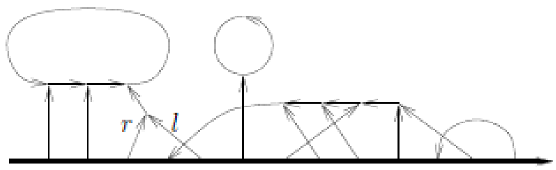


Figure 10. A w-Jacobi diagram on a long line skeleton of degree 11. It has a skeleton line at the bottom, 13 vertices along the skeleton (of which 2 are incoming and 11 are outgoing), 9 internal vertices (with only one explicitly marked with "left" (l) and "right" (r)) and one bubble. The four quadrivalent vertices that seem to appear in the diagram are just projection artifacts and graph-theoretically, they don't exist.

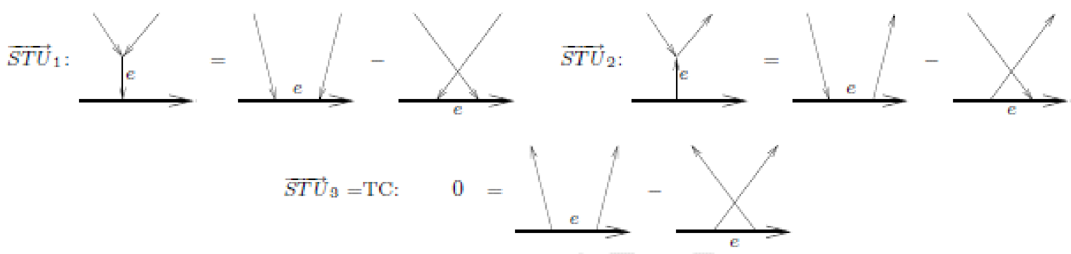


Figure 11. The $\overrightarrow{STU}_{1,2}$ and TC relations with their "central edges" marked e.

Figure 12. The $\overrightarrow{AS}$ and $\overrightarrow{IHX}$ relations.

Theorem 3.15 (bracket-rise). *The obvious inclusion $\iota : \mathcal{D}^v(\uparrow) \to \mathcal{D}^{wt}(\uparrow)$ of arrow diagrams (Definition 3.8) into w-Jacobi diagrams descends to the quotient $\mathcal{A}^w(\uparrow)$ and induces an isomorphism $\bar\iota : \mathcal{A}^w(\uparrow) \xrightarrow{\sim} \mathcal{A}^{wt}(\uparrow)$. Furthermore, the $\overrightarrow{AS}$ and $\overrightarrow{IHX}$ relations of Figure 12 hold in $\mathcal{A}^{wt}(\uparrow)$.*